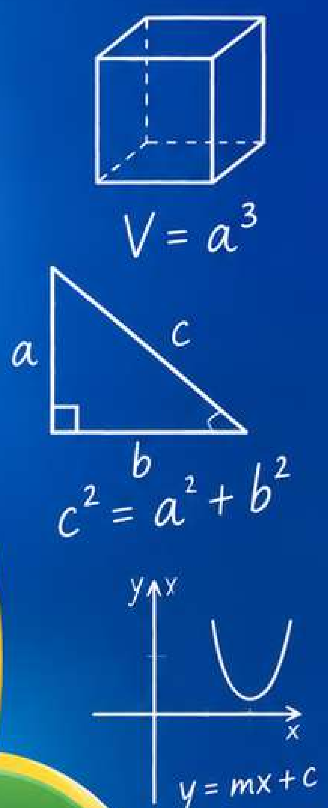
Super Power Series Book



GANITA PRAKASH FOR YOUNG INTELLECTS

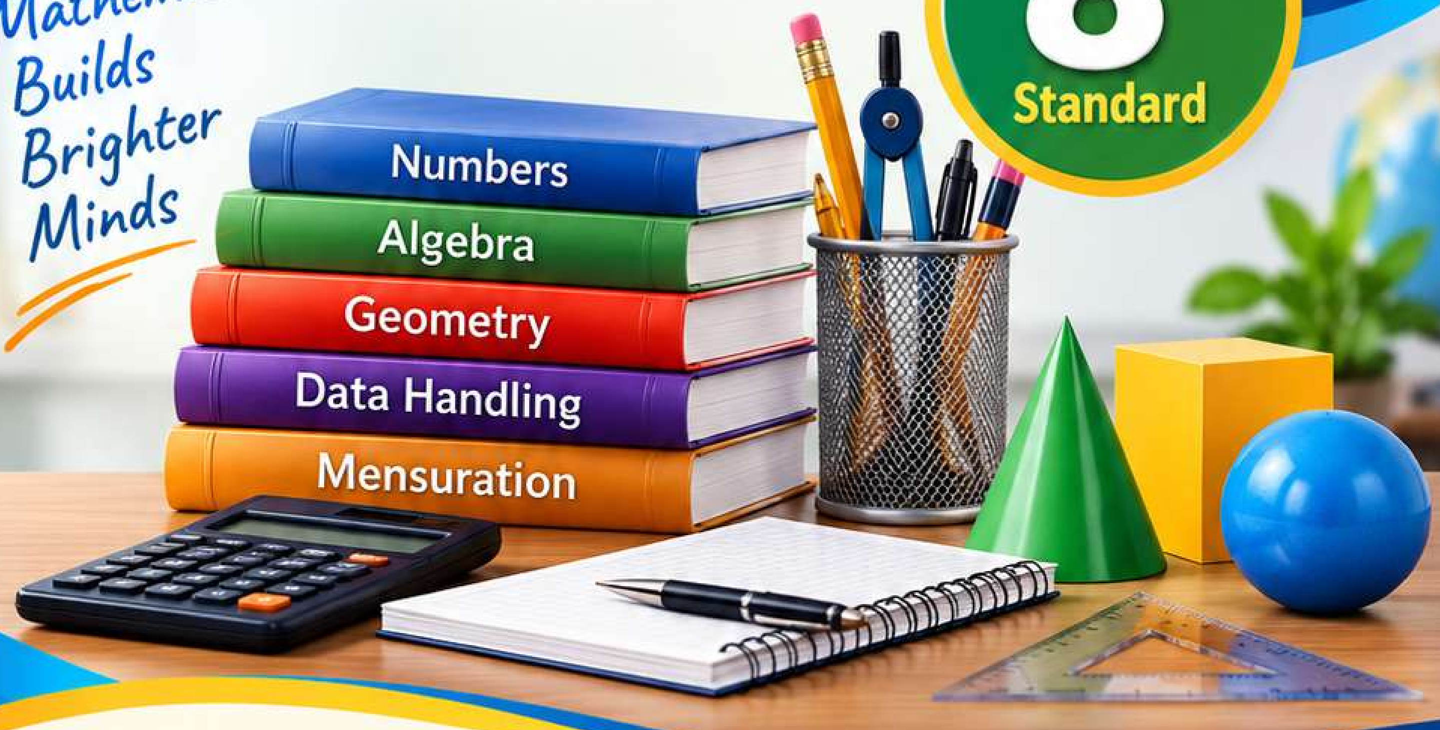
Study Material BOOK for 8th Standard Mathematics Subject

(Part One)

Based on NCERT/CBSE Curriculum

8th Standard

Mathematics Builds Brighter Minds



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Published BY:
Poornaprajna Publication, Udupi, India
Year of Publication: 2026
www.poornaprajnapublication.com

Knowledge Today for a Better Tomorrow

Super Power Series Book

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ISBN: ISBN: 978-81-17181-04-1

DOI: https://doi.org/10.5281/zenodo.22719537

www.poornaprajnapublication.com

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(Prepared as per Poornaprajna Integrated Student Support Model)

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PREFACE

Ganita Prakash for Young Intellects – Study Material Book for 8th Standard Mathematics Subject (Part One) has been developed with the help of AI-driven GPTs, on the **NCERT/CBSE Curriculum** with the objective of making Mathematics meaningful, systematic, and engaging for young learners. The book covers important mathematical concepts through carefully organised chapters and learning activities, helping students build a strong foundation in **conceptual understanding, logical reasoning, computational skills, and problem-solving**.

The book has been prepared in accordance with the **Poornaprajna Integrated Student Support Model**, adopting a comprehensive approach that moves from **chapter summaries and important formulas to LOTS and HOTS questions of different levels**. The learning material includes fill-in-the-blanks, descriptive questions, reasoning and application-oriented exercises, enabling students to progress from basic understanding to higher-order mathematical thinking.

This study material is designed to support **8th Grade students in both CBSE school examinations and competitive examinations such as Olympiads**. By combining regular academic preparation with **analytical, application-based, HOTS, and competitive-examination-oriented practice**, the book aims to develop accuracy, confidence, mathematical curiosity, and independent thinking. We hope *Ganita Prakash for Young Intellects* inspires students not merely to **learn Mathematics, but to explore patterns, reason logically, solve creatively, and experience the joy of Mathematics**.

Dr. P. S. Aithal
(www.psaithal.in)

8th Standard Maths 1 Book Chapter 1

Chapter 1: A SQUARE AND A CUBE

1.1 Summary of the Chapter:

Chapter 1: A Square and a Cube — Summary:

(1) A **square number** is obtained by multiplying a number by itself: $n \times n = n^2$. The squares of natural numbers are called **perfect squares**, such as 1, 4, 9, 16, 25, 36, etc.

(2) Perfect squares have an interesting connection with **factors**: only square numbers have an **odd number of factors**, because one factor pairs with itself.

(3) Perfect squares can end only in **0, 1, 4, 5, 6, or 9**. Therefore, a number ending in **2, 3, 7, or 8 cannot be a perfect square**.

(4) The difference between two consecutive perfect squares follows the sequence of **odd numbers**. Also, the sum of the first $n$consecutive odd numbers is n^2; for example, $1 + 3 + 5 + 7 + 9 = 25 = 5^2$.

(5) A **square root** is the inverse of squaring. If $y = x^2$, then $x$is a square root of y. Every perfect square has a positive and a negative integer square root, though the chapter mainly considers the **positive square root**.

(6) **Prime factorisation** can be used to identify perfect squares and find their square roots. A number is a perfect square when its prime factors can be arranged into **pairs or two identical groups**.

(7) Square roots of numbers that are not perfect squares can be **estimated using nearby perfect squares**. For example, since $15^2 = 225$and $16^2 = 256$, $\sqrt{250}$lies between 15 and 16 and is approximately 16.

(8) A **cube number** is obtained by multiplying a number by itself three times: $n \times n \times n = n^3$. Numbers such as **1, 8, 27, 64 and 125** are perfect cubes.

(9) The chapter connects cubes with geometry: a cube of edge 4 units contains $4 \times 4 \times 4 = 64$**unit cubes**, illustrating why $4^3 = 64$.

(10) Perfect cubes also show patterns involving **consecutive odd numbers**. For example, $3 + 5 = 8 = 2^3$, $7 + 9 + 11 = 27 = 3^3$, and $13 + 15 + 17 + 19 = 64 = 4^3$.

(11) The **cube root** is the inverse operation of cubing. Prime factorisation helps identify perfect cubes: the prime factors of a perfect cube can be grouped into **triplets or three identical groups**.

(12) The chapter also introduces the famous **Hardy–Ramanujan number 1729**, the smallest number expressible as the sum of two positive cubes in two different ways: $1729 = 1^3 + 12^3 = 9^3 + 10^3$. Such numbers are called **taxicab numbers**.

1.2 Important Formulas:

Important Formulas – Chapter 1: A Square and a Cube:

A. Square Numbers and Perfect Squares

1. **Square of a number**

$$\boxed{n^2 = n \times n}$$

Example: $15^2 = 15 \times 15 = 225$.

2. **Square of a sum**

$$\boxed{(a + b)^2 = a^2 + 2ab + b^2}$$

Useful for quickly calculating squares such as $45^2 = (40 + 5)^2$. The chapter demonstrates this expansion while estimating square roots.

3. **Difference between consecutive squares**

$$\boxed{(n+1)^2 - n^2 = 2n+1}$$

Thus, consecutive perfect squares differ by consecutive **odd numbers**.

4. **Sum of first $n$odd natural numbers**

$$\boxed{1+3+5+\cdots+(2n-1) = n^2}$$

Example:

$$1+3+5+7+9 = 25 = 5^2$$

5. n^{th}**odd number**

$$\boxed{2n-1}$$

Example: 36th odd number $= 2(36) - 1 = 71$.

6. **Number of natural numbers between consecutive squares**

$$\boxed{(n+1)^2 - n^2 - 1 = 2n}$$

For example, between $5^2 = 25$and $6^2 = 36$, there are 10natural numbers.

B. Square Roots

7. **Square and square-root relationship**

$$\boxed{x^2 = y \Rightarrow x = \sqrt{y}}$$

For the positive square root used in the chapter:

$$\boxed{\sqrt{x^2} = x}$$

8. **Finding square roots by prime factorisation**

If

$$N = (p \times q \times r \cdots)^2,$$

then

$$\boxed{\sqrt{N} = p \times q \times r \cdots}$$

Example:

$$324 = 2^2 \times 3^2 \times 3^2$$

Therefore,

$$\sqrt{324} = 2 \times 3 \times 3 = 18$$

9. **Area and side of a square**

$$\boxed{A = s^2}$$

Therefore,

$$\boxed{s = \sqrt{A}}$$

where $A$is area and $s$is side length.

C. Cube Numbers

10. **Cube of a number**

$$\boxed{n^3 = n \times n \times n}$$

Example:

$$5^3 = 5 \times 5 \times 5 = 125$$

11. **Volume interpretation of a cube**

$$\boxed{V = a^3}$$

where $a$is the edge length of a cube. For example, a cube of edge 4 units contains $4^3 = 64$unit cubes.

D. Cube Roots

12. **Cube and cube-root relationship**

$$\boxed{x^3 = y \Rightarrow x = \sqrt[3]{y}}$$

Hence,

$$\boxed{\sqrt[3]{x^3} = x}$$

Examples:

$$\sqrt[3]{8} = 2,\ \sqrt[3]{27} = 3,\ \sqrt[3]{1000} = 10$$

13. **Cube root using prime factorisation**

For a perfect cube, prime factors occur in **groups of three**:

$$\boxed{\sqrt[3]{p^3q^3r^3} = pqr}$$

Example:

$$3375 = 3^3 \times 5^3$$

Therefore,

$$\boxed{\sqrt[3]{3375} = 3 \times 5 = 15}$$

E. Important Patterns for Problem Solving

14. **Perfect square through odd-number subtraction**

A natural number $N$is a perfect square if successive subtraction of

$$\boxed{1,3,5,7,9,\ldots}$$

eventually gives **exactly zero**. The number of subtractions gives its positive square root.

15. **Perfect-square prime-factor rule**

$$\boxed{\text{Perfect square} \Rightarrow \text{prime factors can be grouped in pairs}}$$

16. **Perfect-cube prime-factor rule**

$$\boxed{\text{Perfect cube} \Rightarrow \text{prime factors can be grouped in triplets}}$$

These **16 formulas and rules** cover the major quantitative calculations in Chapter 1, including **perfect squares, square roots, consecutive squares, odd-number patterns, perfect cubes, cube roots, prime factorisation, and geometrical applications**.

1.3 Fill-Up the Blank LOTS type questions with Answers: (35 Questions)

Fill in the Blanks – LOTS Type:

1. A number obtained by multiplying a number by itself is called a __________ **number**.
   Answer: Square
2. The squares of natural numbers are called __________ **squares**.
   Answer: Perfect
3. 8^2 =__________.
   Answer: 64
4. 15^2 =__________.
   Answer: 225
5. The only numbers that have an odd number of factors are __________ **numbers**.
   Answer: Square
6. A perfect square can end with the digit 0, 1, 4, 5, 6 or __________.
   Answer: 9
7. A number ending in 2, 3, 7 or 8 cannot be a __________ **square**.
   Answer: Perfect
8. The difference between two consecutive perfect squares is always an __________ **number**.
   Answer: Odd
9. The sum of the first $n$odd natural numbers is __________.
   Answer: n^2
10. The n^{th}odd natural number is __________.
    Answer: $2n - 1$

11. $1 + 3 + 5 + 7 =$__________.
    Answer: 16
12. The positive square root of 49 is __________.
    Answer: 7
13. The positive square root of 100 is __________.
    Answer: 10
14. To identify a perfect square using prime factorisation, its prime factors should be grouped into __________.
    Answer: Pairs
15. $324 = 18^2$; therefore, the positive square root of 324 is __________.
    Answer: 18
16. A number obtained by multiplying a number by itself __________ **times** is called a cube.
    Answer: Three
17. $2^3 =$__________.
    Answer: 8
18. $3^3 =$__________.
    Answer: 27
19. $4^3 =$__________.
    Answer: 64
20. $5^3 =$__________.
    Answer: 125
21. The cube root of 8 is __________.
    Answer: 2
22. The cube root of 27 is __________.
    Answer: 3
23. The cube root of 1000 is __________.
    Answer: 10
24. In the prime factorisation of a perfect cube, the factors can be grouped into __________.
    Answer: Triplets
25. $3375 = 15^3$; therefore, $\sqrt[3]{3375} =$__________.
    Answer: 15
26. 500is __________ a perfect cube.
    Answer: Not
27. The number __________ is known as the Hardy–Ramanujan Number.
    Answer: 1729
28. The Hardy–Ramanujan Number can be written as $1^3 +$__________ ^{3}.
    Answer: 12
29. Numbers that can be expressed as the sum of two cubes in two different ways are called __________ **numbers**.
    Answer: Taxicab
30. The Sanskrit term __________ was used for both the square figure/area and the square power.
    Answer: Varga
31. The Sanskrit term __________ was used for a cube and for the product of a number with itself three times.
    Answer: Ghana

32. The Sanskrit word __________, meaning “root,” was used for mathematical operations involving roots.
Answer: Mula
33. The sum $3 + 5$is equal to __________, which is 2^3.
Answer: 8
34. The sum $7 + 9 + 11$is equal to __________, which is 3^3.
Answer: 27
35. Each prime factor of a number appears __________ **times** in the prime factorisation of its cube.
Answer: Three

These questions focus mainly on **recall, recognition, definitions, basic calculations, and direct application**, making them suitable as **LOTS-type fill-in-the-blank questions** for Grade 8.

1.4 Fill-Up the Blank HOTS type questions with Answers: (30 Questions)

Fill in the Blanks – HOTS Type:

1. A perfect square has an odd number of factors because one of its factor pairs contains __________ factors.
Answer: Equal
2. Locker number 81 in the chapter’s opening puzzle remains open because 81 has an __________ number of factors.
Answer: Odd
3. Since a number ending in 7 cannot be a perfect square, 2027 is __________ a perfect square.
Answer: Not
4. A number ending in 6 is __________ guaranteed to be a perfect square, because numbers such as 26 also end in 6.
Answer: Not
5. If a natural number ends with three zeros, its square will end with __________ zeros.
Answer: Six
6. Since $35^2 = 1225$, we can find $36^2$by adding the 36th odd number, __________, to 1225.
Answer: 71
7. If successive subtraction of $1,3,5,7,...$from a number reaches exactly zero, the original number is a __________.
Answer: Perfect square
8. If 49 is expressed as the sum of consecutive odd numbers beginning with 1, the number of odd numbers required is __________.
Answer: 7
9. There are __________ natural numbers between $12^2$and 13^2.
Answer: 24
10. Since $20^2 = 400$and $21^2 = 441$, the number 420 is __________ a perfect square.
Answer: Not
11. If all the prime factors of a number can be arranged into pairs, the number must be a __________.
Answer: Perfect square
12. The prime factorisation of 156is $2 \times 2 \times 3 \times 13$. Since 3 and 13 cannot be paired, 156 is __________ a perfect square.
Answer: Not

13. Since $15^2 = 225$and $16^2 = 256$, $\sqrt{250}$lies between ___________ and 16.
Answer: 15
14. A square cloth has an area of 125 cm². The largest square with an integer side length that can be cut from it has side ___________ **cm**.
Answer: 11
15. If the prime factorisation of a number is $2^4 \times 3^2$, its positive square root is ___________.
Answer: 12
16. If each prime factor in the prime factorisation of a number occurs in multiples of three, the number is a ___________.
Answer: Perfect cube
17. Since $3375 = 3^3 \times 5^3$, its cube root is ___________.
Answer: 15
18. The number 500 is not a perfect cube because its prime factors cannot be arranged into ___________ **identical groups**.
Answer: Three
19. If $n^3 = 729$, then the value of $n$is ___________.
Answer: 9
20. Since $4^3 = 64$, a cube of edge 4 units can be formed using ___________ unit cubes.
Answer: 64
21. In the pattern $3 + 5 = 2^3$, $7 + 9 + 11 = 3^3$, the next group representing 4^3contains ___________ consecutive odd numbers.
Answer: 4
22. The sum $13 + 15 + 17 + 19$can be written as the cube of ___________.
Answer: 4
23. Without adding individually, $21 + 23 + 25 + 27 + 29$can be recognised as ___________.
Answer: 5^3or 125
24. Since $1729 = 1^3 + 12^3 = 9^3 + 10^3$, 1729 is classified as a ___________ **number**.
Answer: Taxicab
25. If $a^3 = 1728$, then $a =$___________.
Answer: 12
26. A perfect cube has prime factorisation $2^6 \times 3^3$. Its cube root is ___________.
Answer: 12
27. If $40^2 < N < 50^2$, then the positive square root of Nlies between ___________ **and** ___________.
Answer: 40 and 50
28. Since 1936 lies between $40^2$and $45^2$and its square root must end in 4 or 6, $\sqrt{1936} =$___________.
Answer: 44
29. If the square of a number ends with 1, the original number may have ___________ **or** ___________ in its units place.
Answer: 1 or 9
30. If a number is odd, its square will also be ___________.
Answer: Odd
31. If a number is even, its cube will also be ___________.
Answer: Even

32. If a perfect square ends with zeros, the number of trailing zeros in the square must be ___________.
    Answer: Even
33. If $10^2 = 100$and $11^2 = 121$, then there are ___________ natural numbers strictly between these two perfect squares.
    Answer: 20
34. If a number has prime factorisation $2^3 \times 3^3 \times 5^3$, its cube root is ___________.
    Answer: 30
35. In the locker puzzle, lockers numbered $1,4,9,16,\ldots,100$ remain open because their numbers are ___________.
    Answer: Perfect squares

These HOTS fill-ups require students to **analyse patterns, apply properties, use prime factorisation, estimate roots, and draw conclusions** rather than merely recall definitions.

1.5 One-mark questions with Answers of LOTS Type: (35 Questions)

One-Mark Descriptive Questions with Answers – LOTS Type:

1. **What is a square number?**
   Answer: A number obtained by multiplying a number by itself is called a square number.
2. **What is a perfect square?**
   Answer: The square of a natural number is called a perfect square.
3. **Write the first five perfect squares.**
   Answer: The first five perfect squares are 1, 4, 9, 16, and 25.
4. **How is the square of a number $n$represented?**
   Answer: The square of $n$is represented as $n^2 = n \times n$.
5. **Which digits can occur in the units place of a perfect square?**
   Answer: A perfect square can have 0, 1, 4, 5, 6, or 9 in its units place.
6. **Which digits cannot occur in the units place of a perfect square?**
   Answer: A perfect square cannot have 2, 3, 7, or 8 in its units place.
7. **What is the relationship between perfect squares and their number of factors?**
   Answer: Every perfect square has an odd number of factors.
8. **What is the sum of the first $n$odd natural numbers?**
   Answer: The sum of the first $n$odd natural numbers is n^2.
9. **What is the n^{th}odd natural number?**
   Answer: The n^{th}odd natural number is $2n - 1$.
10. **What is the relationship between consecutive perfect squares?**
    Answer: The differences between consecutive perfect squares are successive odd numbers.
11. **What is a square root?**
    Answer: If $y = x^2$, then $x$is called a square root of y.
12. **What is the positive square root of 64?**
    Answer: The positive square root of 64 is 8.
13. **What is the positive square root of 100?**
    Answer: The positive square root of 100 is 10.
14. **How can prime factorisation be used to identify a perfect square?**
    Answer: A number is a perfect square if its prime factors can be divided into two identical groups.
15. **Is 324 a perfect square?**
    Answer: Yes, 324 is a perfect square because $324 = 18^2$.

16. **What is a cube number?**
Answer: A number obtained by multiplying a number by itself three times is called a cube number.
17. **Write the first five perfect cubes.**
Answer: The first five perfect cubes are 1, 8, 27, 64, and 125.
18. **How is the cube of a number $n$represented?**
Answer: The cube of $n$is represented as $n^3 = n \times n \times n$.
19. **What is the cube of 5?**
Answer: The cube of 5 is $5^3 = 125$.
20. **What is a cube root?**
Answer: If $y = x^3$, then $x$is called the cube root of y.
21. **What is the cube root of 27?**
Answer: The cube root of 27 is 3.
22. **What is the cube root of 1000?**
Answer: The cube root of 1000 is 10.
23. **How can prime factorisation be used to identify a perfect cube?**
Answer: A number is a perfect cube if its prime factors can be divided into three identical groups.
24. **Is 3375 a perfect cube?**
Answer: Yes, 3375 is a perfect cube because $3375 = 15^3$.
25. **Is 500 a perfect cube?**
Answer: No, 500 is not a perfect cube because its prime factors cannot be divided into three identical groups.
26. **What is the Hardy–Ramanujan Number?**
Answer: 1729 is known as the Hardy–Ramanujan Number.
27. **Why is 1729 a special number?**
Answer: 1729 is the smallest number that can be expressed as the sum of two positive cubes in two different ways.
28. **Write the two cube-sum representations of 1729.**
Answer: $1729 = 1^3 + 12^3 = 9^3 + 10^3$.
29. **What are taxicab numbers?**
Answer: Numbers that can be expressed as the sum of two positive cubes in two different ways are called taxicab numbers.
30. **What Sanskrit term was used for a square?**
Answer: The Sanskrit term **varga** was used for a square figure, its area, and square power.
31. **What Sanskrit term was used for a cube?**
Answer: The Sanskrit term **ghana** was used for a solid cube as well as the cube of a number.
32. **What Sanskrit word was used for the mathematical concept of root?**
Answer: The Sanskrit word **mula** was used for the mathematical concept of root.
33. **What is meant by varga-mula?**
Answer: Varga-mula is the Sanskrit term used for square root.
34. **What is meant by ghana-mula?**
Answer: Ghana-mula is the Sanskrit term used for cube root.
35. **What happens to each prime factor when a number is cubed?**
Answer: Each prime factor of a number appears three times in the prime factorisation of its cube.

1.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – HOTS Type:

1. **Why does a perfect square have an odd number of factors?**
   Answer: A perfect square has an odd number of factors because one factor pairs with itself while all other factors occur in distinct pairs.
2. **Can a number ending in 3 be a perfect square? Give a reason.**
   Answer: No, because a perfect square can end only in 0, 1, 4, 5, 6, or 9.
3. **Can you conclude that 256 is a perfect square merely because it ends in 6? Why?**
   Answer: No, because ending in 6 is a necessary possibility but does not by itself prove that a number is a perfect square.
4. **If a number ends with four zeros, how many zeros will its square have at the end?**
   Answer: Its square will end with eight zeros because squaring doubles the number of trailing zeros.
5. **If $24^2 = 576$, how can you find 25^2 without direct multiplication?**
   Answer: Add the next odd number 49 to 576 to obtain $25^2 = 625$.
6. **Why is $1 + 3 + 5 + 7 + 9 + 11 = 36$ a perfect square?**
   Answer: It is a perfect square because the sum of the first six odd numbers equals $6^2 = 36$.
7. **How can successive subtraction of odd numbers determine whether 49 is a perfect square?**
   Answer: Successively subtracting 1, 3, 5, 7, 9, 11, and 13 from 49 gives exactly zero, showing that 49 is a perfect square.
8. **If successive subtraction of odd numbers from a number crosses zero instead of reaching exactly zero, what can you conclude?**
   Answer: The number is not a perfect square.
9. **How many natural numbers lie between 20^2 and 21^2?**
   Answer: There are 40 natural numbers between 20^2 and 21^2.
10. **If $30^2 = 900$, what is 31^2 using the consecutive-square pattern?**
    Answer: $31^2 = 900 + 61 = 961$, because the 31st odd number is 61.
11. **Why is 156 not a perfect square according to its prime factorisation?**
    Answer: Since $156 = 2 \times 2 \times 3 \times 13$, all its prime factors cannot be arranged into pairs.
12. **Without finding its exact square root, between which two integers does $\sqrt{250}$ lie?**
    Answer: $\sqrt{250}$ lies between 15 and 16 because $15^2 = 225 < 250 < 256 = 16^2$.
13. **Why can 327 immediately be ruled out as a perfect square?**
    Answer: 327 cannot be a perfect square because its units digit is 7.
14. **A square has an area of 125 cm²; what is the greatest possible integer length of its side that does not exceed this area?**
    Answer: The greatest possible integer side length is 11 cm because $11^2 = 121 < 125 < 144 = 12^2$.
15. **If the prime factorisation of a number is $2^4 \times 3^2$, what is its square root?**
    Answer: Its square root is $2^2 \times 3 = 12$.
16. **Why is $2^4 \times 3^3$ not a perfect square?**
    Answer: It is not a perfect square because the factor 3 occurs an odd number of times and therefore cannot be completely paired.
17. **If a number is odd, what can you conclude about its square?**
    Answer: The square of an odd number is also odd.
18. **If a number is even, what can you conclude about its square?**
    Answer: The square of an even number is also even.

19. **Why is 500 not a perfect cube?**
Answer: Since $500 = 2 \times 2 \times 5 \times 5 \times 5$, its prime factors cannot be divided into three identical groups.
20. **If $3 + 5 = 2^3$ and $7 + 9 + 11 = 3^3$, what does $13 + 15 + 17 + 19$ represent?**
Answer: $13 + 15 + 17 + 19$ represents $4^3 = 64$.
21. **How can you determine from prime factorisation that 3375 is a perfect cube?**
Answer: Since $3375 = 3^3 \times 5^3$, all its prime factors can be grouped into triplets.
22. **If $N = 2^3 \times 3^3$, what is $\sqrt[3]{N}$?**
Answer: $\sqrt[3]{N} = 2 \times 3 = 6$.
23. **If $N = 2^6 \times 5^3$, what is its cube root?**
Answer: Its cube root is $2^2 \times 5 = 20$.
24. **A cube consists of 729 unit cubes; what is the length of each edge?**
Answer: Each edge is 9 units long because $9^3 = 729$.
25. **Why is 1729 called a taxicab number?**
Answer: It is a taxicab number because $1729 = 1^3 + 12^3 = 9^3 + 10^3$, expressing it as the sum of two positive cubes in two different ways.
26. **If $5^3 = 125$, how many consecutive odd numbers are added in the chapter's cube pattern to obtain 125?**
Answer: Five consecutive odd numbers, $21 + 23 + 25 + 27 + 29$, are added to obtain 125.
27. **Why can prime factors of a perfect cube be grouped into triplets?**
Answer: Each prime factor of a number appears three times in the prime factorisation of its cube.
28. **If $10^3 = 1000$, what can you infer about $\sqrt[3]{1000}$?**
Answer: $\sqrt[3]{1000} = 10$ because cube root is the inverse operation of cubing.
29. **If $40^2 < 1936 < 50^2$ and the square root must end in 4 or 6, which possible roots should be tested?**
Answer: The possible roots to test are 44 and 46.
30. **Why do only square-numbered lockers remain open in the chapter's locker puzzle?**
Answer: Square-numbered lockers remain open because only perfect squares have an odd number of factors and hence are toggled an odd number of times.

1.7 Two-mark questions with Answers of LOTS Type: (28 Questions)

Two-Mark Descriptive Questions with Answers – LOTS Type:

1. What is a square number? Give an example.

Answer:
1. A number obtained by multiplying a number by itself is called a **square number**.
2. For example, $6^2 = 6 \times 6 = 36$, so 36 is a square number.

2. What are perfect squares? Write the first five perfect squares.

Answer:
1. The squares of natural numbers are called **perfect squares**.
2. The first five perfect squares are **1, 4, 9, 16, and 25**.

3. State two properties of the units digit of perfect squares.

Answer:
1. Perfect squares can have only **0, 1, 4, 5, 6, or 9** in the units place.
2. Therefore, numbers ending in **2, 3, 7, or 8** cannot be perfect squares.

4. Explain the relationship between perfect squares and factors.
Answer:
1. Factors of a number generally occur in pairs whose product gives the number.
2. A perfect square has an **odd number of factors** because one factor pairs with itself.

5. What pattern is observed in the differences between consecutive perfect squares?
Answer:
1. The differences between consecutive perfect squares are consecutive **odd numbers**.
2. For example, $4-1=3,\ 9-4=5,\ 16-9=7,\ 25-16=9$.

6. Explain the relationship between perfect squares and consecutive odd numbers.
Answer:
1. Every perfect square can be expressed as the sum of successive odd numbers starting from 1.
2. For example, $25=1+3+5+7+9=5^2$.

7. What is a square root? Give an example.
Answer:
1. If $y=x^2$, then $x$is called a **square root** of y.
2. For example, $7^2=49$, so the positive square root of 49 is 7.

8. What are the two integer square roots of 64? Explain.
Answer:
1. The two integer square roots of 64 are **+8 and −8**.
2. This is because $8^2=64$and $(-8)^2=64$.

9. How can successive subtraction of odd numbers be used to identify a perfect square?
Answer:
1. Successively subtract 1,3,5,7, ...from the given natural number.
2. If the subtraction reaches exactly **zero**, the given number is a perfect square.

10. How can prime factorisation be used to identify a perfect square?
Answer:
1. Find the prime factorisation of the given number and arrange the prime factors into **pairs**.
2. If all the factors can be paired, the number is a perfect square.

11. Show that 324 is a perfect square using prime factorisation.
Answer:
1. $324=2\times2\times3\times3\times3\times3=(2\times3\times3)^2$.
2. Therefore, $324=18^2$, and hence $\sqrt{324}=18$.

12. What is a cube number? Give an example.
Answer:
1. A number obtained by multiplying a number by itself three times is called a **cube number**.
2. For example, $4^3=4\times4\times4=64$, so 64 is a perfect cube.

13. Write the first five perfect cubes and their cube forms.
Answer:
1. The first five perfect cubes are **1, 8, 27, 64, and 125**.
2. They can be written as $1^3, 2^3, 3^3, 4^3$,and 5^3, respectively.

14. Explain the geometrical meaning of $4^3=64$.
Answer:
1. Each layer of a cube of edge 4 units contains $4\times4=16$unit cubes.
2. Since there are four such layers, the cube contains $4\times4\times4=64$unit cubes.

15. What is a cube root? Give an example.
Answer:

1. If $y = x^3$, then $x$is called the **cube root** of y.
2. For example, $2^3 = 8$, so $\sqrt[3]{8} = 2$.

16. How can prime factorisation be used to identify a perfect cube?

Answer:

1. Find the prime factorisation and arrange all the prime factors into **triplets** or three identical groups.
2. If all the prime factors can be grouped in this way, the number is a perfect cube.

17. Show that 3375 is a perfect cube.

Answer:

1. $3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$.
2. Therefore, $3375 = (3 \times 5)^3 = 15^3$, so $\sqrt[3]{3375} = 15$.

18. Why is 500 not a perfect cube?

Answer:

1. The prime factorisation of 500 is $2 \times 2 \times 5 \times 5 \times 5$.
2. Its prime factors cannot be split into three identical groups, so 500 is not a perfect cube.

19. Explain the relationship between perfect cubes and consecutive odd numbers.

Answer:

1. Perfect cubes can be represented by suitable groups of consecutive odd numbers.
2. For example, $3 + 5 = 8 = 2^3$and $7 + 9 + 11 = 27 = 3^3$.

20. What is the Hardy–Ramanujan Number? Why is it special?

Answer:

1. **1729** is known as the Hardy–Ramanujan Number.
2. It is the smallest number expressible as the sum of two positive cubes in two different ways: $1729 = 1^3 + 12^3 = 9^3 + 10^3$.

21. What are taxicab numbers? Give an example.

Answer:

1. Numbers that can be expressed as the sum of two positive cubes in two different ways are called **taxicab numbers**.
2. For example, **1729** is a taxicab number.

22. What do the Sanskrit terms *varga* and *ghana* represent?

Answer:

1. **Varga** was used for a square figure or its area and also for the square power.
2. **Ghana** was used for a solid cube as well as the product of a number with itself three times.

23. What are *varga-mula* and *ghana-mula*?

Answer:

1. **Varga-mula** was the Sanskrit term used for the square root.
2. **Ghana-mula** was the Sanskrit term used for the cube root.

24. State two observations about prime factors of perfect squares and perfect cubes.

Answer:

1. In a perfect square, prime factors can be grouped into **pairs**.
2. In a perfect cube, prime factors can be grouped into **triplets**.

25. How can nearby perfect squares be used to estimate a square root?

Answer:

1. Locate the given number between two consecutive or nearby perfect squares.
2. Its square root must lie between the square roots of those two perfect squares.

1.8. Two-mark questions with Answers of HOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – HOTS Type:
Each answer is given in two clear reasoning/calculation steps.

1. Without finding the square root, determine whether 2027 can be a perfect square. Give reasons.

Answer:
1. The units digit of 2027 is **7**, and a perfect square cannot end in 2, 3, 7, or 8.
2. Therefore, **2027 is not a perfect square**.

2. Given $35^2 = 1225$, find 36^2without direct multiplication.

Answer:
1. The 36th odd number is $2(36) - 1 = 71$.
2. Therefore, $36^2 = 35^2 + 71 = 1225 + 71 = 1296$.

3. Determine whether 81 is a perfect square using successive subtraction of odd numbers.

Answer:
1. Successively subtracting $1, 3, 5, \dots, 17$from 81 reaches exactly **0 after 9 subtractions**.
2. Therefore, **81 is a perfect square and** $\sqrt{81} = 9$.

4. Find the number of natural numbers lying between $16^2$and 17^2.

Answer:
1. $17^2 - 16^2 = (17 - 16)(17 + 16) = 33$.
2. Therefore, the number of natural numbers strictly between them is $33 - 1 = 32$.

5. Show that 324 is a perfect square using prime factorisation.

Answer:
1. $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = (2 \times 3 \times 3)^2$.
2. Hence, $324 = 18^2$, so **324 is a perfect square and** $\sqrt{324} = 18$.

6. Explain why 156 is not a perfect square using prime factorisation.

Answer:
1. $156 = 2 \times 2 \times 3 \times 13$, where 3 and 13 remain unpaired.
2. Since all prime factors cannot be grouped into pairs, **156 is not a perfect square**.

7. Estimate $\sqrt{250}$using nearby perfect squares.

Answer:
1. Since $15^2 = 225 < 250 < 256 = 16^2$, we have $15 < \sqrt{250} < 16$.
2. As 250 is closer to 256 than to 225, $\sqrt{250}$is **approximately 16, but slightly less than 16**.

8. A square cloth has an area of 125 cm². Find the maximum integer side length of a square handkerchief that can be cut from it.

Answer:
1. The nearby perfect squares are $11^2 = 121$and $12^2 = 144$, and $121 < 125 < 144$.
2. Therefore, the largest possible square handkerchief with an integer side length has side **11 cm**.

9. Explain why the square of a number ending with three zeros must end with six zeros.

Answer:
1. A number ending with three zeros contains a factor of 10^3.
2. On squaring, $(10^3)^2 = 10^6$, so its square ends with **six zeros**.

10. If $20^2 = 400$, find 21^2using the pattern of consecutive squares.

Answer:
1. The difference $21^2 - 20^2$is the 21st odd number, $2(21) - 1 = 41$.
2. Therefore, $21^2 = 400 + 41 = 441$.

11. Show that $1 + 3 + 5 + 7 + 9 + 11$is a perfect square.

Answer:

1. The expression is the sum of the first **six consecutive odd numbers**.
2. Since the sum of the first $n$odd numbers is n^2, the sum is $6^2 = 36$.

12. Determine whether $2^4 \times 3^2$represents a perfect square and find its square root.
Answer:
1. $2^4 \times 3^2 = (2^2 \times 3)^2$, so all prime factors occur in pairs.
2. Therefore, it is a **perfect square** and its square root is $2^2 \times 3 = 12$.

13. Show that 3375 is a perfect cube using prime factorisation.
Answer:
1. $3375 = 3^3 \times 5^3 = (3 \times 5)^3$.
2. Therefore, **3375 is a perfect cube** and $\sqrt[3]{3375} = 15$.

14. Explain why 500 is not a perfect cube.
Answer:
1. $500 = 2 \times 2 \times 5 \times 5 \times 5$, so only the factors 5 form a complete triplet.
2. Since the two factors 2 cannot form a triplet, **500 is not a perfect cube**.

15. Find the cube root of $2^6 \times 3^3$without multiplying the factors first.
Answer:
1. $2^6 \times 3^3 = (2^2)^3 \times 3^3 = (2^2 \times 3)^3$.
2. Therefore, $\sqrt[3]{2^6 \times 3^3} = 4 \times 3 = 12$.

16. Use the consecutive odd-number pattern to find 4^3.
Answer:
1. According to the cube pattern, $4^3$is represented by $13 + 15 + 17 + 19$.
2. Their sum is 64; therefore, $4^3 = 64$.

17. Without direct multiplication, identify the cube represented by $21 + 23 + 25 + 27 + 29$.
Answer:
1. The chapter's pattern represents the nth cube by a group of $n$consecutive odd numbers, and here there are five terms.
2. Therefore, $21 + 23 + 25 + 27 + 29 = 5^3 = 125$.

18. Explain why 1729 is called a taxicab number.
Answer:
1. 1729can be expressed in two ways as $1^3 + 12^3$and $9^3 + 10^3$.
2. Therefore, it is a **taxicab number**, since it is expressible as the sum of two positive cubes in two different ways.

19. A cube is made using 729 unit cubes. Find its edge length.
Answer:
1. The edge length is the cube root of the total number of unit cubes, so we need $\sqrt[3]{729}$.
2. Since $9^3 = 729$, the edge length is **9 units**.

20. Determine whether $2^3 \times 3^6 \times 5^3$is a perfect cube and find its cube root.
Answer:
1. All exponents 3,6,3are multiples of 3, so the prime factors can be grouped completely into triplets.
2. Hence it is a perfect cube and its cube root is $2 \times 3^2 \times 5 = 90$.

21. Find $\sqrt{1936}$using estimation and the units-digit property.
Answer:
1. Since $40^2 < 1936 < 50^2$and 1936 ends in 6, its square root can be **44 or 46**.
2. Since $44^2 = 1936$, we get $\sqrt{1936} = 44$.

22. In the locker puzzle, explain why locker 36 remains open while locker 6 remains closed.
Answer:

1. Locker 36 has an odd number of factors because **36 is a perfect square**, so it is toggled an odd number of times.
2. Locker 6 has four factors 1,2,3,6, so it is toggled an even number of times and remains **closed**.

23. A student says, "Every number ending in 6 is a perfect square." Is the statement correct? Justify.
Answer:
1. The statement is incorrect because although some perfect squares such as 16 and 36 end in 6, numbers such as **26 are not perfect squares**.
2. Thus, the units digit can sometimes prove that a number is **not** a square, but it cannot always prove that it **is** a square.

24. Why do only square-numbered lockers remain open at the end of the locker puzzle?
Answer:
1. Each locker is toggled once for every factor of its number, and only perfect squares have an **odd number of factors**.
2. Therefore, only lockers numbered $1,4,9,16,\ldots,100$are toggled an odd number of times and remain open.

25. Determine the smallest number by which 72 must be multiplied to make it a perfect square.
Answer:
1. $72 = 2^3 \times 3^2$; one more factor of 2 is required to make all prime factors occur in pairs.
2. Therefore, the smallest multiplier is **2**, giving $72 \times 2 = 144 = 12^2$.

1.9. Three-mark questions with Answers of LOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – LOTS Type:
1. What is a perfect square? Explain with examples.
Answer:
1. A number obtained by multiplying a natural number by itself is called a **perfect square**.
2. It is represented as $n^2 = n \times n$.
3. For example, $4^2 = 16$, $5^2 = 25$, and $6^2 = 36$are perfect squares.

2. Explain the units-digit property of perfect squares.
Answer:
1. A perfect square can have only **0, 1, 4, 5, 6, or 9** in its units place.
2. A number ending in **2, 3, 7, or 8** cannot be a perfect square.
3. However, ending in 0, 1, 4, 5, 6, or 9 does not necessarily mean that the number is a perfect square.

3. Explain why perfect squares have an odd number of factors.
Answer:
1. Factors of a number generally occur in pairs whose product gives the number.
2. In a perfect square, one factor pair consists of the **same factor twice**, such as $6 \times 6 = 36$.
3. Therefore, perfect squares have an **odd number of distinct factors**.

4. Explain the relationship between perfect squares and consecutive odd numbers.
Answer:
1. Every perfect square can be expressed as a sum of consecutive odd numbers starting from 1.
2. The sum of the first $n$odd numbers is n^2.

3. For example, $1 + 3 + 5 + 7 + 9 = 25 = 5^2$.

5. Describe the successive subtraction method for checking a perfect square.
Answer:
1. Start with the given number and successively subtract 1,3,5,7, ...
2. If the subtraction reaches **exactly zero**, the number is a perfect square.
3. For example, repeated subtraction of odd numbers from 25 reaches zero after five steps, so $25 = 5^2$.

6. What is a square root? Explain with the example of 64.
Answer:
1. If $y = x^2$, then $x$is called a **square root** of y.
2. Since $8^2 = 64$and $(-8)^2 = 64$, the integer square roots of 64 are $+8$and -8.
3. In this chapter, only the **positive square root** is considered for calculations.

7. Explain how prime factorisation is used to find the square root of 324.
Answer:
1. The prime factorisation is $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$.
2. Pairing equal factors gives $324 = (2 \times 3 \times 3)^2$.
3. Therefore, $\sqrt{324} = 2 \times 3 \times 3 = 18$.

8. Explain why 156 is not a perfect square.
Answer:
1. The prime factorisation of 156 is $2 \times 2 \times 3 \times 13$.
2. The factors 2 can be paired, but 3 and 13 remain unpaired.
3. Therefore, **156 is not a perfect square**.

9. Explain how the square root of a number can be estimated using nearby perfect squares.
Answer:
1. Find two nearby perfect squares between which the given number lies.
2. The square root of the given number must lie between the square roots of those perfect squares.
3. For example, $225 < 250 < 256$, so $15 < \sqrt{250} < 16$.

10. What is a perfect cube? Explain with examples.
Answer:
1. A perfect cube is obtained by multiplying a natural number by itself **three times**.
2. It is represented as $n^3 = n \times n \times n$.
3. For example, $2^3 = 8$, $3^3 = 27$, and $4^3 = 64$are perfect cubes.

11. Explain the geometrical meaning of $4^3 = 64$.
Answer:
1. A cube of edge 4 units has $4 \times 4 = 16$unit cubes in each layer.
2. There are **4 such layers** in the cube.
3. Therefore, the total number of unit cubes is $4 \times 4 \times 4 = 64$.

12. What is a cube root? Explain with examples.
Answer:
1. If $y = x^3$, then $x$is called the **cube root** of y.
2. Cube root is denoted by the symbol $\sqrt[3]{\ }$.
3. For example, $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, and $\sqrt[3]{1000} = 10$.

13. Explain how prime factorisation is used to find the cube root of 3375.
Answer:
1. $3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$.
2. Grouping the prime factors into triplets gives $3375 = (3 \times 5)^3$.
3. Therefore, $\sqrt[3]{3375} = 3 \times 5 = 15$.

14. Explain the relationship between perfect cubes and consecutive odd numbers.
Answer:
1. Perfect cubes can be represented using suitable groups of consecutive odd numbers.
2. For example, $3 + 5 = 8 = 2^3$and $7 + 9 + 11 = 27 = 3^3$.
3. Similarly, $13 + 15 + 17 + 19 = 64 = 4^3$.

15. Explain why 500 is not a perfect cube.
Answer:
1. The prime factorisation of 500 is $2 \times 2 \times 5 \times 5 \times 5$.
2. The three factors 5 form a triplet, but the two factors 2 cannot form a triplet.
3. Therefore, **500 is not a perfect cube**.

16. What is special about the number 1729?
Answer:
1. **1729** is known as the **Hardy–Ramanujan Number**.
2. It is the smallest number that can be expressed as the sum of two positive cubes in two different ways.
3. These are $1729 = 1^3 + 12^3 = 9^3 + 10^3$.

17. What are taxicab numbers? Explain with an example.
Answer:
1. Numbers that can be expressed as the sum of two positive cubes in two different ways are called **taxicab numbers**.
2. The smallest such number is **1729**.
3. The chapter also gives **4104 and 13832** as the next two taxicab numbers after 1729.

18. Explain the prime-factor property of a cube.
Answer:
1. When a number is cubed, each of its prime factors appears **three times** in the prime factorisation of the cube.
2. For example, $6 = 2 \times 3$, so $6^3 = 2^3 \times 3^3 = 216$.
3. Hence, the prime factors of a perfect cube can be grouped into **triplets**.

19. Explain the meanings of the ancient Indian mathematical terms *varga*, *ghana*, and *mula*.
Answer:
1. **Varga** was used for a square figure or its area as well as the square power.
2. **Ghana** was used for a solid cube and for the product of a number with itself three times.
3. **Mula**, meaning root or origin, was used in the mathematical sense of taking roots.

20. Summarise the main difference between a perfect square and a perfect cube using prime factorisation.
Answer:
1. In a **perfect square**, all prime factors can be divided into **two identical groups or pairs**.
2. In a **perfect cube**, all prime factors can be divided into **three identical groups or triplets**.
3. These properties can be used to identify perfect squares and cubes and to calculate their roots.

1.10. Three-mark questions with Answers of HOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – HOTS Type:
Each answer is presented in three reasoning/calculation steps.

1. Given that $35^2 = 1225$, find 36^2without multiplying 36 by itself.
Answer:
1. The 36th odd number is $2(36) - 1 = 71$.
2. Consecutive squares differ by consecutive odd numbers, so $36^2 = 35^2 + 71$.
3. Therefore, $36^2 = 1225 + 71 = \boxed{1296}$.

2. Determine whether 1156 is a perfect square using prime factorisation and find its square root.
Answer:
1. Prime factorising, $1156 = 2^2 \times 17^2$.
2. All the prime factors occur in pairs, so $1156 = (2 \times 17)^2$.
3. Therefore, 1156 is a **perfect square** and $\sqrt{1156} = \boxed{34}$.

3. Determine whether 2800 is a perfect square using prime factorisation.
Answer:
1. $2800 = 2^4 \times 5^2 \times 7$.
2. The factor 7 occurs only once and cannot be paired with another 7.
3. Therefore, **2800 is not a perfect square**.

4. Find $\sqrt{1936}$using estimation and the units-digit property.
Answer:
1. Since $40^2 = 1600 < 1936 < 2500 = 50^2$, its square root lies between 40 and 50.
2. As 1936 ends in 6, the possible square roots in this interval are 44 and 46; also $45^2 = 2025 > 1936$.
3. Hence, the required value is $\boxed{\sqrt{1936} = 44}$.

5. Find the smallest number by which 9408 must be multiplied to make the product a perfect square, and find its square root.
Answer:
1. Prime factorisation gives $9408 = 2^6 \times 3 \times 7^2$, where only the factor 3 is unpaired.
2. Multiplying by 3 gives $9408 \times 3 = 2^6 \times 3^2 \times 7^2 = 28224$, a perfect square.
3. Therefore, the smallest multiplier is $\boxed{3}$and $\sqrt{28224} = \boxed{168}$.

6. How many natural numbers lie between $16^2$and 17^2? Explain using the pattern of consecutive squares.
Answer:
1. $16^2 = 256$and $17^2 = 289$.
2. Their difference is $289 - 256 = 33$, which is the corresponding odd-number difference between consecutive squares.
3. Excluding the two squares themselves, there are $33 - 1 = \boxed{32}$natural numbers between them.

7. A square cloth has an area of 125 cm². Can a square handkerchief of side 15 cm be cut from it? Find the largest possible integer side length.
Answer:
1. A 15 cm square requires $15^2 = 225\ \text{cm}^2$, which is greater than 125 cm², so it cannot be cut.
2. The neighbouring perfect squares are $11^2 = 121$and $12^2 = 144$, with $121 < 125 < 144$.
3. Therefore, the maximum possible integer side length is $\boxed{11\ \text{cm}}$.

8. Show that 38 is not a perfect square using successive subtraction of odd numbers.
Answer:
1. Successively subtract 1,3,5,7,9,and 11from 38 to obtain 2.

2. Subtracting the next odd number, 13, gives $2-13=-11$, so the process crosses zero instead of reaching exactly zero.
3. Therefore, **38 is not a perfect square**.

9. A student claims that every number ending in 6 is a perfect square. Analyse the claim.

Answer:

1. Some perfect squares do end in 6, for example $4^2=16$and $6^2=36$.
2. However, a number such as 26 also ends in 6 but is not a perfect square.
3. Therefore, the units digit can sometimes rule out a square, but ending in 6 does **not guarantee** that a number is a perfect square.

10. Explain mathematically why only square-numbered lockers remain open in the chapter's locker puzzle.

Answer:

1. A locker is toggled once for every factor of its locker number.
2. Non-square numbers have factors in distinct pairs and hence an even number of factors, while perfect squares have one self-paired factor and hence an odd number of factors.
3. Therefore, only square-numbered lockers such as $1,4,9,\ldots,100$are toggled an odd number of times and remain **open**.

11. Determine whether 3375 is a perfect cube and find its cube root using prime factorisation.

Answer:

1. $3375=3\times3\times3\times5\times5\times5=3^3\times5^3$.
2. The factors form complete triplets, so $3375=(3\times5)^3$.
3. Therefore, **3375 is a perfect cube** and $\sqrt[3]{3375}=\boxed{15}$.

12. Analyse whether 500 is a perfect cube using prime factorisation.

Answer:

1. $500=2\times2\times5\times5\times5=2^2\times5^3$.
2. The three factors 5 form a triplet, but the two factors 2 cannot form a complete triplet.
3. Therefore, **500 is not a perfect cube**.

13. Find the value of $21+23+25+27+29$without ordinary addition, using the cube pattern.

Answer:

1. The chapter shows the pattern $3+5=2^3$, $7+9+11=3^3$, $13+15+17+19=4^3$.
2. The next group consists of five consecutive odd numbers: $21,23,25,27,29$, representing 5^3.
3. Therefore, $21+23+25+27+29=5^3=\boxed{125}$.

14. Without adding individually, find $91+93+95+97+99+101+103+105+107+109$.

Answer:

1. The expression contains **10 consecutive odd numbers** and appears later in the chapter's perfect-cube pattern.
2. Following the pattern, a group of $n$appropriate consecutive odd numbers represents n^3; hence this group represents 10^3.
3. Therefore, the required sum is $\boxed{1000}$.

15. Explain why 1729 is special by verifying its two representations as sums of cubes.

Answer:

1. $1^3+12^3=1+1728=1729$.
2. Also, $9^3+10^3=729+1000=1729$.

3. Therefore, 1729 is the smallest number expressible as the sum of two positive cubes in two different ways and is called the **Hardy–Ramanujan Number**.

16. Find the cube root of $2^6 \times 3^3 \times 5^3$ using prime-factor grouping.

Answer:

1. Rewrite the expression as $(2^2)^3 \times 3^3 \times 5^3$.
2. Combining the triplets gives $(2^2 \times 3 \times 5)^3 = (4 \times 3 \times 5)^3$.
3. Therefore, the cube root is $\boxed{60}$.

17. A cube contains 1728 unit cubes. Find its edge length using the perfect-cube property.

Answer:

1. The edge length of the cube is $\sqrt[3]{1728}$.
2. Since $1728 = 12 \times 12 \times 12 = 12^3$, it is a perfect cube.
3. Therefore, the cube has an edge length of $\boxed{12 \text{ units}}$.

18. Determine the smallest number by which 500 must be multiplied to make it a perfect cube.

Answer:

1. $500 = 2^2 \times 5^3$, so the factors 5 already form a triplet while the two factors 2 need one more 2.
2. Multiplying by 2 gives $1000 = 2^3 \times 5^3 = (2 \times 5)^3$.
3. Therefore, the smallest required multiplier is $\boxed{2}$, and the resulting perfect cube is $1000 = 10^3$.

19. A student says that every perfect cube has an odd number of factors. Test the claim using 8.

Answer:

1. $8 = 2^3$, so 8 is a perfect cube.
2. Its factors are 1,2,4,8, giving **four factors**, which is an even number.
3. Therefore, the statement that every perfect cube has an odd number of factors is **false**.

20. Compare $67^2 - 66^2$ and $43^2 - 42^2$ without calculating the individual squares. Which is greater?

Answer:

1. Using consecutive-square differences, $67^2 - 66^2 = 67 + 66 = 133$.
2. Similarly, $43^2 - 42^2 = 43 + 42 = 85$.
3. Since $133 > 85$, $\boxed{67^2 - 66^2}$ is greater.

1.11 Five-mark questions with Answers of LOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – LOTS Type:

Each answer is given in five clear sentences/points based on the uploaded chapter.

1. Explain perfect squares and their important properties.

Answer:

1. A number obtained by multiplying a natural number by itself is called a **perfect square**.
2. If n is a natural number, its square is written as $n^2 = n \times n$.
3. Examples of perfect squares are 1,4,9,16,25,36,49,64,81,and 100.
4. A perfect square can have only **0, 1, 4, 5, 6, or 9** as its units digit.
5. A perfect square has an **odd number of factors**, because one of its factor pairs consists of the same factor repeated.

2. Explain the relationship between perfect squares and consecutive odd numbers.

Answer:

1. Perfect squares can be obtained by adding consecutive odd numbers beginning with 1.

2. The sum of the first $n$odd numbers is equal to n^2.
3. For example, $1 + 3 + 5 + 7 = 16 = 4^2$.
4. Similarly, $1 + 3 + 5 + 7 + 9 = 25 = 5^2$.
5. Hence, the sequence of perfect squares can be generated by repeatedly adding the next consecutive odd number.

3. Explain how successive subtraction of odd numbers can be used to check whether a number is a perfect square.
Answer:
1. Begin with the given natural number and subtract consecutive odd numbers starting from 1.
2. Continue subtracting 1,3,5,7,9, ...successively.
3. If the process reaches **exactly zero**, the given number is a perfect square.
4. The number of odd numbers subtracted gives the positive square root of the number.
5. If the process does not reach exactly zero, the given natural number is **not a perfect square**.

4. Explain square roots and the methods used in the chapter to identify perfect squares.
Answer:
1. If $y = x^2$, then $x$is called a **square root** of y.
2. For example, $8^2 = 64$and $(-8)^2 = 64$, so 64 has the integer square roots $+8$and -8.
3. The units digit can first be checked because a perfect square can end only in 0, 1, 4, 5, 6, or 9.
4. A number can also be checked by comparing it with a list of known perfect squares or by successively subtracting odd numbers.
5. In this chapter, the symbol $\sqrt{\ }$ is used for the **positive square root**.

5. Explain how prime factorisation can be used to determine whether a number is a perfect square.
Answer:
1. First, express the given number as a product of its **prime factors**.
2. Arrange identical prime factors into pairs.
3. If all the prime factors can be completely grouped into pairs, the number is a perfect square.
4. For example, $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = (2 \times 3 \times 3)^2$.
5. Therefore, $324 = 18^2$and hence $\sqrt{324} = 18$.

6. Explain perfect cubes with suitable examples and their geometrical meaning.
Answer:
1. A number obtained by multiplying a natural number by itself three times is called a **perfect cube**.
2. The cube of $n$is written as $n^3 = n \times n \times n$.
3. The first few perfect cubes are 1,8,27,64,and 125.
4. Geometrically, a cube of edge 4 units contains $4 \times 4 \times 4 = 64$unit cubes.
5. Thus, $4^3 = 64$, showing the connection between the numerical cube and the geometrical cube.

7. Explain the relationship between perfect cubes and consecutive odd numbers.
Answer:
1. Perfect cubes can be represented by suitable groups of **consecutive odd numbers**.
2. The pattern begins with $1 = 1^3$and $3 + 5 = 8 = 2^3$.
3. Similarly, $7 + 9 + 11 = 27 = 3^3$.
4. Also, $13 + 15 + 17 + 19 = 64 = 4^3$and $21 + 23 + 25 + 27 + 29 = 125 = 5^3$.

5. Thus, the chapter demonstrates a systematic pattern connecting groups of consecutive odd numbers with perfect cubes.

8. Explain cube roots and how prime factorisation is used to find the cube root of 3375.

Answer:

1. If $y = x^3$, then $x$is called the **cube root** of y.
2. Cube root is represented using the symbol $\sqrt[3]{\ }$.
3. The prime factorisation of 3375 is $3 \times 3 \times 3 \times 5 \times 5 \times 5$.
4. Grouping the equal prime factors into triplets gives $3375 = 3^3 \times 5^3 = (3 \times 5)^3 = 15^3$.
5. Therefore, $\sqrt[3]{3375} = \boxed{15}$.

9. Explain how prime factorisation helps distinguish a perfect square from a perfect cube.

Answer:

1. Prime factorisation expresses a natural number as a product of prime factors.
2. In a **perfect square**, the prime factors can be divided into **two identical groups**, or paired completely.
3. In a **perfect cube**, the prime factors can be divided into **three identical groups**, or grouped completely into triplets.
4. For example, $324 = 2^2 \times 3^4$is a perfect square because its prime factors can be paired.
5. Similarly, $3375 = 3^3 \times 5^3$is a perfect cube because its prime factors can be grouped into triplets.

10. Describe the Hardy–Ramanujan Number and explain its importance.

Answer:

1. The number **1729** is famously associated with mathematicians **G. H. Hardy and Srinivasa Ramanujan**.
2. Hardy mentioned 1729 as the number of the taxicab in which he had travelled to visit Ramanujan.
3. Ramanujan observed that 1729 has a very interesting mathematical property.
4. It can be expressed as $1729 = 1^3 + 12^3 = 9^3 + 10^3$.
5. Thus, 1729 is the smallest number expressible as the sum of two positive cubes in two different ways and is known as the **Hardy–Ramanujan Number**.

11. Explain the locker puzzle and its connection with perfect squares.

Answer:

1. In the locker puzzle, each person changes the state of lockers whose numbers are multiples of that person's number.
2. Therefore, a locker is toggled once for every **factor** of its locker number.
3. Most factors occur in pairs, so non-square-numbered lockers are toggled an even number of times.
4. Perfect squares have an odd number of factors because one factor is paired with itself.
5. Hence, only the lockers numbered by perfect squares remain **open** at the end.

12. Explain the historical Indian terms associated with squares, cubes, and roots.

Answer:

1. The Sanskrit word **varga** was used for a square figure or its area and later for the square of a number.
2. The term **ghana** referred to a solid cube and also to the product of a number multiplied by itself three times.
3. These mathematical terms were used in India from ancient times.
4. The Sanskrit word **mula**, meaning root or origin, was also used in the mathematical sense of a root.
5. Thus, **varga-mula** means square root and **ghana-mula** means cube root.

1.12 Five-mark questions with Answers of HOTS Type: (5 Questions)

Five-Mark Descriptive Questions with Answers – HOTS Type:
Each answer is given in five analytical/reasoning points.

1. A student claims that 2025 is a perfect square because it ends in 5. Analyse the claim and verify whether 2025 is actually a perfect square.

Answer:

1. A number ending in 5 **may** be a perfect square, but the units digit alone cannot prove that it is one.
2. We know that $40^2 = 1600$and $50^2 = 2500$, so $\sqrt{2025}$must lie between 40 and 50.
3. Since the number ends in 25, we can test 45^2.
4. $45^2 = (40 + 5)^2 = 1600 + 400 + 25 = 2025$.
5. Therefore, **2025 is a perfect square and** $\sqrt{2025} = 45$.

2. Use prime factorisation to determine whether 9408 is a perfect square and find the smallest number by which it must be multiplied to become a perfect square.

Answer:

1. The prime factorisation is $9408 = 2^6 \times 3 \times 7^2$.
2. The factors $2^6$and 7^2can be completely grouped into pairs, but the factor 3 remains unpaired.
3. Therefore, **9408 is not a perfect square**.
4. Multiplying by 3 gives $9408 \times 3 = 2^6 \times 3^2 \times 7^2 = 28224$.
5. Hence, the smallest required multiplier is **3**, and $28224 = 168^2$.

3. A square-shaped cloth has an area of 250 cm². Determine between which two integers its side length lies and identify the greatest integer side length possible.

Answer:

1. The side length of the square is $\sqrt{250}$cm.
2. The nearby perfect squares are $15^2 = 225$and $16^2 = 256$.
3. Since $225 < 250 < 256$, we have $15 < \sqrt{250} < 16$.
4. Thus, the exact side is greater than 15 cm but less than 16 cm.
5. Therefore, the greatest whole-number side length not exceeding it is **15 cm**.

4. Explain why only perfect-square-numbered lockers remain open at the end of the locker puzzle.

Answer:

1. Each locker is toggled once for every factor of its locker number.
2. For a non-square number, factors occur in distinct pairs, giving an **even number of factors**.
3. Hence, a non-square locker is toggled an even number of times and returns to the closed position.
4. A perfect square has one factor paired with itself, so it has an **odd number of factors** and is toggled an odd number of times.
5. Therefore, only lockers numbered 1,4,9,16,25,36,49,64,81,and 100remain open.

5. A student wants to check whether 38 is a perfect square without calculating its square root. Use the successive odd-number subtraction method to justify your answer.

Answer:

1. Start with 38 and successively subtract odd numbers: $38 - 1 = 37$, $37 - 3 = 34$, and $34 - 5 = 29$.
2. Continue: $29 - 7 = 22$, $22 - 9 = 13$, and $13 - 11 = 2$.
3. The next odd number to subtract is 13, which would take the result below zero.

4. Thus, the subtraction process does not reach **exactly zero**.
5. Therefore, **38 is not a perfect square**.

6. Using the pattern of consecutive squares, find $36^2$from $35^2 = 1225$and explain the reasoning.

Answer:

1. The difference between consecutive perfect squares follows the sequence of consecutive odd numbers.
2. The nth odd number is $2n - 1$, so the 36th odd number is $2(36) - 1 = 71$.
3. Therefore, $36^2 - 35^2 = 71$.
4. Hence, $36^2 = 1225 + 71 = 1296$.
5. Thus, $36^2 = 1296$can be obtained without directly multiplying 36×36.

7. Determine whether 3375 is a perfect cube and find its cube root using prime factorisation.

Answer:

1. Prime factorising gives $3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$.
2. The factors can be grouped as $(3 \times 3 \times 3)(5 \times 5 \times 5)$.
3. Thus, $3375 = 3^3 \times 5^3 = (3 \times 5)^3$.
4. Therefore, $3375 = 15^3$, showing that it is a **perfect cube**.
5. Hence, $\boxed{\sqrt[3]{3375} = 15}$.

8. Explain why 500 is not a perfect cube and determine the smallest number by which it should be multiplied to become one.

Answer:

1. The prime factorisation of 500 is $2^2 \times 5^3$.
2. For a perfect cube, each prime factor must occur in complete groups of three.
3. The three factors 5 form a triplet, but 2^2needs one more factor of 2.
4. Multiplying by 2 gives $500 \times 2 = 1000 = 2^3 \times 5^3 = (2 \times 5)^3$.
5. Therefore, the smallest multiplier is **2**, and the resulting perfect cube is $1000 = 10^3$.

9. Use the pattern of consecutive odd numbers to explain how 5^3can be obtained without multiplying $5 \times 5 \times 5$.

Answer:

1. The chapter shows the pattern $1 = 1^3$, $3 + 5 = 2^3$, and $7 + 9 + 11 = 3^3$.
2. The next group, $13 + 15 + 17 + 19$, gives $4^3 = 64$.
3. For 5^3, the corresponding five consecutive odd numbers are 21,23,25,27,and 29.
4. Their sum is $21 + 23 + 25 + 27 + 29 = 125$.
5. Therefore, the pattern gives $5^3 = 125$without directly multiplying $5 \times 5 \times 5$.

10. Ramanujan said that 1729 is an interesting number. Demonstrate mathematically why his observation is correct.

Answer:

1. Consider the cubes $1^3 = 1$and $12^3 = 1728$.
2. Their sum is $1^3 + 12^3 = 1 + 1728 = 1729$.
3. Now consider $9^3 = 729$and $10^3 = 1000$.
4. Their sum is also $9^3 + 10^3 = 729 + 1000 = 1729$.
5. Therefore, **1729 is the smallest number expressible as the sum of two positive cubes in two different ways**, making it the Hardy–Ramanujan Number.

11. Compare perfect squares and perfect cubes using their prime-factorisation properties.

Answer:

1. A perfect square has prime factors that can be divided completely into **pairs**.
2. For example, $324 = 2^2 \times 3^4 = (2 \times 3^2)^2 = 18^2$.
3. A perfect cube has prime factors that can be divided completely into **triplets**.

4. For example, $3375 = 3^3 \times 5^3 = (3 \times 5)^3 = 15^3$.
5. Thus, pairing prime factors identifies perfect squares, while grouping them in threes identifies perfect cubes.

12. A student says, "If a number has 6 as its units digit, it must be a perfect square." Evaluate this statement using the properties of squares.

Answer:

1. Perfect squares can have **0, 1, 4, 5, 6, or 9** as their units digit.
2. Therefore, a number ending in 6 is **capable** of being a perfect square.
3. For example, $4^2 = 16$and $6^2 = 36$, both of which end in 6.
4. However, numbers such as 26 also end in 6 but are not perfect squares.
5. Hence, the student's statement is **incorrect**, because an allowed units digit does not by itself prove that a number is a perfect square.

1.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – Applied Type:

1. A school wants to arrange 625 students in a square formation for an assembly. Find the number of students in each row and explain how you know the arrangement is possible.

Answer:

1. In a square formation, the number of rows and the number of students in each row must be equal.
2. Therefore, we need to find $\sqrt{625}$.
3. Since $625 = 25 \times 25 = 25^2$, it is a perfect square.
4. Hence, $\sqrt{625} = 25$.
5. Therefore, the school can arrange the students in **25 rows with 25 students in each row**.

2. A square-shaped garden has an area of 1936 m². Find the length of its side using the estimation method.

Answer:

1. The side of the square is $\sqrt{1936}$metres.
2. Since $40^2 < 1936 < 50^2$, the side must lie between 40 m and 50 m.
3. Since 1936 ends in 6, the possible roots in this range are 44 and 46.
4. Checking gives $44^2 = 1936$.
5. Therefore, the side of the square garden is **44 m**.

3. A tailor has 125 cm² of cloth and wants to cut the largest possible square handkerchief with a whole-number side length. What should its side length be?

Answer:

1. The side length is related to the area by $s = \sqrt{125}$.
2. The nearby perfect squares are $11^2 = 121$and $12^2 = 144$.
3. Since $121 < 125 < 144$, $\sqrt{125}$lies between 11 and 12.
4. A 12 cm side would require 144 cm² of cloth, which is more than the available cloth.
5. Therefore, the largest square handkerchief with a whole-number side can have a side of **11 cm**.

4. A factory packs 3375 identical small cubes into one large cube. Find the number of small cubes along each edge.

Answer:

1. If $n$small cubes are placed along each edge, the total number of cubes is n^3.
2. Therefore, we need to calculate $\sqrt[3]{3375}$.
3. Prime factorisation gives $3375 = 3^3 \times 5^3$.

4. Hence, $3375 = (3 \times 5)^3 = 15^3$.
5. Therefore, the large cube has **15 small cubes along each edge**.

5. A shop has 500 small cube-shaped boxes and wants to arrange all of them into one complete larger cube. Can this be done? If not, find the minimum number of additional boxes required.

Answer:

1. The prime factorisation of 500 is $2^2 \times 5^3$.
2. For a perfect cube, every prime factor must occur in complete groups of three.
3. The factor 5 forms a complete triplet, but the factor 2 occurs only twice, so 500 is not a perfect cube.
4. One more factor of 2 is needed, giving $500 \times 2 = 1000 = 10^3$.
5. Therefore, **500 boxes cannot form a complete cube; 500 more boxes are required to make 1000 boxes**, which form a $10 \times 10 \times 10$cube.

6. A hall has 900 floor tiles, and the tiles are to be arranged in a perfect square without leaving any tile unused. Find the number of tiles in each row.

Answer:

1. A square arrangement requires the same number of tiles in each row and column.
2. Therefore, the number of tiles in each row is $\sqrt{900}$.
3. Since $900 = 30 \times 30 = 30^2$, it is a perfect square.
4. Thus, $\sqrt{900} = 30$.
5. Therefore, the tiles can be arranged in **30 rows and 30 columns**.

7. A designer wants to make a square mosaic using exactly 156 identical tiles. Determine whether all the tiles can be arranged in a perfect square.

Answer:

1. To form a perfect square, 156 must itself be a perfect square.
2. Its prime factorisation is $156 = 2 \times 2 \times 3 \times 13$.
3. The two factors 2 form a pair, but 3 and 13 remain unpaired.
4. Therefore, all the prime factors cannot be grouped into pairs.
5. Hence, **156 tiles cannot be arranged into a perfect square with equal numbers of rows and columns**.

8. A cube-shaped storage box is built using 729 identical unit cubes. Find the number of unit cubes along each edge and explain your method.

Answer:

1. The total number of unit cubes in a cube with edge $n$units is n^3.
2. Therefore, we need to find $\sqrt[3]{729}$.
3. Since $9 \times 9 \times 9 = 729$, we have $729 = 9^3$.
4. Hence, $\sqrt[3]{729} = 9$.
5. Therefore, there are **9 unit cubes along each edge** of the storage box.

9. During a mathematics activity, students add $21 + 23 + 25 + 27 + 29$. Use the cube pattern to identify the result without ordinary addition.

Answer:

1. The chapter connects groups of consecutive odd numbers with perfect cubes.
2. It shows $3 + 5 = 2^3$, $7 + 9 + 11 = 3^3$, and $13 + 15 + 17 + 19 = 4^3$.
3. The next group contains five consecutive odd numbers: 21,23,25,27,and 29.
4. Therefore, this group represents 5^3.
5. Hence, $21 + 23 + 25 + 27 + 29 = \boxed{125}$.

10. In a locker activity, lockers are toggled according to their factors. Predict whether lockers numbered 36 and 40 will finally remain open or closed.

Answer:

1. A locker is toggled once for each factor of its locker number.
2. Locker 36 is a perfect square because $36 = 6^2$, so it has an odd number of factors.
3. Therefore, locker 36 is toggled an odd number of times and finally remains **open**.
4. Locker 40 is not a perfect square, so its factors occur in distinct pairs and it has an even number of factors.
5. Therefore, locker 40 is toggled an even number of times and finally remains **closed**.

1.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams:

Olympiad-Type Multiple Choice Questions:
The following questions emphasize **logical reasoning, numerical patterns, perfect squares, square roots, perfect cubes, cube roots, prime factorisation, and application** from the uploaded chapter.

1. Which of the following numbers cannot be a perfect square?
A) 324
B) 441
C) 578
D) 625
Answer: C) 578
Reason: A perfect square cannot have 8 as its units digit.

2. If $31^2 = 961$, what is 32^2without directly multiplying 32 by 32?
A) 992
B) 1024
C) 1023
D) 1056
Answer: B) 1024
Reason: $32^2 = 31^2 + 63 = 961 + 63 = 1024$.

3. How many natural numbers lie strictly between $20^2$and 21^2?
A) 20
B) 39
C) 40
D) 41
Answer: C) 40
Reason: $21^2 - 20^2 = 41$, so there are $41 - 1 = 40$numbers between them.

4. Which of the following has an odd number of factors?
A) 30
B) 36
C) 42
D) 50
Answer: B) 36
Reason: Perfect squares have an odd number of factors.

5. The sum $1 + 3 + 5 + \cdots + 19$is equal to:
A) 81
B) 90
C) 100
D) 121
Answer: C) 100
Reason: There are 10 consecutive odd numbers, so their sum is $10^2 = 100$.

6. Which statement about perfect squares is always true?
A) They always end in 1

B) They always have an even number of factors
C) Their prime factors can be grouped into pairs
D) They are always even
Answer: C) Their prime factors can be grouped into pairs.
7. Which is the smallest number that must be multiplied by 72 to make it a perfect square?
A) 2
B) 3
C) 6
D) 8
Answer: A) 2
Reason: $72 = 2^3 \times 3^2$; multiplying by 2 gives $2^4 \times 3^2 = 144 = 12^2$.
8. If $n^2 = 2025$, then $n$is:
A) 35
B) 40
C) 45
D) 55
Answer: C) 45
Reason: $45^2 = 2025$.
9. Which number lies between $15^2$and 16^2?
A) 196
B) 225
C) 250
D) 289
Answer: C) 250
Reason: $225 < 250 < 256$.
10. If a natural number ends in exactly three zeros, how many zeros will its square end in?
A) 3
B) 5
C) 6
D) 9
Answer: C) 6
Reason: Squaring doubles the number of trailing zeros.
11. Which of the following is both a perfect square and a perfect cube?
A) 16
B) 27
C) 36
D) 64
Answer: D) 64
Reason: $64 = 8^2 = 4^3$.
12. If $\sqrt{x} = 18$, what is x?
A) 36
B) 162
C) 324
D) 648
Answer: C) 324
Reason: $x = 18^2 = 324$.
13. Which of the following prime factorisations represents a perfect square?
A) $2^3 \times 3^2$

B) $2^4 \times 3^2$
C) $2^2 \times 3^3$
D) $2^5 \times 3$
Answer: B) $2^4 \times 3^2$
Reason: Every prime factor has an even exponent.
14. The difference $51^2 - 50^2$**is:**
A) 99
B) 100
C) 101
D) 102
Answer: C) 101
Reason: Consecutive squares differ by the corresponding odd number, so $51^2 - 50^2 = 101$.
15. Which of the following is not a perfect cube?
A) 64
B) 125
C) 216
D) 500
Answer: D) 500
Reason: $500 = 2^2 \times 5^3$, so its prime factors cannot all be grouped into triplets.
16. The cube root of 3375 is:
A) 12
B) 15
C) 25
D) 35
Answer: B) 15
Reason: $3375 = 3^3 \times 5^3 = (15)^3$.
17. If $N = 2^6 \times 3^3$**, then** $\sqrt[3]{N}$**is:**
A) 6
B) 9
C) 12
D) 18
Answer: C) 12
Reason: $\sqrt[3]{2^6 \times 3^3} = 2^2 \times 3 = 12$.
18. What is the smallest number by which 500 must be multiplied to make it a perfect cube?
A) 2
B) 4
C) 5
D) 10
Answer: A) 2
Reason: $500 \times 2 = 1000 = 10^3$.
19. According to the cube pattern in the chapter, $21 + 23 + 25 + 27 + 29$**equals:**
A) 4^3
B) 5^3
C) 6^3
D) 7^3
Answer: B) 5^3
Reason: $21 + 23 + 25 + 27 + 29 = 125 = 5^3$.

20. Which prime factorisation represents a perfect cube?
A) $2^2 \times 3^3$
B) $2^3 \times 3^2$
C) $2^3 \times 3^6$
D) $2^4 \times 3^3$
Answer: C) $2^3 \times 3^6$
Reason: Every exponent is a multiple of 3.
21. A cube contains 729 unit cubes. How many unit cubes lie along one edge?
A) 7
B) 8
C) 9
D) 11
Answer: C) 9
Reason: $9^3 = 729$.
22. Which expression is equal to 1729?
A) $1^3 + 12^3$only
B) $9^3 + 10^3$only
C) Both $1^3 + 12^3$and $9^3 + 10^3$
D) Neither
Answer: C) Both $1^3 + 12^3$**and** $9^3 + 10^3$
Reason: $1729 = 1^3 + 12^3 = 9^3 + 10^3$.
23. Which of the following numbers will remain open in the 100-locker puzzle?
A) 48
B) 64
C) 72
D) 96
Answer: B) 64
Reason: Only perfect-square-numbered lockers remain open because they have an odd number of factors.
24. A number is successively reduced by 1,3,5,7, ...**and reaches exactly zero after 12 subtractions. The original number is:**
A) 121
B) 132
C) 144
D) 169
Answer: C) 144
Reason: The sum of the first 12 odd numbers is $12^2 = 144$.
25. Which of the following statements is false?
A) Every perfect square has an odd number of factors.
B) Every perfect cube has prime factors grouped in triplets.
C) Every number ending in 9 is a perfect square.
D) 1is both a perfect square and a perfect cube.
Answer: C) Every number ending in 9 is a perfect square.
Reason: Ending in 9 is possible for a perfect square, but it does not guarantee that the number is a square.
26. If $a^2 = 441$**and** $b^3 = 343$**, then** $a + b$**is:**
A) 21
B) 24
C) 28
D) 49

Answer: C) 28
Reason: $a = 21, b = 7$, so $a + b = 28$.
27. Which is the next number in the pattern $1,4,9,16,25,36, \ldots$**?**
A) 42
B) 47
C) 49
D) 51
Answer: C) 49
Reason: These are consecutive perfect squares $1^2, 2^2, \ldots, 7^2$.
28. The value of $13 + 15 + 17 + 19$**is:**
A) 36
B) 49
C) 64
D) 81
Answer: C) 64
Reason: According to the chapter's cube pattern, $13 + 15 + 17 + 19 = 4^3 = 64$.
29. A number has prime factorisation $2^4 \times 3^2 \times 5^2$**. Its positive square root is:**
A) 30
B) 45
C) 60
D) 90
Answer: C) 60
Reason: $\sqrt{2^4 \times 3^2 \times 5^2} = 2^2 \times 3 \times 5 = 60$.
30. Which statement best distinguishes a perfect square from a perfect cube using prime factors?
A) Square factors occur once and cube factors twice.
B) Square factors form pairs and cube factors form triplets.
C) Both always have exactly two prime factors.
D) Both must have an odd number of factors.
Answer: B) Square factors form pairs and cube factors form triplets.
Reason: A perfect square's prime factors split into two identical groups, whereas a perfect cube's split into three identical groups.

1.15 Best Practices Opportunity List from the Chapter: (10 Ideas)

"Best Practices" Opportunities for Teaching:
Chapter 1: A Square and a Cube
Based on the chapter's emphasis on **patterns, factor reasoning, visual models, prime factorisation, square/cube roots, and puzzles**, the following best practices can make learning more conceptual and engaging.

(1) Begin with the Locker Puzzle – Inquiry-Based Learning: Introduce the chapter through the 100-locker activity and allow students to predict which lockers remain open. Guide them to connect the number of toggles with factors and finally discover that perfect-square-numbered lockers have an odd number of factors.

(2) Use Concrete and Visual Models: Let students arrange counters, square tiles, or grid paper into 1×1, 2×2, $3 \times 3, \ldots$square formations and use unit cubes to construct $2 \times 2 \times 2$, $3 \times 3 \times 3, \ldots$cubes. This helps students connect $n^2$with area and $n^3$with three-dimensional arrangements.

(3) Encourage Pattern Discovery: Ask students to investigate differences between consecutive squares and sums of consecutive odd numbers rather than simply memorising formulas. Activities such as $1 + 3 + 5 + 7 = 4^2$help them discover the relationship between odd numbers and perfect squares.

(4) **Teach Roots through Multiple Strategies:** Provide opportunities to identify square roots using known squares, successive subtraction of odd numbers, estimation, and prime factorisation. Students can then compare the methods and decide which is most efficient for a given problem.

(5) **Use Prime-Factor Grouping Activities:** Give students number cards and ask them to group prime factors into **pairs for perfect squares** and **triplets for perfect cubes**. This makes the distinction between square roots and cube roots visually clear and reinforces factorisation skills.

(6) **Promote Estimation Before Calculation:** Before calculating a square root, ask students to locate the number between two known perfect squares. For example, $225 < 250 < 256$immediately shows that $\sqrt{250}$lies between 15 and 16.

(7) **Include Error-Analysis Questions:** Present statements such as, "Every number ending in 6 is a perfect square," and ask students to accept or reject them with examples. This reinforces the chapter's important distinction that units digits can sometimes rule out perfect squares but cannot always confirm them.

(8) **Connect Mathematics with Stories and History:** Use the Hardy–Ramanujan story of **1729** to create curiosity about cubes and taxicab numbers. Students can verify $1729 = 1^3 + 12^3 = 9^3 + 10^3$and explore the next examples given in the chapter.

(9) **Use Peer Explanation and Think–Pair–Share:** After solving a square or cube problem, ask students to explain *why* their method works to a partner. This is particularly useful for factor-pair reasoning, odd-number patterns, and prime-factor grouping.

(10) **End with Challenge Puzzles:** Use the chapter's **Square Pairs** puzzle, where adjacent numbers must have a perfect-square sum, as a collaborative challenge. Such activities strengthen logical reasoning, number sense, and problem-solving beyond routine computation.

Recommended Classroom Practice:

A strong teaching sequence for this chapter is **Explore → Observe Pattern → Discuss → Form Rule → Practise → Apply → Challenge**. This keeps squares, cubes, and roots connected to reasoning and discovery rather than treating them only as facts to memorise.

1.16 Innovations Opportunity List from the Chapter: (10 Ideas)

"Innovations" Opportunities in Teaching–Learning:

Chapter 1: A Square and a Cube – Adapted for 5th Standard:

Since the chapter uses **number patterns, square arrangements, cubes, puzzles, and hands-on reasoning**, these concepts can be simplified into playful and activity-based learning for Grade 5 students.

(1) **Square Builder Lab:** Give students buttons, seeds, counters, or bottle caps and challenge them to create 1×1, 2×2, 3×3, 4×4, and 5×5arrangements. Students photograph or draw each arrangement and create their own **"Square Number Discovery Chart"** showing 1, 4, 9, 16, 25, etc.

(2) **Human Locker Game:** Convert the chapter's locker puzzle into a classroom game by assigning numbers to students or cards and carrying out simplified rounds of "toggle/open-close." Children can observe which numbers remain open and discover the connection with square numbers through play.

(3) **Odd-Number Staircase:** Use coloured paper strips, blocks, or a digital board to visually build 1, $1 + 3$, $1 + 3 + 5$, $1 + 3 + 5 + 7$, etc. Students discover that adding

successive odd numbers produces **1, 4, 9, 16, 25…**, making the square-number pattern visible rather than merely memorised.

(4) Build-a-Cube Challenge: Provide interlocking cubes, clay blocks, or cardboard unit cubes and ask groups to construct $2 \times 2 \times 2$, $3 \times 3 \times 3$, and $4 \times 4 \times 4$models. Students count layers and unit cubes to understand visually why $2^3 = 8$, $3^3 = 27$, and $4^3 = 64$.

(5) Square-or-Not Digital Quiz: Create a rapid interactive quiz in which numbers flash on the screen and students classify them as **"Square," "Not a Square," or "Need to Check."** Include numbers ending in 2, 3, 7, and 8 so learners discover how the units digit can help eliminate impossible square numbers.

(6) Math Detective – Find the Hidden Root: Set up clue cards such as "My square is 49," "My cube is 27," or "I am between $5^2$and $7^2$and I am a perfect square." Students solve the clues in teams to reveal hidden numbers, turning square and cube roots into a mathematical treasure hunt.

(7) Cube Pattern Art: Let children represent the chapter's patterns $3 + 5 = 2^3$, $7 + 9 + 11 = 3^3$, and $13 + 15 + 17 + 19 = 4^3$using coloured number cards or blocks. Groups can then design a classroom **"Amazing Cube Patterns"** display.

(8) Ramanujan 1729 Story Corner: Present the Hardy–Ramanujan story as a short mathematical story and let students verify the two combinations $1^3 + 12^3$and $9^3 + 10^3$. Students can create a mini comic strip titled **"The Amazing Number 1729."**

(9) Square Hunt Around School: Students identify real-life square objects—floor tiles, chessboard cells, windows, sticky notes, or playground markings—and measure or count their sides. They can prepare a **"Squares Around Us"** scrapbook connecting mathematical squares with everyday objects.

(10) Student-Created Math Game: Ask groups to design their own board/card game using square and cube challenges—for example, "Land on 36: say its square root," or "Land on 27: identify its cube root." Creating the game makes students both **learners and designers**, while peer play provides repeated practice.

For **5th Standard**, these innovations should emphasize **seeing, touching, building, playing, predicting, and discovering patterns**, while keeping formal prime-factorisation and complex root calculations secondary to conceptual understanding.

1.17: Practicing Questions/Question Bank

A. Fill in the Blanks – 1 Mark Each:

1. The square of 5 is ________. **Answer: 25**
2. $8^2 =$________. **Answer: 64**
3. $10^2 =$________. **Answer: 100**
4. The square root of 49 is ________. **Answer: 7**
5. The square root of 81 is ________. **Answer: 9**
6. $1 + 3 + 5 + 7 =$________. **Answer: 16**
7. The sum of the first five odd numbers is ________. **Answer: 25**
8. A perfect square cannot end in the digit ________. **Answer: 2/3/7/8 (any one)**
9. The cube of 2 is ________. **Answer: 8**
10. $3^3 =$________. **Answer: 27**

11. $4^3 =$________. **Answer: 64**
12. The cube root of 125 is ________. **Answer: 5**
13. A cube of edge 4 units contains ________ unit cubes. **Answer: 64**
14. $21 + 23 + 25 + 27 + 29 =$________. **Answer: 125**
15. The Hardy–Ramanujan Number is ________. **Answer: 1729**

B. One-Mark Descriptive Questions:

1. **What is a square number?**
   Answer: A number obtained by multiplying a number by itself is called a square number.

2. **Write the first five perfect squares.**
   Answer: 1, 4, 9, 16, and 25.

3. **What is 7^2?**
   Answer: $7^2 = 49$.

4. **What is the positive square root of 64?**
   Answer: The positive square root of 64 is 8.

5. **Which digits cannot occur in the units place of a perfect square?**
   Answer: A perfect square cannot end in 2, 3, 7, or 8.

6. **What is a cube number?**
   Answer: A number obtained by multiplying a number by itself three times is called a cube number.

7. **What is 5^3?**
   Answer: $5^3 = 125$.

8. **What is the cube root of 27?**
   Answer: The cube root of 27 is 3.

9. **Write the first four perfect cubes.**
   Answer: 1, 8, 27, and 64.

10. **Why is 1729 special?**
    Answer: It is the smallest number expressible as the sum of two positive cubes in two different ways.

C. Two-Mark Descriptive Questions:

1. **What is a perfect square? Give one example.**
   Answer:
    1. The square of a natural number is called a perfect square.
    2. For example, $6^2 = 36$, so 36 is a perfect square.

2. **Explain the odd-number pattern for square numbers.**
   Answer:
    1. Adding consecutive odd numbers beginning with 1 produces perfect squares.
    2. For example, $1 + 3 + 5 + 7 = 16 = 4^2$.

3. **What is a square root? Give an example.**
   Answer:
    1. A square root is a number which, when multiplied by itself, gives the original number.
    2. For example, $\sqrt{49} = 7$because $7 \times 7 = 49$.
4. **What is a perfect cube? Give an example.**
   Answer:
    1. A perfect cube is obtained by multiplying a natural number by itself three times.
    2. For example, $3^3 = 3 \times 3 \times 3 = 27$.
5. **Explain $4^3 = 64$using unit cubes.**
   Answer:
    1. A cube with edge 4 units has 4×4unit cubes in each layer.
    2. With four layers, it contains $4 \times 4 \times 4 = 64$unit cubes.
6. **What is a cube root? Give an example.**
   Answer:
    1. The cube root of a number is the number which, when cubed, gives the original number.
    2. For example, $\sqrt[3]{125} = 5$because $5^3 = 125$.

D. Three-Mark Descriptive Questions:

1. **Explain three important properties of perfect squares.**
   Answer:
    1. Perfect squares are obtained by multiplying natural numbers by themselves.
    2. Their units digits can only be 0, 1, 4, 5, 6, or 9.
    3. Examples are 1, 4, 9, 16, 25, and 36.
2. **Show how consecutive odd numbers produce square numbers.**
   Answer:
    1. $1 = 1^2$.
    2. $1 + 3 = 4 = 2^2$.
    3. $1 + 3 + 5 = 9 = 3^2$.
3. **Explain how to identify whether a number can be a perfect square using its units digit.**
   Answer:
    1. Observe the units digit of the given number.
    2. If it is 2, 3, 7, or 8, the number cannot be a perfect square.
    3. If it is 0, 1, 4, 5, 6, or 9, further checking is still required.
4. **Explain perfect cubes using 2, 3, and 4.**
   Answer:

1. $2^3 = 2 \times 2 \times 2 = 8$.
2. $3^3 = 3 \times 3 \times 3 = 27$.
3. $4^3 = 4 \times 4 \times 4 = 64$.

5. **Explain the cube pattern using consecutive odd numbers.**
Answer:
    1. $3 + 5 = 8 = 2^3$.
    2. $7 + 9 + 11 = 27 = 3^3$.
    3. $13 + 15 + 17 + 19 = 64 = 4^3$.

6. **Explain why 1729 is called the Hardy–Ramanujan Number.**
Answer:
    1. $1^3 + 12^3 = 1729$.
    2. $9^3 + 10^3 = 1729$.
    3. Thus, 1729 is the smallest number that is the sum of two positive cubes in two different ways.

E. Five-Mark Descriptive Questions:

1. **Explain square numbers and their patterns with suitable examples.**
Answer:
    1. A square number is obtained by multiplying a number by itself.
    2. The first few square numbers are 1, 4, 9, 16, and 25.
    3. Consecutive square numbers can be formed by adding successive odd numbers.
    4. For example, $1 + 3 + 5 + 7 = 16 = 4^2$.
    5. Thus, square numbers show a clear relationship with consecutive odd numbers.

2. **Explain square roots with suitable examples.**
Answer:
    1. A square root is the inverse idea of squaring a number.
    2. If $y = x^2$, then $x$is a square root of y.
    3. For example, $7^2 = 49$, so the positive square root of 49 is 7.
    4. Similarly, $9^2 = 81$, so $\sqrt{81} = 9$.
    5. Square roots help us find the side length of a square when its area is known.

3. **Explain cubes using a three-dimensional model.**
Answer:
    1. A cube number is obtained by multiplying a number by itself three times.
    2. A $2 \times 2 \times 2$arrangement contains 8 unit cubes.
    3. A $3 \times 3 \times 3$arrangement contains 27 unit cubes.

4. A $4 \times 4 \times 4$arrangement contains 64 unit cubes.
5. Thus, $2^3 = 8$, $3^3 = 27$,and $4^3 = 64$.

4. **Compare square numbers and cube numbers with examples.**
Answer:
    1. A square number is obtained by multiplying a number by itself twice.
    2. A cube number is obtained by multiplying a number by itself three times.
    3. For example, $4^2 = 16$, while $4^3 = 64$.
    4. Square numbers can be represented using square arrangements, while cube numbers can be represented using unit cubes.
    5. Therefore, squares relate naturally to two-dimensional arrangements and cubes to three-dimensional arrangements.

5. **Describe the Hardy–Ramanujan Number and its special property.**
Answer:
    1. The Hardy–Ramanujan Number is **1729**.
    2. It is associated with mathematicians G. H. Hardy and Srinivasa Ramanujan.
    3. $1729 = 1^3 + 12^3$.
    4. It can also be written as $1729 = 9^3 + 10^3$.
    5. Hence, 1729 is the smallest number expressible as the sum of two positive cubes in two different ways.

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8th Standard Maths 1 Book Chapter 2

Chapter 2: POWER PLAY

2.1 Summary of the Chapter:

Chapter 2: Power Play — Summary:

(1) Exponential Growth: The chapter introduces exponential or multiplicative growth through the example of repeatedly folding a sheet of paper. Since its thickness **doubles after every fold**, a very small initial thickness can become extraordinarily large after many folds.

(2) Power of Multiplicative Growth: Repeated doubling produces much faster growth than ordinary addition. For example, after every 10 additional folds, the paper's thickness becomes **1,024 times** the earlier thickness.

(3) Exponential Notation: Repeated multiplication of the same number can be written compactly using powers. In general, n^ameans that $n$is **multiplied by itself** a**times**, where $n$is the **base** and $a$is the **exponent or power**.

(4) Prime Factorisation Using Powers: Numbers can be expressed as products of their prime factors using exponential notation. For example, repeated occurrences of the same prime factor can be grouped and represented using an exponent.

(5) Laws of Exponents: When powers having the same base are multiplied, their exponents are added:

$$n^a \times n^b = n^{a+b}.$$

Also, a power raised to another power follows $(n^a)^b = n^{ab}$.

(6) Powers with the Same Exponent: When two numbers raised to the same power are multiplied, they can be combined as

$$m^a \times n^a = (mn)^a.$$

Similar ideas are developed for division.

(7) Powers and Combinations: Exponents are useful for counting large numbers of possible combinations. For example, a **5-digit password** with 10 choices for each digit has $10^5 = 1{,}00{,}000$possible combinations.

(8) Division of Powers: When powers having the same non-zero base are divided, their exponents are subtracted:

$$n^a \div n^b = n^{a-b}.$$

This law helps extend the idea of powers to zero and negative exponents.

(9) Zero and Negative Exponents: For every non-zero number x, $x^0 = 1$. Negative powers represent reciprocals, so

$$n^{-a} = \frac{1}{n^a}.$$

(10) Powers of 10: Powers of 10 provide a convenient way of expressing place value for both whole numbers and decimals. Positive powers represent larger place values, while negative powers represent decimal place values.

(11) Scientific Notation: Very large or very small numbers can be written conveniently in **scientific form** as

$$x \times 10^y,\ 1 \le x < 10,$$

where $x$is the coefficient and $y$is an integer exponent.

(12) Linear vs. Exponential Growth and Estimation: The chapter distinguishes **linear growth**, which is additive, from **exponential growth**, which is multiplicative. It also develops mathematical thinking through **guessing, reasonable assumptions, estimation, approximation, and real-life modelling**.

2.2 Important Formulas:

Important Formulas – Chapter 2: Power Play:

The following formulas and mathematical relations are important for solving the **quantitative problems** in Chapter 2.

1. **Basic Exponential Form**

$$\boxed{a^n = a \times a \times a \times \cdots \times a}$$

where a= **base** and n= **exponent/power**.

2. **Exponential Growth / Repeated Doubling**

$$\boxed{\text{Final quantity} = \text{Initial quantity} \times 2^n}$$

For the paper-folding example:

$$\boxed{T_n = T_0 \times 2^n}$$

where $T_0$is the initial thickness and $n$is the number of folds.

3. **Product of Powers with the Same Base**

$$\boxed{a^m \times a^n = a^{m+n}}$$

Example:

$$3^4 \times 3^3 = 3^7$$

4. **Quotient of Powers with the Same Base**

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}, \ a \neq 0$$

or

$$\boxed{a^m \div a^n = a^{m-n}}$$

5. **Power of a Power**

$$\boxed{(a^m)^n = a^{mn}}$$

The chapter also shows that

$$(a^m)^n = (a^n)^m = a^{mn}.$$

6. **Product of Powers Having the Same Exponent**

$$\boxed{a^n \times b^n = (ab)^n}$$

Example:

$$2^4 \times 3^4 = (2 \times 3)^4 = 6^4$$

7. **Quotient of Powers Having the Same Exponent**

$$\boxed{\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n}$$

where the denominator is non-zero.

8. **Zero Exponent**

$$\boxed{a^0 = 1}, \ a \neq 0$$

This follows from

$$a^m \div a^m = a^{m-m} = a^0 = 1.$$

9. **Negative Exponent**

$$\boxed{a^{-n} = \frac{1}{a^n}}, \ a \neq 0$$

and equivalently,

$$\boxed{a^n = \frac{1}{a^{-n}}}$$

10. **Powers of 10**

$$\boxed{10^n = 10 \times 10 \times \cdots \times 10}$$

For positive integer n, $10^n$is **1 followed by** n**zeros**. Negative powers give decimal place values:

$$10^{-1} = \frac{1}{10}, \; 10^{-2} = \frac{1}{100}, \; 10^{-3} = \frac{1}{1000}.$$

11. **Expanded Form Using Powers of 10**

For a whole number:

$$\boxed{N = a_n 10^n + a_{n-1} 10^{n-1} + \cdots + a_1 10^1 + a_0 10^0}$$

For example:

$$47561 = 4(10^4) + 7(10^3) + 5(10^2) + 6(10^1) + 1(10^0).$$

12. **Scientific / Standard Form**

$$\boxed{N = a \times 10^n}$$

where

$$\boxed{1 \le a < 10}$$

and $n$is an integer.

Example:

$$5900 = 5.9 \times 10^3$$
$$20{,}800 = 2.08 \times 10^4$$

13. **Counting Combinations – Multiplication Principle**

If there are $a$choices for one stage, $b$choices for another, and $c$choices for a third:

$$\boxed{\text{Total combinations} = a \times b \times c}$$

For $n$positions with $r$choices at every position:

$$\boxed{\text{Number of combinations} = r^n}$$

For example, a 5-digit password using digits 0–9 has:

$$10^5 = \boxed{1{,}00{,}000}$$

possible passwords.

14. **Linear Growth**

$$\boxed{\text{Final quantity} = \text{Initial quantity} + n \times (\text{fixed increase})}$$

The chapter contrasts this **additive growth** with exponential growth, which is multiplicative.

Quick Formula Box

$$\boxed{\begin{aligned} a^m \times a^n &= a^{m+n} \\ a^m \div a^n &= a^{m-n} \\ (a^m)^n &= a^{mn} \\ a^n b^n &= (ab)^n \\ \frac{a^n}{b^n} &= \left(\frac{a}{b}\right)^n \\ a^0 &= 1 \\ a^{-n} &= \frac{1}{a^n} \\ \text{Exponential growth} &= \text{Initial value} \times r^n \\ \text{Scientific form} &= a \times 10^n, \; 1 \le a < 10 \\ \text{Equal-choice combinations} &= r^n \end{aligned}}$$

2.3 Fill-Up the Blank LOTS type questions with Answers: (35 Questions)

LOTS – Fill in the Blanks with Answers:

1. The thickness of a sheet of paper _________ after every fold.
   Answer: doubles

2. Multiplicative growth is also called ________ growth.
   Answer: exponential
3. After any three folds, the thickness of paper increases ________ times.
   Answer: 8
4. $2^{10}$is equal to ________.
   Answer: 1024
5. In 5^4, the number 5 is called the ________.
   Answer: base
6. In 5^4, the number 4 is called the ________.
   Answer: exponent (or power)
7. $4 \times 4 \times 4$can be written in exponential form as ________.
   Answer: 4^3
8. $n \times n \times n$is written as ________.
   Answer: n^3
9. $a \times a \times a \times b \times b$can be written as ________.
   Answer: $a^3 b^2$
10. When powers having the same base are multiplied, their exponents are ________.
    Answer: added
11. $n^a \times n^b =$________.
    Answer: n^{a+b}
12. $p^4 \times p^6 =$________.
    Answer: p^{10}
13. A power raised to another power follows the rule $(n^a)^b =$________.
    Answer: n^{ab}
14. $m^a \times n^a =$________.
    Answer: $(mn)^a$
15. A 2-digit lock using the digits 0 to 9 has ________ possible passwords.
    Answer: 100
16. A 3-digit lock using the digits 0 to 9 has ________ possible passwords.
    Answer: 1000
17. A 5-digit lock using the digits 0 to 9 has ________ possible passwords.
    Answer: $10^5 = 1{,}00{,}000$
18. When powers having the same base are divided, their exponents are ________.
    Answer: subtracted
19. $n^a \div n^b =$________, for $n \neq 0$.
    Answer: n^{a-b}
20. For every non-zero number x, $x^0 =$________.
    Answer: 1
21. $2^{-1} =$________.
    Answer: $\frac{1}{2}$
22. $n^{-a} =$________, where $n \neq 0$.
    Answer: $\frac{1}{n^a}$
23. $10^{-3} =$________.
    Answer: $\frac{1}{1000}$
24. The number 47561can be expressed using powers of ________ in its expanded form.
    Answer: 10
25. In scientific notation, a number is written in the form ________.
    Answer: $x \times 10^y$

26. In scientific notation $x \times 10^y$, the coefficient $x$is greater than or equal to ________ and less than 10.
    Answer: 1
27. 5900in scientific notation is ________.
    Answer: 5.9×10^3
28. 80,00,000in scientific notation is ________.
    Answer: 8×10^6
29. A lakh can be expressed as the power of 10 ________.
    Answer: 10^5
30. A crore can be expressed as the power of 10 ________.
    Answer: 10^7
31. A million is equal to ________ in exponential notation.
    Answer: 10^6
32. A billion is equal to ________ in exponential notation.
    Answer: 10^9
33. Growth involving a fixed additive increase is called ________ growth.
    Answer: linear
34. Linear growth is ________, whereas exponential growth is multiplicative.
    Answer: additive
35. The chapter uses the repeated folding of paper to demonstrate the power of ________ growth.
    Answer: exponential

2.4 Fill-Up the Blank HOTS type questions with Answers: (30 Questions)

HOTS – Fill in the Blanks with Answers:

1. If the thickness of a paper doubles after every fold, then after 6 folds it becomes ________ times its original thickness.
   Answer: $2^6 = 64$times
2. If a paper is initially 0.001 cm thick, after 10 folds its thickness will be ________ cm.
   Answer: $0.001 \times 2^{10} = 1.024$cm
3. If a quantity doubles every day and its value today is 320, its value yesterday was ________.
   Answer: 160
4. A magical pond becomes completely covered with lotuses on the 30th day. If the number of lotuses doubles daily, the pond was half covered on the ________ day.
   Answer: 29th day
5. If $2^5 \times 2^7 = 2^x$, then the value of $x$is ________.
   Answer: 12
6. If $3^4 \times 3^x = 3^{10}$, then $x =$________.
   Answer: 6
7. If $p^4 \times p^6 = p^x$, then $x =$________.
   Answer: 10
8. If $5^{12} \div 5^7 = 5^x$, then $x =$________.
   Answer: 5
9. If $7^9 \div 7^4 = 7^x$, then the value of $x$is ________.
   Answer: 5
10. The expression $(4^3)^2$can be written as 4—.
    Answer: 6

11. If $(2^3)^4 = 2^x$, then $x =$________.
Answer: 12
12. If $3^5 \times 2^5$is written as a single power, it becomes ________.
Answer: 6^5
13. If $10^8 \div 10^3 = 10^x$, then $x =$________.
Answer: 5
14. If $a^7 \div a^7 = a^x$, where $a \neq 0$, then $x =$________.
Answer: 0
15. Since $5^0 = 1$, the value of $125 \times 5^0$is ________.
Answer: 125
16. If $2^{-4} = \frac{1}{2^x}$, then $x =$________.
Answer: 4
17. The reciprocal of 7^3can be expressed using a negative exponent as ________.
Answer: 7^{-3}
18. If $3^{-2} \times 3^5 = 3^x$, then $x =$________.
Answer: 3
19. If $2^{-4} \times 2^7 = 2^x$, then $x =$________.
Answer: 3
20. A 4-digit lock in which every position can contain any digit from 0 to 9 has ________ possible passwords.
Answer: $10^4 = 10{,}000$
21. If a lock has 6 slots and each slot can contain any one of the 26 English letters, the number of possible passwords is ________.
Answer: 26^6
22. Roxie has 7 dresses, 2 hats and 3 pairs of shoes. The total number of different dress combinations is ________.
Answer: $7 \times 2 \times 3 = 42$
23. If one more independent choice of 4 bags is added to 42 dress combinations, the new number of combinations becomes ________.
Answer: $42 \times 4 = 168$
24. The number 63,74,000written in scientific notation is ________.
Answer: 6.374×10^6
25. If $5.9 \times 10^3$is written in ordinary numerical form, the number is ________.
Answer: 5900
26. If the exponent in $2 \times 10^7$is increased by 1 while the coefficient remains unchanged, the resulting number becomes ________ times the original number.
Answer: 10 times
27. Among $1.496 \times 10^{11}$and 1.4335×10^{12}, the smaller number is ________.
Answer: 1.496×10^{11}
28. If 561.903is expanded using powers of 10, the digit 9 contributes ________.
Answer: 9×10^{-1}
29. If a lakh is $10^5$and a crore is 10^7, then one crore is ________ times one lakh.
Answer: $10^2 = 100$times
30. If a million is $10^6$and a billion is 10^9, then a billion is ________ times a million.
Answer: $10^3 = 1000$times
31. If a quantity increases from 20 to 40 to 60 to 80, the pattern represents ________ growth.
Answer: linear

32. If a quantity increases from 5 to 10 to 20 to 40, the pattern represents ________ growth.
    Answer: exponential
33. A ladder gains 20 cm in height with every additional step. This is an example of ________ growth because an equal amount is repeatedly added.
    Answer: linear
34. Paper thickness doubles with every fold. This is exponential rather than linear growth because the quantity is repeatedly ________ by 2.
    Answer: multiplied
35. The chapter compares about 192 crore 20 lakh ladder steps with only 46 paper folds to illustrate that ________ growth can become much faster than linear growth.
    Answer: exponential

2.5 One-mark questions with Answers of LOTS Type: (35 Questions)

One-Mark Descriptive Questions with Answers — LOTS Type:

1. **What is exponential growth?**
   Answer: Growth in which a quantity increases by repeated multiplication by a fixed factor is called exponential growth.
2. **What happens to the thickness of a sheet of paper after every fold?**
   Answer: The thickness of the sheet of paper doubles after every fold.
3. **What is exponential notation?**
   Answer: Exponential notation is a short way of representing repeated multiplication of the same number.
4. **What does n^a represent?**
   Answer: n^a represents n multiplied by itself a times.
5. **What is the base in 5^4?**
   Answer: The base in 5^4 is **5**.
6. **What is the exponent in 5^4?**
   Answer: The exponent in 5^4 is **4**.
7. **How is $n \times n$ written in exponential form?**
   Answer: $n \times n$ is written as n^2.
8. **How is $n \times n \times n$ written in exponential form?**
   Answer: $n \times n \times n$ is written as n^3.
9. **What is the value of 2^{10}?**
   Answer: The value of 2^{10} is **1024**.
10. **State the product rule for powers having the same base.**
    Answer: The product rule is $n^a \times n^b = n^{a+b}$.
11. **Simplify $p^4 \times p^6$.**
    Answer: $p^4 \times p^6 = p^{10}$.
12. **State the power-of-a-power rule.**
    Answer: The power-of-a-power rule is $(n^a)^b = n^{ab}$.
13. **State the rule for multiplying powers with the same exponent.**
    Answer: The rule is $m^a \times n^a = (mn)^a$.
14. **How many passwords are possible in a 2-digit lock using digits 0 to 9?**
    Answer: A 2-digit lock using digits 0 to 9 has **100 possible passwords**.
15. **How many passwords are possible in a 5-digit lock using digits 0 to 9?**
    Answer: A 5-digit lock has 10^5 =**1,00,000 possible passwords**.
16. **State the quotient rule for powers having the same base.**
    Answer: The quotient rule is $n^a \div n^b = n^{a-b}$, where $n \neq 0$.

17. **What is the value of x^0, where $x \neq 0$?**
Answer: The value of x^0 is **1**.
18. **What does a negative exponent represent?**
Answer: A negative exponent represents the reciprocal of the corresponding positive power.
19. **Write n^{-a} using a positive exponent.**
Answer: $n^{-a} = \frac{1}{n^a}$, where $n \neq 0$.
20. **What is 10^{-3} in fractional form?**
Answer: $10^{-3} = \frac{1}{1000}$.
21. **What is meant by scientific notation?**
Answer: Scientific notation expresses a number in the form $x \times 10^y$, where $1 \leq x < 10$ and y is an integer.
22. **Write 5900 in scientific notation.**
Answer: $5900 = 5.9 \times 10^3$.
23. **Write 80,00,000 in scientific notation.**
Answer: $80{,}00{,}000 = 8 \times 10^6$.
24. **What is a lakh in terms of a power of 10?**
Answer: One lakh is 10^5 or **1,00,000**.
25. **What is a crore in terms of a power of 10?**
Answer: One crore is 10^7 or **1,00,00,000**.
26. **What is one million in terms of a power of 10?**
Answer: One million is 10^6 or **1,000,000**.
27. **What is one billion in terms of a power of 10?**
Answer: One billion is 10^9 or **1,000,000,000**.
28. **What is linear growth?**
Answer: Linear growth is growth in which a fixed quantity is added repeatedly.
29. **How is linear growth different from exponential growth?**
Answer: Linear growth is **additive**, whereas exponential growth is **multiplicative**.
30. **Which example in the chapter illustrates linear growth?**
Answer: The ladder with steps placed at a fixed distance of **20 cm** illustrates linear growth.
31. **Which example in the chapter illustrates exponential growth?**
Answer: The repeated folding of a sheet of paper illustrates exponential growth.
32. **How many paper folds are used in the chapter to illustrate reaching beyond the Earth–Moon distance?**
Answer: The chapter uses **46 folds** to illustrate the rapid effect of exponential growth.
33. **What is estimation?**
Answer: Estimation is the process of finding a reasonably close value rather than an exact value.
34. **Why are reasonable assumptions used in estimation problems?**
Answer: Reasonable assumptions are used when some information needed to solve a problem is unavailable.
35. **What mathematical skill can be developed through regular guessing and estimation?**
Answer: Regular guessing and estimation can develop **intuition about numbers and quantities**.

2.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers — HOTS Type:

1. **A paper doubles in thickness after every fold. How many times thicker will it be after 5 folds?**
   Answer: After 5 folds, the paper will be $2^5 = 32$times its original thickness.
2. **Why does repeatedly folding paper represent exponential growth rather than linear growth?**
   Answer: It represents exponential growth because the thickness is repeatedly **multiplied by 2** instead of increasing by a fixed amount.
3. **If a paper is 0.001 cm thick initially, what will its thickness be after 3 folds?**
   Answer: Its thickness will be $0.001 \times 2^3 = 0.008$cm.
4. **If the paper is 0.016 cm thick after 4 folds, what will its thickness be after the next fold?**
   Answer: Its thickness will become $0.016 \times 2 = 0.032$cm.
5. **If $2^6 \times 2^4 = 2^x$, what is the value of x?**
   Answer: The value of $x$is **10** because exponents are added when powers with the same base are multiplied.
6. **If $3^8 \div 3^5 = 3^x$, find x.**
   Answer: $x = 3$because $3^8 \div 3^5 = 3^{8-5} = 3^3$.
7. **How can $5^3 \times 2^3$be written as a single power?**
   Answer: $5^3 \times 2^3 = (5 \times 2)^3 = 10^3$.
8. **If $(4^2)^3 = 4^x$, what is x?**
   Answer: $x = 6$because $(4^2)^3 = 4^{2\times3} = 4^6$.
9. **If $p^7 \times p^{-3} = p^x$, what is x?**
   Answer: $x = 4$because $7 + (-3) = 4$.
10. **Why is $7^5 \div 7^5$equal to 1?**
    Answer: It equals 1 because $7^5 \div 7^5 = 7^0 = 1$.
11. **How can $\frac{1}{5^3}$be expressed using a negative exponent?**
    Answer: $\frac{1}{5^3}$can be expressed as 5^{-3}.
12. **If $2^{-4}$is written using a positive exponent, what is its value?**
    Answer: $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$.
13. **A pond is completely covered with lotuses on the 30th day and the lotuses double every day; when was it half covered?**
    Answer: The pond was half covered on the **29th day** because the number of lotuses doubles the next day.
14. **If a pond contains 2^8lotuses today after daily doubling, how many did it contain yesterday?**
    Answer: It contained $2^7 = 128$lotuses yesterday.
15. **Why does increasing the exponent in $4 \times 10^6$from 6 to 7 cause a major increase?**
    Answer: Increasing the exponent by one multiplies the number by **10**.
16. **Write 65,950 in scientific notation.**
    Answer: $65{,}950 = 6.595 \times 10^4$.
17. **Which is greater, 8×10^6or 9×10^5, and why?**
    Answer: $8 \times 10^6$is greater because its power of 10 is larger.
18. **Which is smaller, 1.496×10^{11}or 1.4335×10^{12}?**
    Answer: $1.496 \times 10^{11}$is smaller because $10^{11} < 10^{12}$.
19. **If a number in scientific notation is 2.08×10^4, what is the number in ordinary form?**
    Answer: The number is **20,800**.

20. **How many times greater is one crore (10^7)than one lakh (10^5)?**
Answer: One crore is $10^{7-5} = 10^2 = 100$times greater than one lakh.
21. **How many times greater is one billion (10^9)than one million (10^6)?**
Answer: One billion is $10^{9-6} = 1000$times greater than one million.
22. **A lock has three slots with 10 choices for each slot; how many passwords are possible?**
Answer: The lock has $10^3 = 1000$possible passwords.
23. **Why does adding another digit to a decimal password lock greatly increase the number of possible passwords?**
Answer: Adding another digit multiplies the number of possible passwords by **10**.
24. **Roxie has 7 dresses, 2 hats and 3 pairs of shoes; how many different combinations can she make?**
Answer: Roxie can make $7 \times 2 \times 3 = 42$different combinations.
25. **A sequence grows as 10, 20, 40, 80; what type of growth does it show?**
Answer: It shows **exponential growth** because each term is twice the preceding term.
26. **A sequence grows as 10, 30, 50, 70; what type of growth does it show?**
Answer: It shows **linear growth** because a fixed amount of 20 is added each time.
27. **Why does a ladder with equally spaced steps illustrate linear growth?**
Answer: It illustrates linear growth because every additional step increases the height by the same fixed distance.
28. **Why can exponential growth reach a very large value much faster than linear growth?**
Answer: Exponential growth repeatedly multiplies a quantity, while linear growth repeatedly adds a fixed quantity.
29. **If a 20-cm gap is maintained between successive ladder steps, how much height is gained after 100 steps?**
Answer: The height gained is $100 \times 20 = 2000$cm or **20 m**.
30. **Why is scientific notation useful for extremely large numbers?**
Answer: Scientific notation makes extremely large numbers easier to **write, read, compare, and understand using powers of 10**.
31. **If 10^{-2}represents $\frac{1}{100}$, what does 10^{-4}represent?**
Answer: 10^{-4}represents $\frac{1}{10,000}$.
32. **If the exponent of a power of 10 decreases from 5 to 4, how does its value change?**
Answer: Its value becomes **one-tenth** of the original value.
33. **Why should we make reasonable assumptions while solving estimation problems?**
Answer: Reasonable assumptions allow us to obtain a useful approximate answer when exact information is unavailable.
34. **Why does the chapter encourage students to guess before calculating an estimate?**
Answer: Guessing before calculation helps students develop **number sense and intuition about quantities**.
35. **If your estimated answer differs from another student's answer, must one of them necessarily be wrong?**
Answer: No, different reasonable assumptions can produce different but acceptable approximate answers.

2.7 Two-mark questions with Answers of LOTS Type: (28 Questions)

Two-Mark Descriptive Questions with Answers — LOTS Type:

1. What is exponential growth? Give one example from the chapter.

Answer:
1. Growth in which a quantity increases by repeated multiplication by a fixed factor is called **exponential growth**.
2. The doubling of the thickness of paper after every fold is an example of exponential growth.

2. What are the base and exponent in 5^4?

Answer:
1. In 5^4, **5 is the base**.
2. **4 is the exponent or power**, indicating that 5 is multiplied by itself four times.

3. Explain 2^{10}using repeated multiplication and find its value.

Answer:
1. $2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
2. Therefore, $2^{10} = 1024$.

4. Express $a \times a \times a \times b \times b$in exponential form and explain it.

Answer:
1. The expression can be written as a^3b^2.
2. Here, $a$occurs three times and $b$occurs two times as factors.

5. State the rule for multiplying powers having the same base with an example.

Answer:
1. The rule is $n^a \times n^b = n^{a+b}$.
2. For example, $p^4 \times p^6 = p^{4+6} = p^{10}$.

6. State the power-of-a-power rule with an example.

Answer:
1. The power-of-a-power rule is $(n^a)^b = n^{ab}$.
2. For example, $(2^5)^2 = 2^{10}$.

7. State the rule for multiplying powers having the same exponent.

Answer:
1. Powers having the same exponent can be multiplied using $m^a \times n^a = (mn)^a$.
2. For example, $2^4 \times 3^4 = (2 \times 3)^4 = 6^4$.

8. How many combinations are possible for a 2-digit lock using digits 0–9? Explain.

Answer:
1. Each position has 10 choices, from 0 to 9.
2. Therefore, the number of combinations is $10 \times 10 = 100$.

9. How many passwords are possible for a 5-digit lock using digits 0–9?

Answer:
1. Each of the five positions has 10 possible choices.
2. Therefore, $10^5 = 1{,}00{,}000$different passwords are possible.

10. State the quotient rule for powers with the same base and give an example.

Answer:
1. For the same non-zero base, $n^a \div n^b = n^{a-b}$.
2. For example, $2^4 \div 2^3 = 2^{4-3} = 2^1 = 2$.

11. Explain the meaning of a zero exponent.

Answer:
1. For any non-zero number x, $x^0 = 1$.
2. This follows because $x^a \div x^a = x^{a-a} = x^0 = 1$.

12. What is a negative exponent? Give an example.
Answer:
1. A negative exponent represents the reciprocal of the corresponding positive power, so $n^{-a} = \frac{1}{n^a}$.
2. For example, $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$.

13. How are powers of 10 used to write a number in expanded form?
Answer:
1. Each digit is multiplied by the power of 10 corresponding to its place value.
2. For example, $47561 = (4 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$.

14. What is scientific notation? Give an example.
Answer:
1. Scientific notation expresses a number as $x \times 10^y$, where $1 \leq x < 10$and $y$is an integer.
2. For example, $5900 = 5.9 \times 10^3$.

15. Write 561.903 using powers of 10.
Answer:
1. The whole-number part is $(5 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$.
2. The decimal part is $(9 \times 10^{-1}) + (0 \times 10^{-2}) + (3 \times 10^{-3})$.

16. What is linear growth? Give an example from the chapter.
Answer:
1. **Linear growth** occurs when a fixed quantity is added repeatedly.
2. A ladder in which every step adds a fixed height of 20 cm is an example of linear growth.

17. Distinguish between linear growth and exponential growth.
Answer:
1. Linear growth is **additive**, as a fixed amount is repeatedly added.
2. Exponential growth is **multiplicative**, as a quantity is repeatedly multiplied by a fixed factor.

18. What are the values of one lakh and one crore as powers of 10?
Answer:
1. One lakh is $10^5 = 1{,}00{,}000$.
2. One crore is $10^7 = 1{,}00{,}00{,}000$.

19. What are the values of one million and one billion as powers of 10?
Answer:
1. One million is $10^6 = 1{,}000{,}000$.
2. One billion is $10^9 = 1{,}000{,}000{,}000$.

20. What steps does the chapter suggest for solving estimation and approximation problems?
Answer:
1. First, describe the relationships among the required quantities and make reasonable assumptions for unavailable information.
2. Then calculate an approximate answer and compare it with the initial guess.

21. Why are guessing and estimation useful in mathematics?
Answer:
1. Guessing provides an instinctive idea of the possible size or range of an answer before calculation.
2. Regular estimation develops better intuition about numbers and different quantities.

22. Explain the 'Magical Pond' example of exponential growth.
Answer:
1. The number of lotuses in the magical pond doubles every day.
2. Hence, if the pond is completely covered on the 30th day, it must be half covered on the 29th day.

23. What is meant by the multiplication principle in counting combinations?
Answer:
1. When a choice is made in successive independent stages, the number of possibilities at each stage is multiplied.
2. Thus, 4 dresses and 3 caps give $4 \times 3 = 12$possible combinations.

24. Why is scientific notation useful for representing large numbers?
Answer:
1. Scientific notation makes numbers containing many digits and zeroes easier to read and write.
2. It also makes the **order of magnitude** clear through the exponent of 10.

25. What happens to the thickness of paper after every 10 folds?
Answer:
1. Since the thickness doubles with every fold, 10 folds multiply the thickness by 2^{10}.
2. Therefore, after every 10 folds, the thickness becomes **1024 times** the earlier thickness.

2.8. Two-mark questions with Answers of HOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers — HOTS Type:

1. A paper is 0.001 cm thick and its thickness doubles after every fold. Find its thickness after 5 folds.
Answer:
1. Thickness after 5 folds $= 0.001 \times 2^5$cm.
2. Therefore, the thickness is $0.001 \times 32 = 0.032$ cm.

2. Why does the thickness of folded paper increase much faster than the number of folds?
Answer:
1. Each fold multiplies the previous thickness by 2 instead of adding a fixed thickness.
2. Hence, repeated folding produces **exponential growth**, which becomes very rapid.

3. Without expanding completely, simplify $3^7 \times 3^5$. Explain the rule used.
Answer:
1. Powers with the same base are multiplied by adding their exponents.
2. Therefore, $3^7 \times 3^5 = 3^{7+5} = 3^{12}$.

4. If $5^x \times 5^4 = 5^{11}$, find $x$and justify your answer.
Answer:
1. Using $5^x \times 5^4 = 5^{x+4}$, we get $x + 4 = 11$.
2. Therefore, $x = 7$.

5. Simplify $(2^3)^4$without multiplying 2 repeatedly twelve times.
Answer:
1. Using the power-of-a-power rule, $(2^3)^4 = 2^{3\times4}$.
2. Therefore, the simplified exponential form is 2^{12}.

6. Express $2^5 \times 5^5$as a single power and find its value.
Answer:
1. Since the exponents are equal, $2^5 \times 5^5 = (2 \times 5)^5 = 10^5$.
2. Therefore, its value is 1,00,000.

7. Simplify $7^9 \div 7^4$ and explain why the exponent decreases.
Answer:
1. Division cancels four common factors of 7, leaving five factors of 7.
2. Hence, $7^9 \div 7^4 = 7^{9-4} = 7^5$.

8. Why is $6^0 = 1$ rather than 0?
Answer:
1. $6^4 \div 6^4 = 1$ because any non-zero number divided by itself equals 1.
2. Using the exponent rule, $6^4 \div 6^4 = 6^{4-4} = 6^0$, so $6^0 = 1$.

9. Express 4^{-3} using a positive exponent and explain its meaning.
Answer:
1. A negative exponent represents the reciprocal of the corresponding positive power.
2. Therefore, $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$.

10. Simplify $2^{-3} \times 2^7$ and write the answer with a positive exponent.
Answer:
1. Adding the exponents gives $2^{-3} \times 2^7 = 2^{-3+7}$.
2. Therefore, the answer is $2^4 = 16$.

11. A pond is fully covered with lotuses on the 30th day, and the lotuses double every day. Explain when it was one-fourth covered.
Answer:
1. The pond was half covered on the 29th day and half of that, or one-fourth covered, on the 28th day.
2. Therefore, the pond was **one-fourth covered on the 28th day**.

12. Roxie has 7 dresses, 2 hats and 3 pairs of shoes. Find the number of different ways she can dress up.
Answer:
1. Each dress can be combined independently with any hat and any pair of shoes.
2. Therefore, the total number of combinations is $7 \times 2 \times 3 = 42$.

13. Why does a 5-digit lock have 10^5 possible passwords when digits 0–9 are allowed?
Answer:
1. Each of the five positions independently has 10 possible choices.
2. Therefore, the number of passwords is $10 \times 10 \times 10 \times 10 \times 10 = 10^5 = 1{,}00{,}000$.

14. A 5-digit lock is changed to a 6-digit lock using digits 0–9. How does the number of possible passwords change?
Answer:
1. The possibilities increase from 10^5 to 10^6.
2. Hence, the 6-digit lock has **10 times as many possible passwords**.

15. Write 34,30,000 in scientific notation and explain the position of the decimal point.
Answer:
1. Moving the decimal point six places to the left gives 3.43.
2. Therefore, $34{,}30{,}000 = 3.43 \times 10^6$.

16. Compare 1.496×10^{11} and 1.4335×10^{12} without writing them in ordinary form.
Answer:
1. The exponent 12 is greater than 11, so the second number has the greater order of magnitude.
2. Hence, $1.4335 \times 10^{12} > 1.496 \times 10^{11}$.

17. One lakh is 10^5 and one crore is 10^7. How many times larger is one crore than one lakh?
Answer:
1. The ratio is $10^7 \div 10^5 = 10^{7-5} = 10^2$.

2. Therefore, **one crore is 100 times one lakh**.

18. A sequence is 20, 40, 60, 80, while another is 20, 40, 80, 160. Identify the type of growth in each.

Answer:

1. The first sequence shows **linear growth** because 20 is added each time.
2. The second shows **exponential growth** because each term is multiplied by 2.

19. Why can 46 paper folds represent a much greater increase than 46 equally spaced ladder steps?

Answer:

1. Each ladder step adds a fixed distance, so it represents linear growth.
2. Each paper fold doubles the thickness, so exponential growth increases far more rapidly.

20. A ladder has steps 20 cm apart. How many steps are required to cover a vertical distance of 100 m?

Answer:

1. Converting 100 m into centimetres gives $100 \times 100 = 10{,}000$cm.
2. Therefore, the number of 20-cm intervals required is $10{,}000 \div 20 = 500$.

21. Why is 8×10^7greater than 9×10^6, even though 9 is greater than 8?

Answer:

1. The exponent of 10 has a major effect on the size of a number in scientific notation.
2. Since $10^7$is ten times 10^6, $8 \times 10^7 > 9 \times 10^6$.

22. Write 561.903using powers of 10 and identify why negative exponents are needed.

Answer:

1. $561.903 = 5(10^2) + 6(10^1) + 1(10^0) + 9(10^{-1}) + 0(10^{-2}) + 3(10^{-3})$.
2. Negative exponents are required to represent the **decimal place values** to the right of the decimal point.

23. A student says $3^4 + 3^2 = 3^6$. Is the student correct? Give a reason.

Answer:

1. No, the rule of adding exponents applies when powers with the same base are **multiplied**, not added.
2. Thus, $3^4 \times 3^2 = 3^6$, but $3^4 + 3^2 \neq 3^6$.

24. A student claims that $5^{-2}$is a negative number because its exponent is negative. Is the claim correct?

Answer:

1. No, a negative exponent indicates a reciprocal and does not make a positive base negative.
2. Hence, $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$, which is positive.

25. Why can two students obtain different answers to the same estimation problem and still both be reasonable?

Answer:

1. Students may make different reasonable assumptions about quantities for which exact information is unavailable.
2. Therefore, their approximate answers can differ while both remain logically acceptable.

2.9. Three-mark questions with Answers of LOTS Type: (18 Questions)

Three-Mark Descriptive Questions with Answers — LOTS Type

1. Explain exponential growth using the paper-folding example.

Answer:

1. The initial thickness of the paper considered in the chapter is **0.001 cm**.
2. Every time the paper is folded, its thickness becomes **twice** its previous thickness.
3. Thus, repeated doubling produces **multiplicative or exponential growth**.

2. Explain exponential notation with the example 5^4.

Answer:

1. Exponential notation is a short way of representing repeated multiplication of the same number.
2. 5^4means $5 \times 5 \times 5 \times 5$, where **5 is the base** and **4 is the exponent**.
3. Therefore, $5^4 = 625$.

3. Express 32400 as a product of its prime factors in exponential form.

Answer:

1. The prime factors are $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3$.
2. Grouping identical prime factors gives four 2s, two 5s and four 3s.
3. Therefore, $32400 = \boxed{2^4 \times 5^2 \times 3^4}$.

4. Explain the product rule for powers having the same base.

Answer:

1. When powers with the same base are multiplied, their exponents are added.
2. For example, $p^4 \times p^6 = p^{4+6}$.
3. Hence, in general, $\boxed{n^a \times n^b = n^{a+b}}$.

5. Explain the power-of-a-power rule with an example.

Answer:

1. When a power is raised to another power, the two exponents are multiplied.
2. For example, $(2^5)^2 = 2^{5\times2}$.
3. Hence, the general rule is $\boxed{(n^a)^b = n^{ab}}$.

6. Explain the rule for multiplying powers having the same exponent.

Answer:

1. When two powers have the same exponent, their bases can be multiplied.
2. For example, $2^4 \times 3^4 = (2 \times 3)^4$.
3. Hence, the general rule is $\boxed{m^a \times n^a = (mn)^a}$.

7. Explain how the number of passwords for a 3-digit lock is calculated.

Answer:

1. Each position of the lock can contain any digit from **0 to 9**, giving 10 choices.
2. For three positions, the total number of combinations is $10 \times 10 \times 10$.
3. Therefore, a 3-digit lock has $\boxed{1000}$possible passwords.

8. Explain how the number of possible passwords for a 5-digit lock is obtained.

Answer:

1. Each of the five positions has **10 choices**, from 0 to 9.
2. Hence, the total number of passwords is $10 \times 10 \times 10 \times 10 \times 10 = 10^5$.
3. Therefore, there are $\boxed{1{,}00{,}000}$possible passwords.

9. Explain the quotient rule for powers having the same base.

Answer:

1. When powers with the same non-zero base are divided, their exponents are subtracted.
2. For example, $2^4 \div 2^3 = 2^{4-3} = 2^1$.
3. Thus, the general rule is $\boxed{n^a \div n^b = n^{a-b}}$.

10. Explain why the zeroth power of a non-zero number is equal to 1.

Answer:

1. Dividing a non-zero power by itself gives $x^a \div x^a = 1$.
2. By the quotient rule, $x^a \div x^a = x^{a-a} = x^0$.

3. Therefore, $\boxed{x^0 = 1}$ for $x \neq 0$.

11. Explain negative exponents with an example.

Answer:

1. A negative exponent represents the reciprocal of the corresponding positive power.
2. For example, $2^{-6} = \frac{1}{2^6}$.
3. Therefore, in general, $\boxed{n^{-a} = \frac{1}{n^a}}$, where $n \neq 0$.

12. Write 561.903 using powers of 10.

Answer:

1. The whole-number part is $5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$.
2. The decimal part is $9 \times 10^{-1} + 0 \times 10^{-2} + 3 \times 10^{-3}$.
3. Therefore, $561.903 = (5 \times 10^2) + (6 \times 10^1) + (1 \times 10^0) + (9 \times 10^{-1}) + (0 \times 10^{-2}) + (3 \times 10^{-3})$.

13. Explain scientific notation and give two examples.

Answer:

1. Scientific notation represents a number as $x \times 10^y$, where $1 \leq x < 10$and $y$is an integer.
2. For example, $5900 = 5.9 \times 10^3$.
3. Similarly, $20{,}800 = 2.08 \times 10^4$.

14. Explain the 'Magical Pond' example of exponential growth.

Answer:

1. The number of lotuses in the magical pond **doubles every day**.
2. If the pond is completely covered on the 30th day, it must have been half covered on the 29th day.
3. This demonstrates how repeated doubling leads to rapid **exponential growth**.

15. Explain the difference between linear growth and exponential growth.

Answer:

1. In **linear growth**, a fixed quantity is repeatedly added.
2. In **exponential growth**, a quantity is repeatedly multiplied by a fixed factor.
3. Thus, the equally spaced ladder steps show linear growth, whereas repeated paper folding shows exponential growth.

16. Explain the steps suggested in the chapter for solving estimation problems.

Answer:

1. First, make an instinctive **guess** about the possible answer.
2. Next, identify the relationships among the quantities and make reasonable assumptions or approximations.
3. Finally, calculate the approximate answer and compare it with the original guess.

17. Explain how powers of 10 are used in the expanded form of 47561.

Answer:

1. Each digit is multiplied by a power of 10 according to its place value.
2. Thus, $47561 = (4 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$.
3. This representation expresses the place values compactly using **powers of 10**.

18. Explain the values of lakh, crore and billion using powers of 10.

Answer:

1. One lakh is $10^5 = 1{,}00{,}000$.
2. One crore is $10^7 = 1{,}00{,}00{,}000$.
3. One billion is $10^9 = 1{,}000{,}000{,}000$.

2.10. Three-mark questions with Answers of HOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers — HOTS Type:

1. A sheet of paper is 0.001 cm thick and doubles in thickness after every fold. Find its thickness after 10 folds and explain the growth.

Answer:

1. After 10 folds, thickness $= 0.001 \times 2^{10}$cm.
2. Since $2^{10} = 1024$, the thickness becomes **1.024 cm**.
3. This is **exponential growth** because the thickness is multiplied by 2 after every fold.

2. A magical pond is completely covered with lotuses on the 30th day, and the lotuses double every day. On which days was it half and one-fourth covered?

Answer:

1. Since the lotuses double each day, the pond was **half covered on the 29th day**.
2. One day earlier, on the 28th day, it was half of that amount.
3. Therefore, it was **one-fourth covered on the 28th day**.

3. Simplify $2^7 \times 2^5 \div 2^4$using the laws of exponents.

Answer:

1. First, $2^7 \times 2^5 = 2^{7+5} = 2^{12}$.
2. Then, $2^{12} \div 2^4 = 2^{12-4}$.
3. Therefore, the simplified answer is $\boxed{2^8}$.

4. If $3^x \times 3^4 = 3^{10}$, find $x$and justify your answer.

Answer:

1. Using the product rule, $3^x \times 3^4 = 3^{x+4}$.
2. Therefore, $x + 4 = 10$.
3. Hence, $\boxed{x = 6}$.

5. Simplify $(5^3)^4 \div 5^7$using exponent laws.

Answer:

1. Using the power-of-a-power rule, $(5^3)^4 = 5^{12}$.
2. Therefore, $5^{12} \div 5^7 = 5^{12-7}$.
3. Hence, the simplified expression is $\boxed{5^5}$.

6. A student writes $2^4 \times 3^4 = 6^8$. Identify the mistake and give the correct answer.

Answer:

1. The student has incorrectly added the exponents while multiplying powers with different bases but the same exponent.
2. The correct rule is $m^a \times n^a = (mn)^a$.
3. Therefore, $2^4 \times 3^4 = (2 \times 3)^4 = \boxed{6^4}$.

7. A student claims that $7^0 = 0$. Explain why the claim is incorrect.

Answer:

1. Consider $7^4 \div 7^4$, which equals 1 because a non-zero number divided by itself is 1.
2. By the quotient rule, $7^4 \div 7^4 = 7^{4-4} = 7^0$.
3. Therefore, $\boxed{7^0 = 1}$, not 0.

8. A student says that $4^{-2}$is a negative number. Examine the statement and correct it.

Answer:

1. A negative exponent does not make the value negative; it indicates a **reciprocal**.
2. Thus, $4^{-2} = \frac{1}{4^2}$.
3. Therefore, $4^{-2} = \boxed{\frac{1}{16}}$, which is positive.

9. A lock has 6 positions, and each position can contain any digit from 0 to 9. Find the number of possible passwords.

Answer:

1. Each position has 10 possible choices.
2. Hence, the total number of passwords is 10^6.
3. Therefore, $\boxed{10{,}00{,}000}$ different passwords are possible.

10. Roxie has 7 dresses, 2 hats and 3 pairs of shoes. How many different outfits can she make, and what principle is used?

Answer:

1. For every dress, Roxie can choose any of the 2 hats and any of the 3 pairs of shoes.
2. The total number of outfits is $7 \times 2 \times 3 = 42$.
3. Therefore, she can make **42 different outfits**, using the multiplication principle of counting.

11. Compare 8×10^6 and 9×10^5 without writing them in ordinary form.

Answer:

1. Compare the powers of 10 first: 10^6 is ten times 10^5.
2. Hence, the larger exponent makes 8×10^6 greater even though 8 is less than 9.
3. Therefore, $\boxed{8 \times 10^6 > 9 \times 10^5}$.

12. The distances are 1.496×10^{11} m and 1.4335×10^{12} m. Which distance is greater and why?

Answer:

1. The first number has 10^{11}, while the second has 10^{12}.
2. Since 10^{12} represents a greater order of magnitude than 10^{11}, the second distance is larger.
3. Therefore, $\boxed{1.4335 \times 10^{12}\ \text{m}}$ is the greater distance.

13. One lakh is 10^5 and one billion is 10^9. Determine how many times larger one billion is than one lakh.

Answer:

1. Divide the larger quantity by the smaller: $10^9 \div 10^5$.
2. Using the quotient rule, $10^9 \div 10^5 = 10^{9-5} = 10^4$.
3. Therefore, one billion is $\boxed{10{,}000}$ times one lakh.

14. Compare the sequences 10, 20, 30, 40 and 10, 20, 40, 80 in terms of their growth.

Answer:

1. In the first sequence, a fixed quantity of 10 is added each time, so it shows **linear growth**.
2. In the second sequence, each term is multiplied by 2, so it shows **exponential growth**.
3. Thus, exponential growth eventually increases much faster than linear growth.

15. A ladder has steps 20 cm apart. Find the number of steps required to cover 1 km and identify the type of growth involved.

Answer:

1. Since 1 km $= 100{,}000$ cm, the number of 20-cm intervals is $100{,}000 \div 20$.
2. Therefore, $\boxed{5{,}000}$ such intervals are required.
3. This illustrates **linear growth** because every interval adds the same 20 cm.

16. Why can repeated paper folding theoretically cover a huge distance with far fewer repetitions than climbing equally spaced ladder steps?

Answer:

1. Each paper fold **multiplies** the thickness by 2, producing exponential growth.
2. Each ladder step adds only a fixed distance, producing linear growth.
3. Hence, exponential growth reaches very large quantities far faster than linear growth.

17. Convert 65,950 into scientific notation and explain the process.
Answer:
1. Move the decimal point four places to the left to obtain 6.595.
2. Compensate for this movement by multiplying by 10^4.
3. Therefore, $65{,}950 = \boxed{6.595 \times 10^4}$.

18. A student writes $3^5 + 3^2 = 3^7$. Analyse the error and state the correct result.
Answer:
1. The rule $3^a \times 3^b = 3^{a+b}$ applies to **multiplication**, not addition.
2. Here, $3^5 + 3^2 = 243 + 9$.
3. Therefore, the correct value is $\boxed{252}$, not 3^7.

19. Two students obtain different estimates for the same real-life problem. Can both estimates be acceptable? Explain.
Answer:
1. Estimation problems may require students to make assumptions when exact information is unavailable.
2. Different students may choose different but reasonable assumptions and approximations.
3. Hence, both estimates may be acceptable if their assumptions and calculations are reasonable.

20. A quantity triples at every stage. Starting with 1, find its value after 4 stages and identify the growth pattern.
Answer:
1. Repeated tripling for four stages can be represented as 3^4.
2. Since $3^4 = 3 \times 3 \times 3 \times 3 = 81$, the quantity becomes **81**.
3. The pattern represents **exponential growth** because the quantity is repeatedly multiplied by 3.

2.11 Five-mark questions with Answers of LOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers — LOTS Type:
Each answer is given in five clear stages/points.

1. Explain the paper-folding activity and how it demonstrates exponential growth.
Answer:
1. The chapter considers a sheet of paper having an initial thickness of **0.001 cm**.
2. After every fold, the thickness of the paper becomes **twice** its previous thickness.
3. Thus, after n folds, the thickness can be represented as 0.001×2^n cm.
4. For example, after 10 folds, the thickness becomes $0.001 \times 2^{10} = 1.024$ cm.
5. Since the thickness increases by repeated multiplication rather than repeated addition, it is called **exponential or multiplicative growth**.

2. Explain exponential notation, base and exponent with suitable examples.
Answer:
1. Exponential notation is a compact method of representing the repeated multiplication of the same number.
2. In general, n^a means that n is multiplied by itself a times.
3. In 5^4, **5 is the base** and **4 is the exponent**.
4. Thus, $5^4 = 5 \times 5 \times 5 \times 5$.
5. Therefore, $5^4 = 625$, showing how exponential notation simplifies repeated multiplication.

3. Explain the important laws of exponents for multiplication and powers.
Answer:

1. When powers with the same base are multiplied, their exponents are added: $n^a \times n^b = n^{a+b}$.
2. For example, $2^3 \times 2^4 = 2^{3+4} = 2^7$.
3. When a power is raised to another power, the exponents are multiplied: $(n^a)^b = n^{ab}$.
4. For example, $(3^2)^4 = 3^{2\times 4} = 3^8$.
5. When powers have the same exponent, their bases can be multiplied: $m^a \times n^a = (mn)^a$.

4. Explain the quotient rule, zero exponent and negative exponents.

Answer:

1. When powers having the same non-zero base are divided, their exponents are subtracted: $n^a \div n^b = n^{a-b}$.
2. For example, $2^7 \div 2^4 = 2^{7-4} = 2^3$.
3. When the exponents are equal, $n^a \div n^a = n^0 = 1$, so $n^0 = 1$for $n \neq 0$.
4. A negative exponent represents the reciprocal of the corresponding positive power.
5. Therefore, $n^{-a} = \frac{1}{n^a}$; for example, $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$.

5. Explain how powers of 10 are used to represent whole numbers and decimal numbers.

Answer:

1. Powers of 10 can be used to express the place value of every digit in a number.
2. For example, $47561 = (4 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$.
3. The places to the right of the decimal point are represented using **negative powers of 10**.
4. Thus, $561.903 = (5 \times 10^2) + (6 \times 10^1) + (1 \times 10^0) + (9 \times 10^{-1}) + (0 \times 10^{-2}) + (3 \times 10^{-3})$.
5. Hence, positive, zero and negative powers of 10 provide a compact way of representing place values.

6. Explain scientific notation with suitable examples.

Answer:

1. Scientific notation is a convenient way of expressing very large or very small numbers using powers of 10.
2. A number in scientific notation is written as $x \times 10^y$, where $1 \leq x < 10$and $y$is an integer.
3. For example, $5900$is written as 5.9×10^3.
4. Similarly, $20{,}800$is written as 2.08×10^4, and $80{,}00{,}000$as 8×10^6.
5. The exponent of 10 helps indicate the magnitude of the number and makes large numbers easier to represent and compare.

7. Explain how combinations and powers are connected using the password example.

Answer:

1. If a password uses digits 0–9, each position has **10 possible choices**.
2. A 2-digit password therefore has $10 \times 10 = 10^2 = 100$possibilities.
3. A 3-digit password has $10 \times 10 \times 10 = 10^3 = 1000$possibilities.
4. Similarly, a 5-digit password has $10^5 = 1{,}00{,}000$possibilities.
5. Thus, powers provide a convenient method for counting possibilities when the same number of choices is available at each position.

8. Explain the Magical Pond example and its connection with exponential growth.

Answer:

1. In the Magical Pond example, the number of lotuses **doubles every day**.
2. The pond becomes completely covered with lotuses on the **30th day**.
3. Therefore, it must have been half covered on the **29th day**.

4. Going backwards another day, it would have been one-fourth covered on the **28th day**.
5. This demonstrates exponential growth because the quantity changes by repeated multiplication by 2.

9. Explain the difference between linear growth and exponential growth using examples from the chapter.

Answer:

1. **Linear growth** occurs when the same fixed quantity is repeatedly added.
2. A ladder whose successive steps are separated by a fixed distance of **20 cm** illustrates linear growth.
3. **Exponential growth** occurs when a quantity is repeatedly multiplied by the same factor.
4. Repeated folding of paper illustrates exponential growth because its thickness doubles with every fold.
5. Therefore, exponential growth becomes much faster than linear growth as the number of stages increases.

10. Explain the process of estimation suggested in the chapter.

Answer:

1. First, make an initial **guess** about the possible value of the required quantity.
2. Identify the relationships between the quantities involved in the problem.
3. Make **reasonable assumptions and approximations** when exact information is not available.
4. Use these assumptions to calculate a reasonable approximate answer.
5. Finally, compare the calculated estimate with the initial guess to develop better intuition about numbers and quantities.

11. Explain the multiplication principle of counting with an example involving dresses, caps and shoes.

Answer:

1. When several independent choices are made, the total number of combinations is obtained by multiplying the number of choices at each stage.
2. For example, suppose there are **7 dresses, 2 hats and 3 pairs of shoes**.
3. For each dress, there are 2 possible choices of hat.
4. For every dress-hat combination, there are 3 possible choices of shoes.
5. Therefore, the total number of combinations is $7 \times 2 \times 3 = \boxed{42}$.

12. Explain how Indian and international large-number units can be represented using powers of 10.

Answer:

1. **One lakh** is $1{,}00{,}000 = 10^5$.
2. **One million** is $1{,}000{,}000 = 10^6$.
3. **One crore** is $1{,}00{,}00{,}000 = 10^7$.
4. **One arab** and **one billion** are each represented by 10^9.
5. Expressing these quantities as powers of 10 makes it easier to understand and compare very large numbers.

2.12 Five-mark questions with Answers of HOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers — HOTS Type:

Each answer is given in five clear sentences/points.

1. A sheet of paper is 0.001 cm thick and doubles in thickness after every fold. Find its thickness after 15 folds and explain why this represents exponential growth.

Answer:

1. The initial thickness of the paper is **0.001 cm**, and each fold doubles its thickness.
2. After 15 folds, thickness $= 0.001 \times 2^{15}$cm.
3. Since $2^{15} = 32{,}768$, the thickness becomes $0.001 \times 32{,}768 = 32.768$cm.
4. The thickness does not increase by a fixed amount; instead, each new value is twice the previous value.
5. Hence, paper folding demonstrates **exponential growth**, where repeated multiplication produces rapid growth.

2. A magical pond is completely covered with lotuses on the 30th day, and the number of lotuses doubles every day. Determine when it was half, one-fourth and one-eighth covered, and explain your reasoning.

Answer:

1. Since the lotuses double daily, the pond must have been **half covered on the 29th day**.
2. Going back one more day halves the amount again, so it was **one-fourth covered on the 28th day**.
3. Going back another day, it was **one-eighth covered on the 27th day**.
4. Thus, moving backwards by one day repeatedly divides the covered area by 2.
5. This demonstrates the rapid change produced by **exponential growth**.

3. Simplify $\frac{(2^3)^4 \times 2^5}{2^7}$ using the laws of exponents and explain each step.

Answer:

1. Using the power-of-a-power rule, $(2^3)^4 = 2^{3\times4} = 2^{12}$.
2. Multiplying powers with the same base gives $2^{12} \times 2^5 = 2^{17}$.
3. Dividing by 2^7 gives $2^{17} \div 2^7 = 2^{17-7}$.
4. Therefore, the expression simplifies to 2^{10}.
5. Since $2^{10} = 1024$, the value of the expression is **1024**.

4. A student claims that $3^4 \times 5^4 = 15^8$. Analyse the claim and obtain the correct answer.

Answer:

1. The two powers 3^4 and 5^4 have **different bases but the same exponent**.
2. For equal exponents, the correct rule is $m^a \times n^a = (mn)^a$.
3. Therefore, $3^4 \times 5^4 = (3 \times 5)^4$.
4. Hence, the correct simplified expression is 15^4, not 15^8.
5. The student's mistake was **adding the exponents**, a rule that applies when multiplying powers with the same base.

5. A student says that 5^{-3} is a negative number because its exponent is negative. Examine the statement and find the correct value.

Answer:

1. A negative exponent does not mean that the value of the power is negative.
2. It indicates the **reciprocal** of the corresponding positive power.
3. Therefore, $5^{-3} = \frac{1}{5^3}$.
4. Since $5^3 = 125$, we get $5^{-3} = \frac{1}{125}$.
5. Hence, the student's statement is incorrect because $\frac{1}{125}$ is a **positive number**.

6. A 5-digit password uses digits 0–9. If one more digit is added to the password, analyse how the number of possible passwords changes.

Answer:

1. Each position can contain any of the 10 digits from 0 to 9.
2. A 5-digit password therefore has $10^5 = 1{,}00{,}000$ possibilities.
3. A 6-digit password has $10^6 = 10{,}00{,}000$ possibilities.
4. The ratio is $10^6 \div 10^5 = 10$.

5. Therefore, adding just one position makes the number of possible passwords **10 times greater**.

7. Compare 8×10^6 and 9×10^5 and explain why comparing only the first numbers can lead to a wrong conclusion.

Answer:

1. Looking only at the coefficients may suggest that 9 is greater than 8.
2. However, the powers of 10 must first be considered because 10^6 is ten times 10^5.
3. We can write $8 \times 10^6 = 80 \times 10^5$.
4. Since $80 \times 10^5 > 9 \times 10^5$, we have $8 \times 10^6 > 9 \times 10^5$.
5. Thus, when comparing numbers in scientific notation, the **exponents should be compared before the coefficients** when the exponents differ.

8. Compare a ladder with steps 20 cm apart and a paper whose thickness doubles after every fold. Explain why their growth patterns are different.

Answer:

1. Every additional ladder step increases the height by the same fixed amount of **20 cm**.
2. Therefore, the height of the ladder shows **linear or additive growth**.
3. In paper folding, every fold makes the paper twice as thick as before.
4. Therefore, paper thickness shows **exponential or multiplicative growth**.
5. Exponential growth eventually becomes much faster than linear growth because it repeatedly multiplies the existing quantity.

9. A student writes $7^5 \div 7^5 = 7$. Analyse the error and explain why the correct answer is 1.

Answer:

1. The quotient rule states that $n^a \div n^b = n^{a-b}$ for a non-zero base.
2. Therefore, $7^5 \div 7^5 = 7^{5-5}$.
3. This gives 7^0.
4. Since any non-zero number divided by itself is 1, $7^0 = 1$.
5. Hence, the correct answer is **1**, and this also explains the zero-exponent rule.

10. One lakh is 10^5, one crore is 10^7, and one billion is 10^9. Compare these quantities using powers of 10.

Answer:

1. One crore compared with one lakh gives $10^7 \div 10^5 = 10^2 = 100$.
2. Therefore, **one crore is 100 times one lakh**.
3. One billion compared with one crore gives $10^9 \div 10^7 = 10^2 = 100$.
4. Therefore, **one billion is 100 times one crore**.
5. Powers of 10 make comparison of very large quantities easier by allowing their exponents to be compared.

11. Two students obtain different answers while estimating the value of objects used in a Tulabhara. Explain how both estimates could still be reasonable.

Answer:

1. Estimation problems may not provide every quantity needed for an exact calculation.
2. Each student may therefore make different **reasonable assumptions**, such as the person's weight or the price per kilogram.
3. They can then use the relationship between weight and cost to calculate an approximate value.
4. Different reasonable assumptions naturally produce somewhat different numerical estimates.
5. Hence, both answers can be acceptable if the **assumptions, method and calculations are reasonable**.

12. A student claims that $2^5 + 2^3 = 2^8$. Examine the claim and give the correct result.
Answer:
1. The rule $n^a \times n^b = n^{a+b}$ applies only when powers with the same base are **multiplied**.
2. The given expression contains addition, so the exponents cannot simply be added.
3. We calculate $2^5 = 32$ and $2^3 = 8$.
4. Therefore, $2^5 + 2^3 = 32 + 8 = 40$.
5. Hence, the student's claim is incorrect, and the correct answer is **40**.

2.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers — Applied Type:
1. A sheet of paper is 0.001 cm thick. If its thickness doubles after every fold, calculate its thickness after 12 folds and explain the application of exponential growth.
Answer:
1. The initial thickness of the paper is **0.001 cm**, and every fold doubles the thickness.
2. After 12 folds, thickness $= 0.001 \times 2^{12}$ cm.
3. Since $2^{12} = 4096$, the thickness becomes $0.001 \times 4096 = 4.096$ cm.
4. Each new thickness is obtained by multiplying the previous thickness by 2 rather than adding a fixed amount.
5. Hence, paper folding is a practical illustration of **exponential growth**, showing how repeated multiplication can rapidly produce a large value.

2. A mobile phone uses a 6-digit PIN in which each position can contain any digit from 0 to 9. Find the total number of possible PINs and compare it with a 5-digit PIN.
Answer:
1. Each position of the PIN has **10 possible choices**, from 0 to 9.
2. A 6-digit PIN therefore has $10^6 = 10{,}00{,}000$ possible combinations.
3. A 5-digit PIN has $10^5 = 1{,}00{,}000$ possible combinations.
4. Since $10^6 \div 10^5 = 10$, the 6-digit PIN has **10 times as many possibilities**.
5. This shows how powers can be applied to count possibilities when the same number of choices is available at every position.

3. A water plant doubles the area it covers every day. If a pond is completely covered on the 24th day, determine when it was half, one-fourth and one-eighth covered.
Answer:
1. Since the covered area doubles daily, it must have been **half covered on the 23rd day**.
2. One day earlier, on the 22nd day, it was **one-fourth covered**.
3. On the 21st day, it was **one-eighth covered**.
4. Moving backwards one day repeatedly divides the covered area by 2.
5. This real-life situation applies the idea of **exponential growth through repeated doubling**.

4. A student has 5 shirts, 4 pairs of trousers and 3 pairs of shoes. How many different outfits can the student make? Explain the method.
Answer:
1. For each of the **5 shirts**, the student can choose any of the 4 pairs of trousers.
2. For each shirt-trouser combination, any of the **3 pairs of shoes** can be selected.
3. Using the multiplication principle, the number of outfits is $5 \times 4 \times 3$.
4. Therefore, the student can make 60 different outfits.
5. This applies the chapter's method of multiplying the number of choices at successive stages to find the total number of combinations.

5. The population of a species is approximately 3,450,000. Express it in scientific notation and explain why this form is useful.

Answer:

1. The given population is **3,450,000**.
2. Moving the decimal point six places to the left gives **3.45**.
3. Therefore, $3{,}450{,}000 = 3.45 \times 10^6$.
4. The exponent 6 indicates the order of magnitude of the population.
5. Scientific notation makes large quantities easier to **write, read and compare**.

6. A staircase has steps that raise a person by 20 cm each, while another quantity doubles at every stage. Compare their growth after 10 stages.

Answer:

1. In the staircase, 10 stages give an increase of $10 \times 20 = 200$ cm.
2. This is **linear growth** because the same 20 cm is added at every stage.
3. A quantity that doubles for 10 stages grows by a factor of $2^{10} = 1024$.
4. This is **exponential growth** because the quantity is repeatedly multiplied by 2.
5. Thus, the example shows why exponential growth can become much faster than fixed additive growth.

7. A school wants to estimate the cost of distributing 2 kg of jaggery to each of 500 students. If jaggery costs ₹60 per kg, estimate the total cost using the approach discussed in the chapter.

Answer:

1. The total quantity required is $500 \times 2 = 1000$ kg.
2. The assumed cost of jaggery is **₹60 per kg**.
3. Therefore, the estimated total cost is $1000 \times ₹60$.
4. The estimated expenditure is ₹60,000.
5. This applies the chapter's estimation approach of identifying relationships between quantities and using reasonable values to calculate an approximate result.

8. A science report gives two distances as $\mathbf{4.5 \times 10^7}$m and $\mathbf{8.2 \times 10^6}$m. Which distance is greater, and approximately how many times greater is it?

Answer:

1. The first distance contains 10^7, while the second contains 10^6.
2. Rewrite 4.5×10^7 as 45×10^6 for easier comparison.
3. Thus, the ratio is $\frac{45 \times 10^6}{8.2 \times 10^6} = \frac{45}{8.2}$.
4. This is approximately **5.5**, so the first distance is about 5.5 times the second distance.
5. This demonstrates how scientific notation can be applied to efficiently **compare very large measurements**.

9. A computer file occupies $\mathbf{2^5}$units of storage, and a larger file occupies $\mathbf{2^9}$units. How many times larger is the second file?

Answer:

1. The required comparison is $\frac{2^9}{2^5}$.
2. Using the quotient rule, $\frac{2^9}{2^5} = 2^{9-5}$.
3. Therefore, the ratio is 2^4.
4. Since $2^4 = 16$, the second file is **16 times larger**.
5. This illustrates how the laws of exponents can simplify comparisons between quantities expressed as powers.

10. A student wants to estimate the number of 20 cm steps needed to cover a distance of 2 km. Find the number of steps and identify the type of growth involved.

Answer:

1. Convert 2 km into centimetres: 2 km $= 2000$ m $= 200{,}000$ cm.
2. Each step covers a fixed distance of **20 cm**.

3. Therefore, the number of steps required is $200{,}000 \div 20$.
4. Hence, approximately $10{,}000$steps are required.
5. This represents **linear growth** because every additional step adds the same fixed distance of 20 cm.

2.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams:

Olympiad-Type Multiple Choice Questions:

Choose the correct answer.

1. A sheet of paper is 0.001 cm thick. If its thickness doubles after every fold, what will be its thickness after 10 folds?
A) 0.1024 cm
B) 1.024 cm
C) 10.24 cm
D) 102.4 cm
Answer: B) 1.024 cm

2. A sheet of paper doubles in thickness after every fold. After how many folds will it become 2^{12}times its original thickness?
A) 6
B) 10
C) 12
D) 24
Answer: C) 12

3. If $3^5 \times 3^x = 3^{12}$, then $x$is:
A) 5
B) 6
C) 7
D) 17
Answer: C) 7

4. Which of the following is equal to $2^4 \times 5^4$?
A) 7^4
B) 10^4
C) 10^8
D) 2^8
Answer: B) 10^4

5. The value of $(3^2)^4$is:
A) 3^6
B) 3^8
C) 3^{16}
D) 9^8
Answer: B) 3^8

6. Simplify $\frac{5^{12}}{5^7}$.
A) 5^{19}
B) 5^7
C) 5^5
D) 1^5
Answer: C) 5^5

7. Which expression is equivalent to 7^{-3}?
A) -7^3

B) $\frac{1}{7^3}$
C) $\frac{-1}{7^3}$
D) 7^3
Answer: B) $\frac{1}{7^3}$
8. If $x \neq 0$, the value of $\frac{x^8}{x^8}$is:
A) 0
B) x
C) 1
D) x^{16}
Answer: C) 1
9. Which statement is **incorrect**?
A) $a^3 \times a^4 = a^7$
B) $(a^3)^4 = a^{12}$
C) $a^7 \div a^3 = a^4$
D) $a^3 + a^4 = a^7$
Answer: D) $a^3 + a^4 = a^7$
10. Simplify $\frac{2^8 \times 2^5}{2^9}$.
A) 2^2
B) 2^3
C) 2^4
D) 2^{22}
Answer: C) 2^4
11. A magical pond is completely covered with lotuses on the 30th day, and the lotuses double each day. On which day was it one-eighth covered?
A) 27th day
B) 28th day
C) 29th day
D) 26th day
Answer: A) 27th day
12. A quantity triples at every stage. If it starts at 1, what will it become after 6 stages?
A) 243
B) 486
C) 729
D) 2187
Answer: C) 729
13. A lock has 4 positions, each of which can contain any digit from 0 to 9. How many possible passwords are there?
A) 40
B) 400
C) 1,000
D) 10,000
Answer: D) 10,000
14. A password system is increased from 4 digits to 6 digits, with 10 choices for each position. By what factor does the number of possible passwords increase?
A) 10
B) 20
C) 100

D) 1,000
Answer: C) 100
15. A student has 5 shirts, 3 pairs of trousers and 2 pairs of shoes. How many different outfits can be formed?
A) 10
B) 15
C) 30
D) 60
Answer: C) 30
16. Which of the following is the correct scientific notation for 8,050,000?
A) 8.05×10^5
B) 8.05×10^6
C) 80.5×10^6
D) 0.805×10^6
Answer: B) 8.05×10^6
17. Which is the greatest number?
A) 9.5×10^5
B) 1.2×10^6
C) 8.9×10^5
D) 9.9×10^4
Answer: B) 1.2×10^6
18. Which is the smallest number?
A) 4.5×10^7
B) 7.2×10^6
C) 9.1×10^5
D) 3.8×10^8
Answer: C) 9.1×10^5
19. One crore is how many times one lakh?
A) 10
B) 100
C) 1,000
D) 10,000
Answer: B) 100
20. One billion is how many times one million?
A) 10
B) 100
C) 1,000
D) 10,000
Answer: C) 1,000
21. Which sequence represents **exponential growth**?
A) 2, 4, 6, 8, 10
B) 5, 10, 15, 20, 25
C) 3, 6, 12, 24, 48
D) 10, 20, 30, 40, 50
Answer: C) 3, 6, 12, 24, 48
22. Which sequence represents **linear growth**?
A) 2, 4, 8, 16
B) 5, 15, 45, 135
C) 10, 30, 50, 70

D) 3, 9, 27, 81
Answer: C) 10, 30, 50, 70
23. A staircase rises 20 cm with every step. What vertical distance is covered in 250 steps?
A) 5 m
B) 50 m
C) 500 m
D) 5,000 m
Answer: B) 50 m
24. If $10^x = 1,00,000$, then $x$equals:
A) 4
B) 5
C) 6
D) 7
Answer: B) 5
25. Which of the following represents 0.001using a power of 10?
A) 10^3
B) 10^{-1}
C) 10^{-2}
D) 10^{-3}
Answer: D) 10^{-3}
26. The number 561.903contains the term 3×10^nin its expanded form. What is n?
A) -1
B) -2
C) -3
D) 3
Answer: C) -3
27. If $2^x = 32$and $4^y = 64$, what is $x + y$?
A) 6
B) 7
C) 8
D) 9
Answer: C) 8
28. If $3^x = 81$, then the value of $3^{x+2}$is:
A) 243
B) 729
C) 2187
D) 6561
Answer: B) 729
29. Which expression has the greatest value?
A) 2^8
B) 4^4
C) 8^3
D) 16^2
Answer: C) 8^3
Note: $2^8 = 4^4 = 16^2 = 256$, *while* $8^3 = 512$.
30. If $a^3 = 125$, what is the value of a^5?
A) 625
B) 1,250
C) 3,125

D) 15,625
Answer: C) 3,125
31. Which statement best explains why exponential growth eventually overtakes linear growth?
A) Exponential growth repeatedly subtracts a fixed quantity.
B) Linear growth always begins with a smaller number.
C) Exponential growth repeatedly multiplies the existing quantity by a fixed factor.
D) Linear growth has no pattern.
Answer: C) Exponential growth repeatedly multiplies the existing quantity by a fixed factor.
32. A quantity doubles at each stage. If its value at the 8th stage is 256, what was its value at the 5th stage?
A) 16
B) 32
C) 64
D) 128
Answer: B) 32
33. If $x = 2^3$, what is x^4?
A) 2^7
B) 2^{12}
C) 2^{24}
D) $8^4 = 2^8$
Answer: B) 2^{12}
34. Which of the following is equal to $\frac{1}{100000}$?
A) 10^{-3}
B) 10^{-4}
C) 10^{-5}
D) 10^5
Answer: C) 10^{-5}
35. A student estimates the cost of 50 kg of an item priced at ₹70 per kg. Which is the correct estimate?
A) ₹350
B) ₹700
C) ₹3,500
D) ₹35,000
Answer: C) ₹3,500

2.15 Best Practices Opportunity List from the Chapter: (10 Ideas)

"Best Practices" Opportunities for Teaching:
Based on the chapter's emphasis on **powers, exponential growth, laws of exponents, scientific notation, estimation, combinations, and comparison of linear and exponential growth**, the following best practices can make teaching more effective.

(1) **Begin with the Paper-Folding Activity:** Let students fold paper and record its theoretical thickness after each fold in a table. Use the activity to move naturally from repeated doubling to 0.001×2^n and the idea of **exponential growth**.
(2) **Use "Pattern → Rule → Formula":** Before giving exponent laws directly, ask students to expand expressions such as $2^3 \times 2^4$, observe what happens to the number

of factors, and then derive $a^m \times a^n = a^{m+n}$. This encourages understanding rather than memorisation.

(3) **Maintain an Exponent-Laws Concept Chart:** Display the major laws—product of powers, quotient of powers, power of a power, zero exponent, and negative exponent—with one worked example for each. Students can refer to the chart while solving problems and gradually learn to select the correct rule independently.

(4) **Use Error Analysis Regularly:** Present statements such as $2^3 + 2^4 = 2^7$, $5^0 = 0$, or $4^{-2} = -16$and ask students to identify and correct the error. This is particularly useful for distinguishing conditions under which different exponent rules apply.

(5) **Connect Powers with Real-Life Counting:** Use passwords, clothing combinations, seating choices, or similar situations to demonstrate the multiplication principle and powers. The chapter's password activity is especially useful for showing how adding one more position can dramatically increase the number of possibilities.

(6) **Teach Scientific Notation through Large-Number Comparisons:** Give students large measurements and ask them first to convert them into scientific notation and then compare them. Emphasise the role of the **power of 10** in indicating the magnitude of a number.

(7) **Compare Linear and Exponential Growth Visually:** Create two tables on the board—one showing a fixed increase at every stage and another showing repeated doubling. Students can compare the values and explain why exponential growth eventually becomes much faster than linear growth.

(8) **Encourage "Estimate First, Calculate Next":** Before solving real-life numerical problems, ask every student to make a reasonable guess and state the assumptions used. After calculation, students should compare their estimates with their original guesses, reflecting the estimation approach encouraged in the chapter.

(9) **Use Think–Pair–Share for Challenging Problems:** Give a problem such as the **Magical Pond** situation, allow students to think individually, discuss their reasoning with a partner, and then share it with the class. This promotes mathematical reasoning and communication while reinforcing exponential growth.

(10) **Use a Mixed LOTS–HOTS Exit Ticket:** End the lesson with three short questions—one recalling an exponent law, one applying it, and one requiring reasoning or error correction. This provides immediate feedback on whether students can **remember, apply, and reason with** the chapter concepts.

2.16 Innovations Opportunity List from the Chapter: (10 Ideas)

"Innovations" Opportunities in Teaching–Learning for 5th Standard Students:
The chapter introduces ideas such as **repeated doubling, powers, patterns, combinations, estimation, powers of 10, and linear versus exponential growth**. For 5th Standard students, these can be simplified into playful, activity-oriented learning experiences.

(1) **Paper-Folding Power Lab:** Let students fold a sheet of paper and create a "doubling table" showing 1, 2, 4, 8, 16, 32, etc. They can predict the next value before calculating it, turning the chapter's paper-folding idea into an experiential introduction to rapid growth.

(2) **Human Power Chain:** Give students number cards and ask them to stand in sequence representing $2^1, 2^2, 2^3, ...$ Each child explains how their number is obtained from the previous one, making repeated multiplication visible and memorable.

(3) **Magical Pond Simulation:** Use counters, paper lotuses, or digital objects to simulate a pond where the number of lotuses doubles at every stage. Students can work

backwards from a “fully covered pond” to discover half, one-fourth and one-eighth coverage.

(4) **Password Combination Game:** Create a classroom “secret lock” with cards containing digits or symbols and challenge teams to determine how many passwords can be formed with two, three, or more positions. This transforms the chapter’s password problem into a hands-on exploration of combinations.

(5) **Dress-Up Combination Challenge:** Provide picture cards of shirts, caps, trousers and shoes and ask students to create as many different outfits as possible. They can then discover that multiplying the number of choices gives the total number of combinations.

(6) **Linear vs Doubling Race:** Divide the class into two teams—one sequence grows by adding 2 each round, while the other doubles each round. Recording both sequences on the board allows students to visually discover how **multiplicative growth eventually races ahead of additive growth**.

(7) **Power-of-10 Place-Value Floor:** Create floor cards labelled $10^3, 10^2, 10^1, 10^0, 10^{-1}$, ..., and let students physically position digit cards on them. This makes the chapter’s connection between **place value and powers of 10** more concrete.

(8) **“Guess First” Estimation Station:** Set up stations with familiar objects such as books, pencils, coins or water bottles and ask students to estimate quantities before measuring or calculating. Award points not merely for exact answers but for **reasonable assumptions and sensible reasoning**, reflecting the chapter’s estimation approach.

(9) **Exponent Treasure Hunt:** Hide question cards around the classroom containing simple power patterns and exponent puzzles. Each correct solution provides a clue to the next location, converting practice into a collaborative mathematical treasure hunt.

(10) **Student-Created Math Comic:** Ask students to create a short comic titled **“The Power of Doubling”**, featuring a character whose coins, plants, chocolates, or magical objects double at every stage. The final panel should compare repeated addition with repeated multiplication, helping students communicate the central idea of the chapter creatively.

2.17: Practicing Questions/Question Bank

A. Fill in the Blanks:

1. The thickness of a paper ________ after every fold.
   Answer: doubles

2. $2 \times 2 \times 2 \times 2$can be written as ________.
   Answer: 2^4

3. In 5^3, the number 5 is called the ________.
   Answer: base

4. In 7^4, the number 4 is called the ________.
   Answer: exponent

5. 2^5 =________.
   Answer: 32

6. 10^3 =________.
   Answer: 1000

7. $a^m \times a^n = a$—.
   Answer: $m + n$

8. For $a \neq 0$, a^0 =________.
   Answer: 1

9. One lakh can be written as 10—.
   Answer: 5

10. Growth caused by repeatedly adding the same amount is called ________ growth.
    Answer: linear

B. One-Mark Descriptive Questions:

1. **What is a power?**
   Answer: A power is a short way of representing repeated multiplication of the same number.

2. **What is the base in 4^5?**
   Answer: The base is **4**.

3. **What is the exponent in 4^5?**
   Answer: The exponent is **5**.

4. **Write $3 \times 3 \times 3 \times 3$in exponential form.**
   Answer: 3^4.

5. **Find 2^6.**
   Answer: $2^6 = 64$.

6. **What is exponential growth?**
   Answer: Exponential growth occurs when a quantity repeatedly increases by multiplication by a fixed factor.

7. **What is the value of 8^0?**
   Answer: $8^0 = 1$.

8. **Write 5900 in scientific notation.**
   Answer: $5900 = 5.9 \times 10^3$.

9. **What is linear growth?**
   Answer: Linear growth occurs when a fixed amount is repeatedly added.

10. **How many possible passwords are there in a 2-digit lock using digits 0–9?**
    Answer: There are **100 possible passwords**.

C. Two-Mark Descriptive Questions:

1. Explain 5^4.
Answer:

1. 5^4means $5 \times 5 \times 5 \times 5$.

2. Its value is **625**.

2. State the product rule of exponents with an example.
Answer:

1. For the same base, $a^m \times a^n = a^{m+n}$.

2. Thus, $2^3 \times 2^4 = 2^7$.

3. What is the difference between linear and exponential growth?
Answer:

1. Linear growth involves repeatedly adding a fixed quantity.
2. Exponential growth involves repeatedly multiplying by a fixed factor.

4. How many combinations can be made using 4 shirts and 3 caps?
Answer:

1. Each shirt can be combined with any of the 3 caps.
2. Therefore, $4 \times 3 = 12$combinations are possible.

5. Explain why $6^0 = 1$.
Answer:

1. $6^3 \div 6^3 = 6^{3-3} = 6^0$.
2. Since a non-zero number divided by itself is 1, $6^0 = 1$.

D. Three-Mark Descriptive Questions:

1. Explain the paper-folding example of exponential growth.
Answer:

1. The paper begins with a certain thickness.
2. Its thickness doubles after every fold.
3. Therefore, repeated folding illustrates **exponential growth**.

2. Explain the multiplication rule for powers with the same base.
Answer:

1. When powers have the same base, their exponents are added during multiplication.
2. Thus, $a^m \times a^n = a^{m+n}$.
3. For example, $3^2 \times 3^4 = 3^6$.

3. Explain the Magical Pond problem.
Answer:

1. The number of lotuses in the pond doubles every day.
2. If the pond is full on the 30th day, it was half full on the 29th day.
3. This happens because the lotuses show **exponential growth**.

4. Explain how powers of 10 are connected with place value.
Answer:

1. Hundreds can be represented using 10^2.
2. Tens can be represented using 10^1, and ones using 10^0.
3. Thus, powers of 10 provide a compact representation of place values.

5. Explain how to solve an estimation problem.
Answer:

1. First, make a reasonable guess about the answer.
2. Make suitable assumptions and calculate an approximate value.
3. Finally, compare the calculated result with the original guess.

E. Five-Mark Descriptive Questions:

1. Explain exponential growth using the paper-folding activity.
Answer:

1. The chapter begins with a paper thickness of **0.001 cm**.
2. Every fold doubles the thickness of the paper.
3. After $n$folds, the thickness can be represented by 0.001×2^ncm.
4. After 10 folds, the thickness becomes $0.001 \times 2^{10} = 1.024$cm.
5. This rapid increase through repeated multiplication is called **exponential growth**.

2. Explain the important laws of exponents with examples.
Answer:

1. Same-base multiplication: $a^m \times a^n = a^{m+n}$.
2. Same-base division: $a^m \div a^n = a^{m-n}$.
3. Power of a power: $(a^m)^n = a^{mn}$.
4. Zero exponent: $a^0 = 1$, for $a \neq 0$.
5. Negative exponent: $a^{-n} = \frac{1}{a^n}$, for $a \neq 0$.

3. Explain scientific notation and its usefulness.
Answer:

1. Scientific notation represents numbers using powers of 10.
2. Its standard form is $x \times 10^y$, where $1 \leq x < 10$.
3. For example, $5900 = 5.9 \times 10^3$.
4. Similarly, $80{,}00{,}000 = 8 \times 10^6$.
5. It makes very large numbers easier to **write, read and compare**.

4. Compare linear and exponential growth with examples.
Answer:

1. Linear growth occurs through repeated addition of a fixed amount.
2. Equally spaced ladder steps are an example of linear growth.
3. Exponential growth occurs through repeated multiplication by a fixed factor.
4. Doubling the thickness of paper after every fold is an example of exponential growth.

5. Exponential growth becomes much faster than linear growth as the number of stages increases.

5. Explain how powers can be used to count possible passwords.
Answer:

1. Suppose each position can contain any digit from 0 to 9.
2. Each position therefore has **10 choices**.
3. A 2-digit password has $10^2 = 100$possibilities.
4. A 3-digit password has $10^3 = 1000$possibilities.
5. Thus, an n-digit password has 10^npossible combinations.

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8th Standard Maths 1 Book Chapter 3

Chapter 3: A STORY OF NUMBERS

3.1 Summary of the Chapter:

Chapter 3 – A Story of Numbers: Summary:

(1) **Need for Counting:** Humans have needed counting since the Stone Age to keep track of food, livestock, trade, rituals, days, seasons, and other quantities.

(2) **Number Systems:** A **number system** is a standard sequence of objects, names, or written symbols arranged in a fixed order for representing and counting quantities. The written symbols used to represent numbers are called **numerals**.

(3) **Early Methods of Counting:** People initially counted using **body parts, sticks, pebbles, tally marks, sounds, names, and symbols**. Counting involved making a **one-to-one correspondence** between objects and the chosen counting representation.

(4) **Counting in Groups:** Instead of representing every object separately, people began counting in groups such as **2, 5, 10, or 20**. This made the representation of larger quantities more efficient.

(5) **Landmark Numbers:** More advanced systems introduced **landmark numbers**—easily recognisable reference numbers used to represent and understand larger quantities. The Roman system, for example, uses symbols such as **I, V, X, L, C, D and M**.

(6) **Roman Number System:** Roman numerals were more efficient than simple tally systems because they used several landmark numbers. However, performing arithmetic operations, especially **multiplication and division**, was difficult.

(7) **Idea of a Base:** A major development was choosing landmark numbers as **powers of a fixed number**. Such a system is called a **base-n number system**; its landmark numbers are $1, n, n^2, n^3, \ldots$.

(8) **Egyptian Number System:** The Egyptian system developed around **3000 BCE** and used powers of 10 as landmark numbers. Thus, it is an example of a **base-10 or decimal system**.

(9) **Place Value System:** The next important breakthrough was the **place value system**, where the position of a symbol determines the landmark number associated with it. Place-value representations appeared in Mesopotamian, Mayan, Chinese, and Indian civilisations.

(10) **Hindu/Indian Number System:** Our modern system is a **base-10 place value system** using ten digits—**0, 1, 2, 3, 4, 5, 6, 7, 8 and 9**. For example, $375 = 3 \times 10^2 + 7 \times 10 + 5 \times 1$.

(11) **Importance of Zero:** The introduction of **0 both as a positional digit and as a number in its own right** was a revolutionary development. Indian mathematicians such as **Aryabhata and Brahmagupta** contributed greatly to the mathematical use of zero.

(12) **Evolution of Modern Numerals:** The chapter traces the development from **counting in groups → landmark numbers → base systems → place value → zero**. The Hindu number system originated in India, spread through the Arab world to Europe and eventually became the globally used system because of its simple representation and efficient computation.

3.2 Important Formulas:

Important Formulas / Rules – Chapter 3: A Story of Numbers:

This chapter contains fewer conventional "formulas" and more **number-representation rules and relationships** needed for quantitative problems.

1. **Base-n Landmark Numbers**

$$n^0 = 1,\ n^1 = n,\ n^2,\ n^3,\ \ldots$$

In a base-n system, the landmark numbers are successive powers of n.

2. **Decimal (Base-10) Landmark Numbers**

$$10^0 = 1,\ 10^1 = 10,\ 10^2 = 100,\ 10^3 = 1000, \ldots$$

The Egyptian and Hindu number systems use powers of 10 as landmark numbers.

3. **Base-5 Landmark Numbers**

$$5^0 = 1,\ 5^1 = 5,\ 5^2 = 25,\ 5^3 = 125,\ 5^4 = 625, \ldots$$

This relationship is used in the chapter to explain the general idea of a base.

4. **Place Value Expansion in the Hindu Number System**

For a numeral with digits $a, b, c, \ldots,$

$$abc \ldots = a \times 10^2 + b \times 10^1 + c \times 10^0 + \cdots$$

Example:

$$375 = 3 \times 10^2 + 7 \times 10^1 + 5 \times 10^0$$
$$= 300 + 70 + 5$$

5. **General Place Value Formula in Base-n**

$$(a_k a_{k-1} \ldots a_1 a_0)_n = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$$

This is the general mathematical form of the chapter's place-value idea.

6. **Regrouping Rule in Base-n**

$$n \times n^k = n^{k+1}$$

So, n**units of one place can be replaced by 1 unit of the next higher place**.

For base 10:

$$10 \times 10^k = 10^{k+1}$$

For base 5:

$$5 \times 5^k = 5^{k+1}$$

7. **Product of Landmark Numbers**

$$n^a \times n^b = n^{a+b}$$

Therefore, the product of two landmark numbers in a base-n system is also a landmark number. This is one reason base systems simplify multiplication.

8. **Multiplication by the Base**

$$N \times n$$

moves each place value to the next higher landmark position.
In decimal:

$$N \times 10$$

shifts each place one position higher.

9. **Distributive Property**

$$(a + b) \times n = an + bn$$

More generally:

$$(a + b + c) \times n = an + bn + cn$$

This rule is used in the chapter while explaining multiplication in base systems.

10. **Grouping a Number Using Landmark Numbers**

A number can be expressed as the sum of its landmark-number components:

$$N = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$$

For example:

$$143 = 125 + 5 + 5 + 5 + 1 + 1 + 1$$

in the chapter's base-5 representation.

11. **Roman Numeral Additive Representation**

A number may be expressed as the sum of Roman landmark numbers.

Example:

$$27 = 10 + 10 + 5 + 1 + 1$$

Therefore:

$$27 = \text{XXVII}$$

Similarly, Roman numerals use landmark values such as:

$$I = 1,\ V = 5,\ X = 10,\ L = 50,\ C = 100,\ D = 500,\ M = 1000$$

12. **Roman Numeral Subtractive Rule**
    A smaller landmark placed before a larger one can indicate subtraction.

Examples:

$$IV = 5 - 1 = 4$$
$$IX = 10 - 1 = 9$$
$$XL = 50 - 10 = 40$$

Key Formula Sheet for Quick Revision

$$\boxed{\text{Base-}n \text{ landmark numbers} = n^0, n^1, n^2, n^3, \ldots}$$
$$\boxed{n^a \times n^b = n^{a+b}}$$
$$\boxed{n \times n^k = n^{k+1}}$$
$$\boxed{N = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0}$$
$$\boxed{375 = 3(10^2) + 7(10) + 5}$$
$$\boxed{(a + b)n = an + bn}$$

These are the **most important quantitative rules and formulas** students should remember for solving problems from Chapter 3, **"A Story of Numbers."**

3.3 Fill-Up the Blank LOTS type questions with Answers: (35 Questions)

LOTS Fill-in-the-Blank Questions with Answers:

1. Humans had the need to count even as early as the ________ **Age**.
   Answer: Stone
2. A standard sequence of objects, names, or written symbols having a fixed order is called a ________ **system**.
   Answer: number
3. Associating each object being counted with exactly one counting object is called ________ **mapping**.
   Answer: one-to-one
4. The symbols occurring in a written number system are called ________.
   Answer: numerals
5. Marks cut on surfaces such as bones or cave walls for counting are called ________ **marks**.
   Answer: tally
6. The ________ **bone** has 29 notches and is considered one of the oldest mathematical artefacts.
   Answer: Lebombo
7. The Gumulgal people formed their number names by counting mainly in groups of ________.
   Answer: two (2)
8. Some commonly used group sizes in different number systems were 2, 5, 10 and ________.
   Answer: 20
9. In Roman numerals, the symbol **V** represents the number ________.
   Answer: 5

10. In Roman numerals, the symbol **X** represents ________.
    Answer: 10
11. In Roman numerals, **L** represents ________.
    Answer: 50
12. In Roman numerals, **C** represents ________.
    Answer: 100
13. In Roman numerals, **M** represents ________.
    Answer: 1000
14. The Roman numeral for 27 is ________.
    Answer: XXVII
15. Numbers that have new basic symbols and serve as reference points in a number system are called ________ **numbers**.
    Answer: landmark
16. The Egyptian number system is a base-________ number system.
    Answer: 10
17. A base-10 number system is also called a ________ **number system**.
    Answer: decimal
18. In a base-n number system, the landmark numbers are the ________ of n.
    Answer: powers
19. The Mesopotamian number system later became a base-________ system.
    Answer: 60
20. The base-60 Mesopotamian number system is also called the ________ **system**.
    Answer: sexagesimal
21. One hour contains ________ minutes, showing the continuing influence of the Mesopotamian base-60 system.
    Answer: 60
22. A number system in which the position of a symbol determines its value is called a ________ **value system**.
    Answer: place
23. The Hindu number system is a base-________ or decimal system.
    Answer: 10
24. The Hindu number system uses ________ basic digits from 0 to 9.
    Answer: ten (10)
25. The digit ________ plays an important role both as a positional digit and as a number in the Hindu number system.
    Answer: 0
26. The first known use of ten digits including zero, then represented by a dot, occurs in the ________ **manuscript**.
    Answer: Bakhshali
27. ________ explained and performed elaborate scientific computations using the Indian system of ten symbols around 499 CE.
    Answer: Aryabhata
28. The Persian mathematician ________ helped popularise Hindu numerals in the Arab world.
    Answer: Al-Khwārizmī
29. The Italian mathematician ________ strongly promoted the adoption of Indian numerals in Europe around 1200 CE.
    Answer: Fibonacci

30. ________ codified the arithmetic use of zero as a number in his *Brāhmasphuṭasiddhānta* in 628 CE.
Answer: Brahmagupta

Answer Key:
1. Stone | 2. number | 3. one-to-one | 4. numerals | 5. tally | 6. Lebombo | 7. two | 8. 20 | 9. 5 | 10. 10 | 11. 50 | 12. 100 | 13. 1000 | 14. XXVII | 15. landmark | 16. 10 | 17. decimal | 18. powers | 19. 60 | 20. sexagesimal | 21. 60 | 22. place | 23. 10 | 24. ten | 25. 0 | 26. Bakhshali | 27. Aryabhata | 28. Al-Khwārizmī | 29. Fibonacci | 30. Brahmagupta.

3.4 Fill-Up the Blank HOTS type questions with Answers: (30 Questions)

HOTS Fill-in-the-Blank Questions with Answers:

1. If every cow in a herd is matched with exactly one stick, the counting principle being applied is called ________.
   Answer: one-to-one mapping
2. If a counting system uses only the 26 English letters, it cannot directly represent a collection containing 30 objects because the sequence is ________.
   Answer: finite
3. A counting system must have a fixed order because objects are counted by matching them with a ________ **sequence**.
   Answer: standard
4. If five tally marks are repeatedly replaced by one new symbol, the system is using the idea of counting in ________.
   Answer: groups of five
5. The Gumulgal representation *ukasar-ukasar-urapon* corresponds to $2 + 2 + 1$, and therefore represents ________.
   Answer: 5
6. The Roman numeral XXVII represents $10 + 10 + 5 + 1 + 1$, which is ________.
   Answer: 27
7. If a Roman numeral contains two M's, three C's, one L, one X and two I's, its value is ________.
   Answer: 2362
8. Roman numerals are less convenient for multiplication and division because their landmark numbers do not follow the simple pattern of ________ **of a fixed number**.
   Answer: powers
9. The transition from using one group size to using several reference group sizes led to the concept of ________ **numbers**.
   Answer: landmark
10. If the first landmark number is 1 and every succeeding landmark number is five times the previous one, the number system has base ________.
    Answer: 5
11. In a base-7 system, the landmark number immediately after 7^2 is ________.
    Answer: 7^3 or 343
12. If the landmark numbers are $1, 4, 16, 64, 256, \ldots$, the number system has base ________.
    Answer: 4
13. In a base-n system, n collections of n^2 can be regrouped into one collection of ________.
    Answer: n^3

14. Since the Egyptian landmark numbers are 1,10,100,1000, ..., the Egyptian system is a ________ system.
    Answer: base-10
15. In the chapter's base-5 system, $143 = 125 + 5 + 5 + 5 + 1 + 1 + 1$. Therefore, the largest landmark number required to represent 143 is ________.
    Answer: 125
16. If two landmark numbers in a base-n system are $n^3$and n^2, their product is the landmark number ________.
    Answer: n^5
17. When $n$units of one landmark number are regrouped, they produce ________ **unit of the next landmark number**.
    Answer: one
18. A system in which the same symbol has different values depending upon its position is called a ________ **number system**.
    Answer: positional (or place value)
19. In the Hindu number 375, the digit 3 represents 3 ×________.
    Answer: 10^2or 100
20. In 375, the digit 7 contributes ________ to the total value of the numeral.
    Answer: 70
21. If a digit moves one position to the left in a base-10 place value system, its place value becomes ________ **times** its previous place value.
    Answer: 10
22. A positional number system without an effective placeholder can produce ________ in interpreting numerals.
    Answer: ambiguity
23. The digit that removes such ambiguity by marking an empty position in the Hindu number system is ________.
    Answer: 0
24. The sequence $1, 60, 60^2, 60^3$, ...indicates a number system with base ________.
    Answer: 60
25. The continued division of an hour into 60 minutes reflects the influence of the ancient ________ number system.
    Answer: Mesopotamian (Babylonian)
26. A number system using powers of a fixed number as landmark numbers makes arithmetic easier because the product of two landmark numbers is another ________ **number**.
    Answer: landmark
27. The expression $3 \times 10^2 + 7 \times 10 + 5 \times 1$produces the Hindu numeral ________.
    Answer: 375
28. If zero were omitted from a place-value numeral such as 305, it would become difficult to distinguish the empty ________ **place**.
    Answer: tens
29. The historical progression **grouping → landmark numbers → base → place value → zero** resulted in increasingly ________ **number representation and computation**.
    Answer: efficient
30. Because the Hindu number system combines **base 10, place value and zero**, all numbers can be represented efficiently using only ________ **basic digits**.
    Answer: ten

Answer Key:

1. one-to-one mapping | 2. finite | 3. standard | 4. groups of five | 5. 5 | 6. 27 | 7. 2362 | 8. powers | 9. landmark | 10. 5 | 11. 7^3/343 | 12. 4 | 13. n^3| 14. base-10 | 15. 125 | 16. n^5| 17. one | 18. positional/place value | 19. 10^2/100 | 20. 70 | 21. 10 | 22. ambiguity | 23. 0 | 24. 60 | 25. Mesopotamian/Babylonian | 26. landmark | 27. 375 | 28. tens | 29. efficient | 30. ten.

3.5 One-mark questions with Answers of LOTS Type: (35 Questions)

One-Mark Descriptive Questions with Answers – LOTS Type:

1. **Why did humans need to count from ancient times?**
   Answer: Humans needed to count food, livestock, traded goods, ritual offerings, days, and other quantities.
2. **What is a number system?**
   Answer: A number system is a standard sequence of objects, names, or written symbols arranged in a fixed order.
3. **What are numerals?**
   Answer: Numerals are the symbols used to represent numbers in a written number system.
4. **What is one-to-one mapping in counting?**
   Answer: One-to-one mapping is the association of each object being counted with exactly one object or symbol in the counting sequence.
5. **What are tally marks?**
   Answer: Tally marks are notches or marks made on surfaces such as bones or cave walls to record quantities.
6. **Name one ancient bone associated with tally marks.**
   Answer: The Lebombo bone is an ancient artefact containing tally marks.
7. **How did the Gumulgal people form their number names?**
   Answer: The Gumulgal people formed their number names mainly by counting in groups of two.
8. **What is meant by counting in groups?**
   Answer: Counting in groups means organising objects into groups of a fixed size to represent quantities more efficiently.
9. **What are landmark numbers?**
   Answer: Landmark numbers are easily recognisable reference numbers used to represent and understand other numbers.
10. **What does the Roman numeral V represent?**
    Answer: The Roman numeral V represents 5.
11. **What does the Roman numeral X represent?**
    Answer: The Roman numeral X represents 10.
12. **What does the Roman numeral L represent?**
    Answer: The Roman numeral L represents 50.
13. **What does the Roman numeral C represent?**
    Answer: The Roman numeral C represents 100.
14. **What does the Roman numeral M represent?**
    Answer: The Roman numeral M represents 1000.
15. **Write 27 in Roman numerals.**
    Answer: The number 27 is written as **XXVII** in Roman numerals.
16. **Mention one limitation of the Roman number system.**
    Answer: The Roman number system makes arithmetic operations such as multiplication and division difficult.

17. **What is a base-n number system?**
Answer: A base-n number system is one whose landmark numbers are successive powers of n.
18. **What are the landmark numbers in a base-5 system?**
Answer: The landmark numbers in a base-5 system are $1, 5, 25, 125, 625, \ldots$.
19. **What is another name for a base-10 number system?**
Answer: A base-10 number system is also called a **decimal number system**.
20. **What is the base of the Egyptian number system discussed in the chapter?**
Answer: The Egyptian number system discussed in the chapter has base 10.
21. **What is a place value system?**
Answer: A place value system is a number system in which the position of a symbol determines the landmark number associated with it.
22. **What was the base of the later Mesopotamian number system?**
Answer: The later Mesopotamian number system used base 60.
23. **What is another name for the base-60 system?**
Answer: The base-60 system is called the **sexagesimal system**.
24. **Where can we still see the influence of the Mesopotamian base-60 system?**
Answer: Its influence can be seen in time measurement, where an hour has 60 minutes and a minute has 60 seconds.
25. **What is the base of the Hindu number system?**
Answer: The Hindu number system is a base-10 or decimal number system.
26. **How many basic digits are used in the Hindu number system?**
Answer: The Hindu number system uses ten basic digits from 0 to 9.
27. **What is the place value of 7 in the number 375?**
Answer: The place value of 7 in 375 is 70.
28. **What important role does zero play in the Hindu number system?**
Answer: Zero acts as a positional digit and is also treated as a number in its own right.
29. **Which manuscript contains an early known use of ten digits including zero?**
Answer: The **Bakhshali manuscript** contains an early known use of ten digits including zero.
30. **Who used the Indian system of ten symbols for elaborate scientific computations around 499 CE?**
Answer: Aryabhata used the Indian system of ten symbols for elaborate scientific computations around 499 CE.
31. **Who codified the arithmetic use of zero as a number?**
Answer: Brahmagupta codified the arithmetic use of zero as a number in 628 CE.
32. **Who helped popularise Hindu numerals in the Arab world?**
Answer: Al-Khwārizmī helped popularise Hindu numerals in the Arab world.
33. **Who promoted the adoption of Indian numerals in Europe around 1200 CE?**
Answer: Fibonacci promoted the adoption of Indian numerals in Europe around 1200 CE.
34. **Why is a number system with a base useful for arithmetic?**
Answer: A base system simplifies arithmetic because its landmark numbers follow a regular pattern of powers.
35. **What are the five major stages in the evolution of number representation described in the chapter?**
Answer: The stages are **grouping, landmark numbers, base systems, place value, and the use of zero**.

3.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – HOTS Type:

1. **Why is one-to-one mapping useful for checking whether any animal in a herd is missing?**
   Answer: It allows each animal to be matched with one counting object, so an unmatched object indicates that an animal is missing.
2. **Why is using sticks inconvenient for counting a very large collection?**
   Answer: It requires as many sticks as the number of objects being counted, making the method cumbersome.
3. **Why cannot a counting system using only the 26 English letters represent all numbers directly?**
   Answer: It has only a finite sequence of 26 symbols, whereas numbers form an unending sequence.
4. **Why are tally marks more suitable for small quantities than very large quantities?**
   Answer: Large quantities require too many individual tally marks, making them difficult to record and count.
5. **Why is counting in groups more efficient than making one tally mark for every object?**
   Answer: Grouping allows several objects to be represented together and therefore reduces the number of symbols required.
6. **If *ukasar* represents 2 and *urapon* represents 1, what does *ukasar-ukasar-urapon* represent?**
   Answer: It represents $2 + 2 + 1 = 5$.
7. **Why might groups such as 5 have become useful in early counting systems?**
   Answer: Humans find it difficult to recognise larger collections at a glance, so grouping made quantities easier to perceive and represent.
8. **Why are landmark numbers useful in a number system?**
   Answer: Landmark numbers provide convenient reference points for representing and working with larger quantities.
9. **If 27 is written as XXVII, what grouping principle does the Roman representation show?**
   Answer: It shows $27 = 10 + 10 + 5 + 1 + 1$, using Roman landmark numbers.
10. **Why is the Roman system more efficient than a simple tally system?**
    Answer: It uses different landmark symbols to represent groups of quantities instead of representing every unit separately.
11. **Why are multiplication and division comparatively difficult using Roman numerals?**
    Answer: Roman landmark numbers do not follow a uniform pattern based on powers of a fixed number.
12. **If the landmark numbers are** $1, 5, 25, 125, \ldots$**, how can you identify the base of the system?**
    Answer: Since every landmark number is five times the previous one, the system is base 5.
13. **What would be the next landmark number after 343 in a base-7 system?**
    Answer: The next landmark number is $343 \times 7 = 2401$.
14. **Why are powers of a fixed number useful as landmark numbers?**
    Answer: They create a regular grouping pattern that simplifies number representation and arithmetic.

15. **If a system has landmark numbers** $1, 8, 64, 512, \ldots$**, what is its base?**
Answer: The system has base 8 because each landmark number is eight times the previous one.
16. **Why is the Egyptian number system considered a base-10 system?**
Answer: Its landmark numbers are successive powers of 10.
17. **What happens when ten units of one landmark number are collected in a base-10 system?**
Answer: They can be regrouped as one unit of the next higher landmark number.
18. **Why does a base system make multiplication of landmark numbers easier?**
Answer: The product of two landmark numbers that are powers of the base is itself another landmark number.
19. **Why is a place-value system more efficient than assigning a new symbol to every landmark number?**
Answer: The position of a digit can indicate its landmark value, reducing the need for continually inventing new symbols.
20. **In the number 375, why does 3 represent 300 rather than 3?**
Answer: The digit 3 occupies the 10^2or hundreds position, so its value is $3 \times 100 = 300$.
21. **If the digit 7 in 375 were moved one place to the left, how would its place value change?**
Answer: Its place value would become ten times greater because the Hindu system is base 10.
22. **Why is zero important as a placeholder in a place-value system?**
Answer: Zero marks an empty position and prevents ambiguity in interpreting the values of the other digits.
23. **Why are 35 and 305 clearly distinguishable in the Hindu number system?**
Answer: The zero in 305 preserves the empty tens position and therefore gives the digits their correct place values.
24. **Why can the Hindu number system represent arbitrarily large numbers using only ten digits?**
Answer: The repeated use of digits in different place-value positions allows increasingly large powers of 10 to be represented.
25. **What ancient number-system idea is reflected in 60 minutes making one hour?**
Answer: It reflects the base-60 or sexagesimal principle of the Mesopotamian number system.
26. **If a number system uses** $1, 60, 3600, \ldots$**as landmark numbers, why is it called base 60?**
Answer: Each successive landmark number is obtained by multiplying the previous one by 60.
27. **Why was the introduction of zero more than just the invention of another digit?**
Answer: Zero functioned both as a positional digit and as an independent number on which arithmetic operations could be performed.
28. **How did zero improve the efficiency of the Hindu number system?**
Answer: Zero made place-value representations unambiguous while allowing numbers to be written efficiently with a finite set of digits.
29. **Why did the Hindu number system eventually become more useful for scientific computation than Roman numerals?**
Answer: Its base-10 place-value structure and use of zero made both number representation and arithmetic computation much more efficient.

30. **What does the historical progression from grouping to zero tell us about the evolution of number systems?**
    Answer: It shows that number systems gradually developed more efficient ways of representing quantities and performing calculations through grouping, landmark numbers, bases, place value, and zero.

3.7 Two-mark questions with Answers of LOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – LOTS Type:

1. Why did humans need counting in ancient times?

Answer:

1. Humans counted quantities such as **food, livestock, traded goods, and ritual offerings**.
2. They also counted passing days to predict events such as the **new moon, full moon, and onset of seasons**.

2. What is a number system? What are numerals?

Answer:

1. A **number system** is a standard sequence of objects, names, or written symbols arranged in a fixed order.
2. The symbols used to represent numbers in a written number system are called **numerals**.

3. Explain the use of sticks for counting.

Answer:

1. One stick can be kept for every object, such as every cow in a herd.
2. The resulting collection of sticks represents the total number of objects and can help identify whether any are missing.

4. What is one-to-one mapping in counting?

Answer:

1. In one-to-one mapping, each object being counted is associated with exactly one counting object or symbol.
2. This correspondence helps determine the size of a collection systematically.

5. What are tally marks? How were they used?

Answer:

1. **Tally marks** are notches or marks cut on surfaces such as bones or cave walls.
2. One mark was made for each object counted, so the total number of marks represented the quantity.

6. What do the Ishango and Lebombo bones tell us about early counting?

Answer:

1. Both bones contain ancient markings believed to have been used for recording numbers or quantities.
2. They provide evidence that humans used **tally-like methods thousands of years ago**.

7. Explain how the Gumulgal people formed their number names.

Answer:

1. The Gumulgal people formed number names mainly by **counting in groups of two**.
2. For example, 3 was represented as $2 + 1$, while 4 was represented as $2 + 2$.

8. What is meant by counting in groups? Give examples of common group sizes.

Answer:

1. Counting in groups means organising quantities into groups of a fixed size instead of counting every object separately.
2. Common group sizes used in different number systems included **2, 5, 10 and 20**.

9. What are landmark numbers? Why are they useful?
Answer:
1. **Landmark numbers** are easily recognisable reference numbers used for understanding and representing other numbers.
2. They act as anchors that make the representation of larger quantities easier.

10. State the main Roman landmark numbers and their symbols.
Answer:
1. The Roman landmark symbols include **I = 1, V = 5, X = 10 and L = 50**.
2. The higher landmark symbols include **C = 100, D = 500 and M = 1000**.

11. How is 27 represented in Roman numerals? Explain.
Answer:
1. The number 27 can be grouped as $10 + 10 + 5 + 1 + 1$.
2. Therefore, 27 is represented as **XXVII** in Roman numerals.

12. Mention two features of the Roman number system.
Answer:
1. The Roman system uses different symbols for different **landmark numbers**.
2. Although more efficient than simple tally systems, it makes operations such as **multiplication and division difficult**.

13. What is a base-n number system?
Answer:
1. In a base-n system, the first landmark number is 1 and every next landmark number is obtained by multiplying the previous one by n.
2. Hence, its landmark numbers are $n^0, n^1, n^2, n^3, \ldots$.

14. Explain the landmark numbers of a base-5 system.
Answer:
1. In a base-5 system, every landmark number is **five times** the previous landmark number.
2. Thus, the landmark numbers are $1, 5, 25, 125, 625, 3125, \ldots$.

15. Why is the Egyptian number system called a base-10 system?
Answer:
1. The Egyptian system uses landmark numbers obtained by repeatedly grouping **10 collections of the previous landmark number**.
2. Hence, its landmark numbers are powers of 10 such as $1, 10, 100, 1000, \ldots$.

16. What is a decimal number system? Give an example.
Answer:
1. A **decimal number system** is a number system whose base is 10.
2. The Egyptian system discussed in the chapter and the modern Hindu number system are examples of base-10 systems.

17. What is a place value system? Name two civilisations that used it.
Answer:
1. A **place value system** uses the position of a symbol to determine the landmark number associated with it.
2. Place-value representations were used in civilisations including the **Mesopotamian, Mayan, Chinese and Indian** civilisations.

18. Describe the Mesopotamian number system mentioned in the chapter.
Answer:
1. The later Mesopotamian or Babylonian number system developed into a **base-60 or sexagesimal system**.
2. Its influence can still be seen in time measurement, such as **60 minutes in an hour and 60 seconds in a minute**.

19. State two important features of the Hindu number system.
Answer:
1. The Hindu number system is a **base-10 place value system** using the ten digits 0 to 9.
2. It uses **zero both as a positional digit and as a number**, enabling efficient and unambiguous representation.

20. Explain the place values in the number 375.
Answer:
1. In 375, 3 is in the hundreds position, 7 is in the tens position, and 5 is in the ones position.
2. Therefore, $375 = (3 \times 100) + (7 \times 10) + (5 \times 1)$.

21. State two important roles of zero in the Hindu number system.
Answer:
1. Zero serves as a **positional digit or placeholder** in the place value system.
2. Indian mathematics also treated zero as an **independent number** on which arithmetic operations could be performed.

22. What were Aryabhata's contributions mentioned in the chapter?
Answer:
1. Aryabhata explained and used the **Indian system of ten symbols** for elaborate computations around 499 CE.
2. He explicitly used arithmetic properties of **zero** in his scientific calculations.

23. What was Brahmagupta's contribution to the development of zero?
Answer:
1. Brahmagupta treated **zero as a number in its own right**, along with other numbers.
2. He codified arithmetic involving zero in his *Brāhmasphuṭasiddhānta* in **628 CE**.

24. How did Hindu numerals spread beyond India?
Answer:
1. Hindu numerals reached the **Arab world by around 800 CE**, where scholars such as Al-Khwārizmī and Al-Kindi helped popularise them.
2. They later reached **Europe and parts of Africa**, with Fibonacci promoting their adoption in Europe around 1200 CE.

25. State the main stages in the evolution of number representation discussed in the chapter.
Answer:
1. Early development progressed from **counting in groups to using landmark numbers and powers of a number as a base**.
2. This was followed by **place-value representation and the introduction of zero as a positional digit and a number**.

3.8. Two-mark questions with Answers of HOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – HOTS Type:
1. Why was one-to-one mapping an important development in counting?
Answer:
1. Each object could be matched with exactly one counting object, such as a stick.
2. This made it possible to compare and keep track of quantities systematically.

2. Why is counting in groups more efficient than using individual tally marks?
Answer:
1. Tally marks require one mark for every object, which becomes inconvenient for large quantities.

2. Grouping represents several objects together and therefore makes counting quicker and easier.

3. A community counts objects in groups of 5. How would this improve its counting method?

Answer:

1. Every five individual objects can be treated as one group, reducing the number of counting units required.
2. Repeated groups of 5 can then be used to represent larger quantities more efficiently.

4. Why were landmark numbers an improvement over simple grouping?

Answer:

1. Landmark numbers provided easily recognisable reference points for different-sized groups.
2. These reference points allowed larger numbers to be represented more conveniently.

5. Roman numerals use several landmark symbols, yet multiplication is difficult in this system. Why?

Answer:

1. Roman landmark numbers do not follow a uniform pattern of successive powers of one fixed base.
2. Therefore, multiplication and division are less systematic than in a regular base system.

6. If the landmark numbers of a system are $1, 5, 25, 125, \ldots$, identify its base and justify your answer.

Answer:

1. Each landmark number is obtained by multiplying the previous one by 5.
2. Therefore, the number system has **base 5**.

7. If a base-5 system has 25 as a landmark number, what will be its next landmark number? Explain.

Answer:

1. Multiply the current landmark number by the base: $25 \times 5 = 125$.
2. Hence, the next landmark number is **125**.

8. A number system has landmark numbers $1, 3, 9, 27, 81, \ldots$. What is its base? Give a reason.

Answer:

1. Each successive landmark number is three times the preceding landmark number.
2. Therefore, it is a **base-3 number system**.

9. Why is the Egyptian number system described as a base-10 system?

Answer:

1. Its landmark numbers follow the sequence $1, 10, 100, 1000, \ldots$.
2. Each landmark number is obtained by grouping 10 of the previous landmark number.

10. Why does using powers of a fixed base make arithmetic easier?

Answer:

1. Landmark numbers in a base-n system follow the regular pattern $1, n, n^2, n^3, \ldots$.
2. This regular structure makes grouping and arithmetic operations more systematic.

11. In a base-4 system, determine the landmark number after 64.

Answer:

1. Since $64 = 4^3$, the next landmark number is 4^4.
2. Therefore, $4^4 = 256$, so the required landmark number is **256**.

12. Why is a place-value system more efficient than a system requiring a new symbol for every landmark number?

Answer:

1. In a place-value system, the position of a symbol indicates the landmark value associated with it.
2. Therefore, a limited collection of symbols can be reused in different positions to represent many numbers.

13. In 375, explain why the two digits 3 and 7 do not simply represent 3 and 7.
Answer:
1. Their positions give them different place values: $3 \times 100 = 300$and $7 \times 10 = 70$.
2. Thus, the value of each digit depends on its position in the numeral.

14. If the digit 5 moves from the ones place to the tens place in a decimal system, how does its value change?
Answer:
1. Its value changes from $5 \times 1 = 5$to $5 \times 10 = 50$.
2. Thus, moving one place to the left makes its place value **10 times greater**.

15. Why is zero essential for distinguishing 35 from 305?
Answer:
1. In 305, zero indicates that there are no tens while keeping 3 in the hundreds place.
2. Without this positional zero, the intended place values could not be represented unambiguously.

16. Why was treating zero as a number an important mathematical development?
Answer:
1. Zero was no longer used only to indicate an empty position in a numeral.
2. It could also participate in arithmetic as a number in its own right.

17. The later Mesopotamian system used base 60. What modern practice reflects this idea?
Answer:
1. An hour is divided into **60 minutes**, and a minute into **60 seconds**.
2. These divisions preserve the influence of the ancient base-60 or sexagesimal system.

18. Why can the Hindu number system represent very large numbers using only ten basic digits?
Answer:
1. The digits 0–9 can be repeatedly used in different positions having different powers of 10.

2. Therefore, a finite set of ten symbols can represent numbers of arbitrarily large size.

19. Compare 375 and 735 to show the importance of position in the Hindu number system.
Answer:
1. In 375, 3 represents 300 and 7 represents 70, whereas in 735, 7 represents 700 and 3 represents 30.
2. Hence, changing the positions of digits changes their place values and therefore changes the number.

20. Why was the development from landmark numbers to place value a major improvement?
Answer:
1. Landmark-number systems could require additional symbols as increasingly large landmark numbers were introduced.
2. Place value allowed the same limited set of symbols to be reused according to their positions.

21. How does $n$groups of n^2become the next landmark number in a base-n system?
Answer:
1. Combining $n$groups of n^2gives $n \times n^2 = n^3$.

2. Therefore, $n^3$is the next landmark number after n^2.

22. Why is the sequence $1, 6, 36, 216, \ldots$ **evidence of a base-6 system?**

Answer:

1. Every term is obtained by multiplying the previous landmark number by 6.
2. Hence, the landmarks are $6^0, 6^1, 6^2, 6^3, \ldots$, showing that the base is **6**.

23. How does zero prevent ambiguity in a place-value number system?

Answer:

1. Zero occupies a position where there is no quantity of that particular place value.
2. It keeps the other digits in their correct positions and therefore preserves their intended values.

24. Why was the Hindu number system well suited for scientific and mathematical calculations?

Answer:

1. It combined **base 10, place value, ten digits, and zero** into an efficient representation system.
2. These features made both writing numbers and carrying out calculations systematic and efficient.

25. What does the historical development of number systems show about mathematical ideas?

Answer:

1. Number representation progressed through **grouping, landmark numbers, bases, place value, and zero**.
2. Each development made the representation of numbers and mathematical computation increasingly efficient.

3.9. Three-mark questions with Answers of LOTS Type: (18 Questions)

Three-Mark Descriptive Questions with Answers – LOTS Type:

1. Explain why counting was important to humans from ancient times.

Answer:

1. Humans needed counting to keep track of **food, livestock and other possessions**.
2. Counting was also required for **trade and ritual offerings**.
3. People counted days to understand events such as the **new moon, full moon and seasons**.

2. Explain the meaning of a number system and numerals.

Answer:

1. A **number system** is a standard sequence of objects, names or written symbols arranged in a fixed order.
2. This standard sequence helps people represent and communicate quantities.
3. The written symbols used to represent numbers are called **numerals**.

3. Describe the early methods used by humans for counting.

Answer:

1. Early humans used **body parts, sticks and other physical objects** for counting.
2. They also made **tally marks** on surfaces such as bones to record quantities.
3. Later, number names and written symbols were developed to make counting and recording easier.

4. Explain one-to-one mapping with an example.

Answer:

1. In one-to-one mapping, every object being counted is matched with one counting object.

2. For example, a person may keep **one stick for each cow** in a herd.
3. The collection of sticks can then be used to keep track of the number of cows.

5. Explain how grouping improved early counting methods.

Answer:

1. Instead of representing every object separately, people began arranging quantities into **groups of a fixed size**.
2. Different cultures used group sizes such as **2, 5, 10 and 20**.
3. Grouping made large quantities easier to recognise, represent and count.

6. What are landmark numbers? Explain their importance.

Answer:

1. **Landmark numbers** are recognisable reference numbers that serve as anchors in a number system.
2. Other numbers can be represented with the help of these landmark numbers.
3. Their use made number representation more efficient than representing every unit separately.

7. Describe the Roman number system.

Answer:

1. The Roman system uses landmark symbols such as **I, V, X, L, C, D and M** for different values.
2. Numbers are represented by combining these landmark symbols; for example, 2367 can be written using Roman symbols.
3. Although more efficient than simple tallying, operations such as multiplication and division are difficult in this system.

8. Explain the concept of a base-n number system.

Answer:

1. In a base-n system, the first landmark number is **1**.
2. Each succeeding landmark number is obtained by multiplying the previous landmark number by n.
3. Hence, its landmark numbers are $n^0, n^1, n^2, n^3, ...$

9. Describe a base-5 number system using its landmark numbers.

Answer:

1. In a base-5 system, every new landmark number is obtained by multiplying the previous one by **5**.
2. Its landmark numbers begin as **1, 5, 25, 125, 625, ...**.
3. Thus, these landmark numbers are the successive powers of 5.

10. Explain the main features of the Egyptian number system discussed in the chapter.

Answer:

1. The Egyptian number system was in use around **3000 BCE**.
2. It used landmark numbers such as **1, 10, 100, 1000, ...**, obtained by grouping ten of the previous landmark number.
3. Therefore, it is described as a **base-10 or decimal system**.

11. What is a place-value system? Explain its main features.

Answer:

1. In a **place-value or positional system**, the position of a symbol determines the landmark number associated with it.
2. The same symbol can therefore represent different values when placed in different positions.
3. Place-value representations appeared in the **Mesopotamian, Mayan, Chinese and Indian civilisations**.

12. Explain the place values of the digits in 375.

Answer:
1. The digit 3 is in the hundreds position and represents $3 \times 100 = 300$.
2. The digit 7 is in the tens position and represents $7 \times 10 = 70$.
3. The digit 5 is in the ones position and represents $5 \times 1 = 5$, so $375 = 300 + 70 + 5$.

13. Describe three important features of the Hindu number system.
Answer:
1. The Hindu number system is a **base-10 place-value system**.
2. It uses only **ten digits, 0 to 9**, which can be reused in different positions.
3. It includes **zero**, enabling numbers to be written efficiently and unambiguously.

14. Explain the importance of zero in the development of the Hindu number system.
Answer:
1. Zero serves as a **positional digit**, indicating an empty place in a numeral.
2. Indian mathematicians also treated zero as a **number in its own right**.
3. Aryabhata used arithmetic properties of zero, while Brahmagupta later codified arithmetic involving zero.

15. Describe the role of the Bakhshali manuscript and Aryabhata in the development of Indian numerals.
Answer:
1. The **Bakhshali manuscript** contains an early use of ten digits, including zero represented by a dot.
2. The system of digits 0–9 developed in India around **2000 years ago**.
3. Around 499 CE, **Aryabhata** explained and used the Indian system of ten symbols for scientific computations.

16. Explain how the Indian number system spread to other parts of the world.
Answer:
1. The Indian numeral system was transmitted to the **Arab world around 800 CE**.
2. Scholars such as **Al-Khwārizmī and Al-Kindi** helped popularise it, and it later reached Europe and Africa.
3. Around 1200 CE, **Fibonacci** strongly advocated its adoption in Europe.

17. Describe the main stages in the evolution of number representation.
Answer:
1. Number representation developed from **counting in groups to using landmark numbers and powers of a fixed base**.
2. The next major development was **place value**, where positions represented different landmark numbers.
3. Finally, **zero** developed both as a positional digit and as a number in its own right.

18. Why is the modern Hindu number system important?
Answer:
1. It uses a finite set of **ten digits** to represent numbers through base-10 place value.
2. Its use of zero and positional notation enables efficient representation and computation.
3. The system became foundational to fields such as **science, technology, computing and accounting**.

3.10. Three-mark questions with Answers of HOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – HOTS Type:

1. Why was counting in groups a major improvement over simple tally marks?
Answer:
1. Tally marks require one separate mark for every object, making large quantities difficult to handle.

2. Grouping allows several objects to be treated as a single counting unit, reducing the number of representations needed.
3. Thus, grouping made counting and representing large quantities **faster and more efficient**.

2. A community uses the landmark numbers 1, 5, 25, 125 and 625. Identify the base and explain your reasoning.

Answer:

1. Each landmark number is obtained by multiplying the previous landmark number by **5**.
2. These numbers can be written as $5^0, 5^1, 5^2, 5^3, 5^4$.
3. Therefore, the community is using a **base-5 number system**.

3. If a number system has landmark numbers 1, 4, 16, 64 and 256, determine its base and the next landmark number.

Answer:

1. Each landmark number is obtained by multiplying the previous one by **4**, so the system has base 4.
2. The next landmark number is obtained as $256 \times 4 = 1024$.
3. Hence, the system is **base 4**, and its next landmark number is **1024**.

4. Why is a base system more convenient for arithmetic than the Roman number system?

Answer:

1. In a base-n system, landmark numbers follow the regular pattern $1, n, n^2, n^3, \ldots$.
2. In the Roman system, landmark values do not follow such a single regular power pattern, making calculations less systematic.
3. Therefore, operations such as multiplication and division are generally easier to organise in a regular base system.

5. Explain why the sequence 1, 10, 100, 1000, … shows that the Egyptian system is base 10.

Answer:

1. Each landmark number in the sequence is **10 times** the preceding landmark number.
2. The sequence can be expressed as $10^0, 10^1, 10^2, 10^3, \ldots$.
3. Therefore, the Egyptian system described in the chapter is a **base-10 or decimal system**.

6. Why is place value an improvement over having a separate symbol for every landmark number?

Answer:

1. A system with separate landmark symbols needs additional symbols as larger landmark numbers are introduced.
2. In a place-value system, the **position of a symbol** indicates the landmark number associated with it.
3. Hence, the same limited set of symbols can be reused to represent many different numbers.

7. Compare the value of the digit 7 in 375 and 735. What does this show?

Answer:

1. In 375, the digit 7 is in the tens position and has the value **70**.
2. In 735, the digit 7 is in the hundreds position and has the value **700**.
3. This shows that in a place-value system, a digit's value depends on its **position**.

8. A student says that the digit 3 has the same value wherever it appears. Use 35 and 305 to correct the student.

Answer:

1. In 35, the digit 3 is in the tens position, so its value is **30**.
2. In 305, the digit 3 is in the hundreds position, so its value is **300**.

3. Therefore, the same digit can have different values depending on its **place or position**.

9. Why is zero necessary for writing a number such as 305 correctly?

Answer:

1. In 305, zero shows that there are **no tens**.
2. It keeps 3 in the hundreds position and 5 in the ones position.
3. Thus, zero acts as a positional digit that makes the representation **unambiguous**.

10. Why was treating zero as a number, and not merely as a placeholder, an important development?

Answer:

1. As a positional digit, zero indicates the absence of a quantity in a particular place.
2. Treating zero as a number also allowed it to be included systematically in arithmetic.
3. The chapter notes that Aryabhata used properties of zero and Brahmagupta later codified arithmetic involving zero.

11. If ten groups of 100 are combined in a base-10 system, what landmark number is formed? Explain.

Answer:

1. In base 10, the landmark number 100 is 10^2.
2. Ten groups of 100 give $10 \times 100 = 1000 = 10^3$.
3. Therefore, the next landmark number formed is **1000**.

12. If five groups of 25 are combined in a base-5 system, what landmark number is formed?

Answer:

1. In base 5, $25 = 5^2$.
2. Combining five groups gives $5 \times 25 = 125 = 5^3$.
3. Hence, **125** is the next landmark number.

13. Why can the Hindu number system represent very large numbers using only ten digits?

Answer:

1. The Hindu number system uses the ten digits **0 to 9** in a base-10 place-value structure.
2. The same digits can be reused in positions corresponding to increasingly higher powers of 10.
3. Therefore, a finite set of ten digits can represent numbers of arbitrarily large size.

14. How does the number 375 demonstrate the working of the Hindu place-value system?

Answer:

1. The digit 3 represents $3 \times 10^2 = 300$.
2. The digit 7 represents $7 \times 10 = 70$, while 5 represents $5 \times 1 = 5$.
3. Hence, $375 = 300 + 70 + 5$, demonstrating how position determines each digit's value.

15. Why does the modern division of time provide evidence of the influence of an ancient number system?

Answer:

1. The later Mesopotamian system used **60 as its base**.
2. Modern time measurement still divides an hour into **60 minutes** and a minute into **60 seconds**.
3. Therefore, our measurement of time preserves an important feature of the ancient **base-60 system**.

16. A student claims that Roman numerals are just as convenient as Hindu numerals for calculations. How would you evaluate this claim?

Answer:

1. Roman numerals can represent numbers using landmark symbols, but their structure makes operations such as multiplication and division difficult.
2. Hindu numerals use a regular **base-10 place-value system with zero**.
3. Therefore, the Hindu system provides a more systematic structure for arithmetic calculations.

17. Why did the introduction of place value reduce the need for many different numeral symbols?

Answer:

1. Earlier systems could use different symbols for different landmark numbers.
2. Place value allowed the **position** of a symbol to indicate the landmark number it represented.
3. Therefore, a small collection of symbols could be reused across different positions.

18. How did the combination of base 10, place value and zero make the Hindu number system efficient?

Answer:

1. Base 10 provides a regular sequence of landmark values based on powers of 10.
2. Place value allows the same digits to represent different values, while zero marks empty positions.
3. Together, these features allow numbers to be represented **efficiently and unambiguously** with only ten digits.

19. What can you infer from the fact that Hindu numerals spread from India to the Arab world and then to Europe?

Answer:

1. The Indian numeral system provided an efficient way of representing numbers and performing computations.
2. Scholars in the Arab world adopted and transmitted the system, and it later reached Europe and other regions.
3. Its widespread adoption shows the usefulness and lasting influence of the **Indian place-value numeral system**.

20. How does the evolution from grouping to zero show improvement in mathematical thinking?

Answer:

1. **Grouping** made quantities easier to organise, while landmark numbers and bases provided systematic reference values.
2. **Place value** allowed symbols to be reused efficiently according to their positions.
3. The addition of **zero** completed a powerful system for representing numbers clearly and carrying out computations efficiently.

3.11 Five-mark questions with Answers of LOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – LOTS Type:

1. Describe the development of counting methods used by humans in early times.

Answer:

1. Humans needed counting to keep track of **food, livestock, traded goods, ritual offerings, days and seasons**.
2. Early counting involved physical objects and **one-to-one correspondence**, such as keeping one stick for each object.
3. People also used **body parts and tally marks** to record quantities.
4. Later, quantities were organised into groups of fixed sizes such as **2, 5, 10 and 20**.

5. These developments gradually led to more systematic number names, numerals and number systems.

2. Explain the importance of grouping and landmark numbers in the development of number systems.

Answer:

1. Early tally methods represented objects individually and became inconvenient for large quantities.
2. **Grouping** allowed several objects to be considered together as one unit.
3. Different cultures used convenient group sizes such as **2, 5, 10 and 20**.
4. **Landmark numbers** developed as recognisable reference numbers or anchors for representing quantities.
5. The use of grouping and landmark numbers made number representation more efficient and prepared the way for systematic base systems.

3. Describe the Roman number system and explain its limitations.

Answer:

1. The Roman number system uses landmark symbols such as **I, V, X, L, C, D and M**.
2. These symbols represent landmark values such as **1, 5, 10, 50, 100, 500 and 1000**.
3. Numbers are represented by combining these landmark symbols; for example, the chapter represents 2367 as **MMCCCLXVII**.
4. The system is more efficient than representing every object by a separate tally mark.
5. However, arithmetic operations, particularly **multiplication and division**, are difficult to perform using Roman numerals.

4. Explain the concept of a base-n number system with an example.

Answer:

1. In a base-n number system, the first landmark number is **1**.
2. Every succeeding landmark number is obtained by multiplying the previous landmark number by n.
3. Therefore, the landmark numbers are $n^0, n^1, n^2, n^3,$ ....
4. In a base-5 system, the landmark numbers are **1, 5, 25, 125, 625, ...**
5. The Egyptian system discussed in the chapter follows the same principle with base 10 and landmark numbers **1, 10, 100, 1000, ...**

5. Describe the Egyptian number system discussed in the chapter.

Answer:

1. The Egyptian number system was in use from around **3000 BCE**.
2. It used landmark numbers such as **1, 10, 100, 1000 and higher powers of 10**.
3. Each new landmark number was formed by grouping **10 of the preceding landmark number**.
4. Therefore, the Egyptian system is described as a **base-10 or decimal number system**.
5. One limitation was that new symbols were needed for increasingly higher landmark numbers.

6. Explain the concept and importance of place value in number representation.

Answer:

1. In a **place-value or positional system**, the position of a symbol determines the landmark number associated with it.
2. Thus, the same digit can represent different values when it occurs in different positions.
3. For example, in 375, 3represents 300, 7represents 70, and 5represents 5.
4. Place-value representations appeared in the **Mesopotamian, Mayan, Chinese and Indian civilisations**.
5. Place value enables a limited set of symbols to represent numbers efficiently.

7. Describe the main features of the Hindu or Indian number system.

Answer:

1. The Hindu number system is a **base-10 or decimal place-value system**.
2. It uses only **ten digits—0, 1, 2, 3, 4, 5, 6, 7, 8 and 9**.
3. The value of a digit depends on its **position** in the numeral.
4. **Zero** is used as a positional digit and is also treated as a number in its own right.
5. These features allow all numbers to be represented unambiguously and calculations to be performed efficiently.

8. Explain the importance and development of zero in the Indian number system.

Answer:

1. Zero became an essential **positional digit** in the Indian place-value system.
2. It indicates an empty position and helps avoid ambiguity while writing numbers.
3. The **Bakhshali manuscript** contains an early use of ten digits, with zero represented by a dot.
4. **Aryabhata**, around 499 CE, explicitly used arithmetic properties involving zero in scientific computation.
5. **Brahmagupta**, in 628 CE, codified arithmetic involving zero as a number in its own right.

9. Explain how the Indian number system spread to different parts of the world.

Answer:

1. The system of ten digits developed in **India around 2000 years ago**.
2. It was transmitted to the **Arab world around 800 CE**.
3. Scholars such as **Al-Khwārizmī and Al-Kindi** helped popularise the Indian numeral system there.
4. The numerals later reached **Europe and parts of Africa**, and Fibonacci advocated their adoption in Europe around 1200 CE.
5. Over time, the system became widely adopted and developed into the number system used throughout much of the world today.

10. Describe the major stages in the evolution of number representation presented in the chapter.

Answer:

1. The first major development was to **count in groups of a single fixed number**.
2. The next stage involved using several convenient **landmark numbers** as reference points.
3. Using successive **powers of a number as landmark numbers** introduced the concept of a base.
4. The development of **place value** allowed positions to denote different landmark numbers.
5. Finally, **zero** developed as both a positional digit and a number, completing the essential structure of the modern Hindu number system.

3.12 Five-mark questions with Answers of HOTS Type: (5 Questions)

Five-Mark Descriptive Questions with Answers – HOTS Type:

1. Why was the development from tally marks to grouping an important step in the evolution of number systems?

Answer:

1. Tally marks represent objects individually, so a large quantity requires a large number of marks.
2. Grouping allows several objects to be considered together as a **single counting unit**.

3. Group sizes such as **2, 5, 10 and 20** made larger quantities easier to recognise and represent.
4. Grouping reduced the effort needed to count and record large collections.
5. Therefore, grouping was an important step towards more efficient systems based on **landmark numbers and bases**.

2. Compare the Roman number system with a base-n system and explain why the latter is more suitable for calculations.

Answer:

1. The Roman system uses landmark symbols such as **I, V, X, L, C, D and M** to represent different quantities.
2. These landmark values do not form a single regular sequence of successive powers of one fixed base.
3. In a base-n system, landmark numbers follow the regular pattern $1, n, n^2, n^3, ...$
4. This regular pattern makes grouping and arithmetic operations more systematic.
5. Hence, a base system is generally more convenient for calculations, especially **multiplication and division**, than Roman numerals.

3. A newly discovered number system has landmark numbers 1, 6, 36, 216 and 1296. Analyse the system and predict the next landmark number.

Answer:

1. Each landmark number is obtained by multiplying the previous landmark number by **6**.
2. The landmarks can therefore be written as $6^0, 6^1, 6^2, 6^3, 6^4$.
3. Hence, the number system follows the principle of a **base-6 system**.
4. The next landmark number is $6^5 = 1296 \times 6 = 7776$.
5. Therefore, the next landmark number would be **7776**.

4. Explain why place value was a major improvement over number systems requiring separate symbols for landmark numbers.

Answer:

1. Systems that assign separate symbols to landmark numbers require additional symbols as increasingly large landmarks are introduced.
2. In a place-value system, the **position of a symbol determines the landmark number associated with it**.
3. Therefore, the same symbols can be reused in different positions to represent different values.
4. This greatly reduces the number of basic symbols required to write numbers.
5. Hence, place value makes number representation **more compact, systematic and efficient**.

5. Analyse the number 375 to explain how the Hindu place-value system works.

Answer:

1. The Hindu system is a **base-10 place-value system**, so its positions correspond to powers of 10.
2. In 375, the digit 3 occupies the hundreds place and represents $3 \times 10^2 = 300$.
3. The digit 7 occupies the tens place and represents $7 \times 10 = 70$.
4. The digit 5 occupies the ones place and represents $5 \times 1 = 5$.
5. Therefore, $375 = 300 + 70 + 5$, showing that the value of each digit depends on its **position**.

6. Suppose zero did not exist as a positional digit. Explain the difficulties that could arise in a place-value system.

Answer:

1. An empty place between two non-zero digits could not be shown clearly.

2. For example, it would become difficult to distinguish a number such as **305** from a number in which 3 occupies a different position.
3. Zero marks the absence of a quantity at a particular place while keeping the other digits in their correct positions.
4. This makes place-value representations **unambiguous**.
5. Therefore, zero is essential for the efficient functioning of the Hindu place-value system.

7. Why was treating zero as a number in its own right a significant mathematical development?

Answer:

1. Zero initially serves an important role as a **positional digit** in number representation.
2. Indian mathematics also developed the idea of treating zero as a **number in its own right**.
3. Aryabhata used arithmetic properties involving zero in scientific computations around 499 CE.
4. Brahmagupta later codified arithmetic involving zero in **628 CE**.
5. Thus, zero became important not only for writing numbers but also for the development of mathematical computation.

8. Compare 375, 735 and 507 to explain the importance of position and zero in the Hindu number system.

Answer:

1. In 375, the digit 3 represents **300**, while in 735 it represents **30**, showing that position changes a digit's value.
2. Similarly, 7 represents **70** in 375 but **700** in 735.
3. In 507, the digit 5 represents 500 and the digit 7 represents 7.
4. The zero in 507 shows that there are **no tens** while preserving the positions of 5 and 7.
5. Hence, both **place value and zero** are essential for representing numbers accurately and unambiguously.

9. Why did the Hindu number system become more efficient than many earlier number systems?

Answer:

1. It uses only **ten basic digits, 0 to 9**, instead of requiring endlessly new symbols for larger values.
2. Its base-10 structure provides a regular sequence of landmark values based on powers of 10.
3. Place value allows the same digit to have different values according to its position.
4. Zero marks empty positions and is also treated as an independent number.
5. Together, these features allow numbers to be represented unambiguously and arithmetic computations to be performed efficiently.

10. Analyse how the major stages in the evolution of number representation improved mathematical efficiency.

Answer:

1. **Grouping** reduced the need to represent every object separately.
2. **Landmark numbers** provided convenient reference points for representing larger quantities.
3. Using **powers of a fixed number** as landmarks created systematic base systems.
4. **Place value** allowed a limited set of symbols to be reused according to their positions.
5. Finally, **zero** made positional representation clearer and completed the essential structure of the modern Hindu number system.

11. A student says, "A number system only needs symbols; their positions do not matter." Evaluate this statement.

Answer:

1. This statement is not true for a **place-value number system**.
2. In such a system, the position of a digit determines the landmark value associated with it.
3. For example, 3 represents 300 in 375 because it occupies the hundreds position.
4. Moving the same digit to another position changes its value even though the symbol remains unchanged.
5. Therefore, both the **symbol and its position** are essential for interpreting a numeral correctly.

12. Explain how the historical evolution of the Hindu number system contributed to its worldwide adoption.

Answer:

1. India developed the system of **ten digits, including zero**, around 2000 years ago.
2. The Bakhshali manuscript provides early evidence of this system, while Aryabhata used it for elaborate computations.
3. The system was transmitted to the **Arab world around 800 CE**, where scholars such as Al-Khwārizmī and Al-Kindi helped popularise it.
4. It later reached Europe and Africa, and **Fibonacci** strongly advocated its adoption in Europe around 1200 CE.
5. Its efficient place-value structure ultimately helped it become the number system used widely throughout the world.

3.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – Applied Type:

1. A farmer has 375 coconuts. Explain how the Hindu place-value system helps him represent this quantity.

Answer:

1. The number **375** is written using only three digits: 3, 7 and 5.
2. The digit 3 is in the hundreds place, so its value is $3 \times 100 = 300$.
3. The digit 7 is in the tens place, so its value is $7 \times 10 = 70$.
4. The digit 5 is in the ones place, so its value is $5 \times 1 = 5$.
5. Therefore, $375 = 300 + 70 + 5$, demonstrating the efficiency of the Hindu **base-10 place-value system**.

2. A shopkeeper wants to record 305 boxes in his store. Explain how zero helps him represent the number correctly.

Answer:

1. The digit 3 in 305 represents **3 hundreds or 300**.
2. The digit 0 indicates that there are **no tens**.
3. The digit 5 represents **5 ones**.
4. Thus, $305 = (3 \times 100) + (0 \times 10) + (5 \times 1)$.
5. Zero therefore keeps the other digits in their correct positions and makes the numeral **unambiguous**.

3. A community decides to count objects in groups of 5. Explain how it can use the base-5 idea to organise 143 objects.

Answer:

1. In a base-5 system, useful landmark numbers include **1, 5, 25 and 125**.
2. First, 143 contains one group of 125, leaving $143 - 125 = 18$.

3. The remaining 18 contains three groups of 5, accounting for 15.
4. The remaining quantity is 3, represented by three groups of 1.
5. Hence, $143 = 125 + 5 + 5 + 5 + 1 + 1 + 1$, illustrating grouping using base-5 landmark numbers.

4. A child has to count 200 shells. Compare counting them individually with counting them in groups and suggest the better method.

Answer:

1. Counting each shell individually requires keeping track of all **200 separate objects**.
2. Large collections are harder to recognise and manage when every object is treated separately.
3. The shells can instead be arranged into convenient groups, such as groups of **10**.
4. The child can then count the groups rather than repeatedly counting individual shells.
5. Therefore, **grouping is a more efficient method** for organising and counting a large collection.

5. A student discovers a number system with landmark numbers 1, 7, 49, 343 and 2401. Identify its base and explain how you know.

Answer:

1. The first landmark number is 1.
2. Each succeeding landmark is obtained by multiplying the previous landmark by **7**.
3. The sequence can be written as $7^0, 7^1, 7^2, 7^3, 7^4$.
4. Therefore, according to the chapter's definition, this is a **base-7 number system**.
5. The next landmark number would be $2401 \times 7 = 16807$.

6. Your school wants to use Roman numerals to perform all arithmetic calculations. Explain why this may not be practical.

Answer:

1. Roman numerals use several landmark symbols such as **I, V, X, L, C, D and M**.
2. These symbols are useful for representing quantities through landmark values.
3. However, their landmark values do not follow a single regular sequence of powers of a fixed base.
4. As a result, operations such as **multiplication and division** are difficult to perform directly using Roman numerals.
5. Therefore, a regular place-value system is more practical for routine numerical calculations.

7. A student writes 507 and 57 and says that zero makes no difference. Explain why the student is incorrect.

Answer:

1. In **507**, 5 occupies the hundreds place and therefore represents 500.
2. Zero occupies the tens place and indicates that there are **no tens**.
3. The digit 7 occupies the ones place and represents 7.
4. In **57**, however, 5 occupies the tens place and represents only 50.
5. Thus, zero is essential because it preserves the correct **place values** of the other digits.

8. You are asked to design a simple number system for easy calculation. Which ideas from the chapter would you include?

Answer:

1. I would use **grouping** so that large quantities need not be represented one object at a time.
2. I would select landmark numbers that are **successive powers of a fixed base**.
3. I would use **place value** so that the position of a symbol determines its landmark value.
4. I would include **zero** to represent an empty position and avoid ambiguity.

5. These features follow the major developments that made modern number representation efficient.

9. A clock shows 2 hours, 35 minutes and 40 seconds. How does this everyday example connect with an ancient number system?

Answer:

1. The later Mesopotamian number system developed around a **base of 60**.
2. Modern time measurement retains the grouping of **60 minutes into one hour**.
3. It also groups **60 seconds into one minute**.
4. Thus, the use of 60 in measuring time reflects the ancient **sexagesimal or base-60 idea**.
5. This example shows how an ancient mathematical idea continues to influence everyday life.

10. A student has to choose between a system that creates a new symbol for every higher landmark number and a place-value system. Which should the student choose and why?

Answer:

1. A system requiring a new symbol for every higher landmark becomes increasingly difficult as numbers grow.
2. A **place-value system** uses position to indicate the landmark number associated with a symbol.
3. Therefore, the same limited collection of symbols can be reused in different positions.
4. This makes writing large numbers more compact and systematic.
5. Hence, the student should choose the **place-value system** because it provides a more efficient method of number representation.

11. A museum wants to demonstrate the historical evolution of number representation using five stations. What should each station display?

Answer:

1. The first station should demonstrate **counting in groups of a single fixed size**.
2. The second should show the use of different **landmark numbers** as reference points.
3. The third should demonstrate **powers of a fixed number**, introducing the concept of a base.
4. The fourth should illustrate **place value**, where positions correspond to landmark numbers.
5. The fifth should demonstrate **zero as both a positional digit and a number**.

12. A computer system must represent extremely large numbers using a limited number of basic symbols. Which idea from the Hindu number system would be most useful? Explain.

Answer:

1. A **place-value system** would allow the same symbols to be reused at different positions.
2. A fixed base would provide a regular sequence of landmark values for those positions.
3. A symbol such as **zero** would indicate positions where no quantity of a particular landmark value is present.
4. The Hindu system demonstrates how only **ten digits, 0–9**, can represent arbitrarily large numbers.
5. Thus, combining a **fixed base, place value and zero** provides an efficient way to represent very large numbers with a finite set of symbols.

3.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams:

Olympiad-Type Multiple Choice Questions:

1. A tribe represents numbers by making one mark for every object counted. What is the major limitation of this method?

A) It cannot represent 1
B) It requires too many marks for large numbers
C) It requires the use of zero
D) It works only with even numbers
Answer: B) It requires too many marks for large numbers
2. Which development made it easier to represent a large collection without making one mark for every object?
A) Counting backwards
B) Grouping objects
C) Removing numerals
D) Using fractions
Answer: B) Grouping objects
3. The landmark numbers of a number system are $1,4,16,64,256,\ldots$ **What is its base?**
A) 2
B) 4
C) 8
D) 16
Answer: B) 4
4. Which sequence correctly represents the landmark numbers of a base-5 system?
A) 1, 5, 10, 15, 20
B) 1, 5, 25, 125, 625
C) 5, 10, 50, 100, 500
D) 1, 2, 5, 10, 20
Answer: B) 1, 5, 25, 125, 625
5. If $n^0, n^1, n^2, n^3, \ldots$ **are the landmark numbers of a system, the system is said to have base:**
A) n
B) n^2
C) $n+1$
D) $2n$
Answer: A) n
6. In a base-6 system, which of the following is NOT a landmark number?
A) 1
B) 6
C) 36
D) 72
Answer: D) 72
7. The landmark numbers are $1,3,9,27,81,\ldots$ **What will be the next landmark number?**
A) 162
B) 243
C) 324
D) 729
Answer: B) 243
8. Why is the Egyptian number system discussed in the chapter called a decimal system?
A) It uses only ten numbers
B) It was invented in the tenth century
C) Its landmark numbers are successive powers of 10
D) Every numeral has ten symbols
Answer: C) Its landmark numbers are successive powers of 10

9. Which of the following best describes a landmark number?
A) A number that cannot be written
B) A reference number used as an anchor for representing other numbers
C) A number containing zero
D) A number written only in Roman numerals
Answer: B) A reference number used as an anchor for representing other numbers
10. The Roman numeral MMCCCLXII represents:
A) 2262
B) 2352
C) 2362
D) 2562
Answer: C) 2362
11. Which feature makes multiplication and division comparatively difficult in Roman numerals?
A) They contain zero
B) Their landmarks do not form successive powers of one fixed base
C) They use only two symbols
D) Their base is always 60
Answer: B) Their landmarks do not form successive powers of one fixed base
12. Ten groups of 100 are combined in a base-10 system. Which landmark number is obtained?
A) 100
B) 500
C) 1000
D) 10,000
Answer: C) 1000
13. Five groups of 25 are combined in a base-5 system. Which landmark number is formed?
A) 30
B) 100
C) 125
D) 625
Answer: C) 125
14. Which statement correctly describes a place-value system?
A) Every number needs a completely new symbol.
B) The value associated with a symbol depends on its position.
C) All symbols have the same value.
D) Zero cannot be used.
Answer: B) The value associated with a symbol depends on its position.
15. Consider the number 375. Which expression correctly represents it?
A) $3 \times 10 + 7 \times 10 + 5$
B) $3 \times 100 + 7 \times 10 + 5$
C) $3 \times 100 + 7 \times 100 + 5$
D) $3 + 7 + 5$
Answer: B) $3 \times 100 + 7 \times 10 + 5$
16. What happens to the value of a digit when it moves one place to the left in the Hindu decimal system?
A) It becomes half
B) It becomes 5 times greater
C) It becomes 10 times greater

D) It remains unchanged
Answer: C) It becomes 10 times greater
17. The value of 7 in 375 is 70, whereas its value in 735 is 700. This demonstrates:
A) tally counting
B) landmark grouping only
C) place value
D) Roman notation
Answer: C) place value
18. Which digit is essential for distinguishing numbers such as 35 and 305 in positional notation?
A) 1
B) 3
C) 5
D) 0
Answer: D) 0
19. What is the main positional role of zero in 405?
A) It makes 4 equal to 40.
B) It shows that there are no tens.
C) It changes the base to 5.
D) It represents five ones.
Answer: B) It shows that there are no tens.
20. Which combination best explains the efficiency of the Hindu number system?
A) Tally marks + Roman symbols
B) Base 60 + Roman symbols
C) Base 10 + place value + zero
D) Base 5 + tally marks + zero
Answer: C) Base 10 + place value + zero
21. A system has landmark numbers 1,8,64,512, **.... What is the next landmark number?**
A) 1024
B) 2048
C) 4096
D) 8192
Answer: C) 4096
22. Which ancient system has left a clear influence on the way we measure time today?
A) Roman base-5 system
B) Egyptian base-10 system
C) Mesopotamian base-60 system
D) Indian base-20 system
Answer: C) Mesopotamian base-60 system
23. Which present-day fact best demonstrates the influence of the base-60 system?
A) 10 millimetres = 1 centimetre
B) 100 centimetres = 1 metre
C) 60 seconds = 1 minute
D) 1000 metres = 1 kilometre
Answer: C) 60 seconds = 1 minute
24. Which civilisation is NOT listed in the chapter among those that developed place-value representations?
A) Mesopotamian
B) Mayan
C) Chinese

D) Roman
Answer: D) Roman
25. Which manuscript provides early evidence of the use of ten digits, including zero represented as a dot?
A) Bakhshali manuscript
B) Roman manuscript
C) Ishango manuscript
D) Fibonacci manuscript
Answer: A) Bakhshali manuscript
26. Which mathematician used the Indian system of ten symbols for elaborate scientific computations around 499 CE?
A) Fibonacci
B) Aryabhata
C) Al-Kindi
D) Euclid
Answer: B) Aryabhata
27. Who codified arithmetic involving zero as a number in 628 CE?
A) Aryabhata
B) Fibonacci
C) Brahmagupta
D) Al-Khwārizmī
Answer: C) Brahmagupta
28. Which mathematician strongly advocated the adoption of Indian numerals in Europe around 1200 CE?
A) Brahmagupta
B) Fibonacci
C) Aryabhata
D) Al-Kindi
Answer: B) Fibonacci
29. Arrange the following ideas in the developmental order presented in the chapter.
P. Place value
Q. Counting in groups
R. Zero as a positional digit and number
S. Powers of a number as landmark numbers
A) P → Q → S → R
B) Q → S → P → R
C) S → Q → R → P
D) Q → P → R → S
Answer: B) Q → S → P → R
30. Which statement best explains why the Hindu number system can represent arbitrarily large numbers with only ten basic digits?
A) Every large number is assigned a new digit.
B) Digits are repeatedly used at positions corresponding to different powers of 10.
C) Only numbers up to 999 can be represented.
D) Every digit always has the same value.
Answer: B) Digits are repeatedly used at positions corresponding to different powers of 10.

Answer Key:

1-B, 2-B, 3-B, 4-B, 5-A, 6-D, 7-B, 8-C, 9-B, 10-C, 11-B, 12-C, 13-C, 14-B, 15-B, 16-C, 17-C, 18-D, 19-B, 20-C, 21-C, 22-C, 23-C, 24-D, 25-A, 26-B, 27-C, 28-B, 29-B, 30-B

3.15 Best Practices Opportunity List from the Chapter: (10 Ideas)

“Best Practices” Opportunities for Teaching:

(1) Begin with Concrete Counting Activities:
Start the chapter by asking students to count classroom objects using **sticks, pebbles, fingers or tally marks**. This helps them experience why early humans needed systematic ways of counting before introducing formal number systems.

(2) Use a “Journey of Number Systems” Timeline:
Create a classroom timeline showing the progression from **tally marks → grouping → landmark numbers → base systems → place value → zero → Hindu number system**. Students can add examples or illustrations at each stage, helping them understand the chapter as an evolution of mathematical ideas rather than isolated facts.

(3) Teach Grouping Through Manipulatives:
Give students counters, seeds, beads or ice-cream sticks and ask them to organise the same quantity into groups of **2, 5, 10 or 20**. Let them compare which grouping method makes counting easier and connect their observations with the historical development of grouping.

(4) Use Comparative Number-System Activities:
Ask students to represent the same number using **tally marks, Roman numerals, landmark numbers and the Hindu place-value system**. Students can then discuss which representation is easiest to write, read and use for calculations.

(5) Introduce Base Systems Through Discovery:
Instead of directly giving the definition of a base, provide sequences such as $1, 5, 25, 125, \ldots$or $1, 4, 16, 64, \ldots$and ask students to identify the pattern. Guide them to discover that landmark numbers in a base-n system are $n^0, n^1, n^2, n^3, \ldots$.

(6) Use a Place-Value Abacus or Place-Value Chart:
Demonstrate numbers such as **375** using hundreds, tens and ones columns. Ask students to physically move a digit or counter from one position to another and observe how its value changes.

(7) Teach Zero Through a “What If Zero Disappeared?” Activity:
Compare numbers such as **35, 305 and 3005** and ask students what would happen if zero were removed. This encourages them to discover the importance of zero as a **positional digit** rather than simply memorising its role.

(8) Connect Ancient Mathematics with Everyday Life:
Use the example of **60 seconds = 1 minute and 60 minutes = 1 hour** to connect the ancient Mesopotamian base-60 system with students' daily lives. Such connections make the historical development of number systems more meaningful.

(9) Integrate Mathematics with History:
Assign small student groups to prepare short presentations on the **Egyptian, Mesopotamian, Roman, Chinese and Indian number systems** discussed in the chapter. Emphasise how different civilisations contributed ideas such as landmark numbers, bases and place value.

(10) Highlight India’s Contribution Through Mathematical Evidence:
Discuss the **Bakhshali manuscript, Aryabhata and Brahmagupta** while explaining the development of the ten digits and zero. Focus not only on historical names and dates but also on how these developments improved mathematical computation.

(11) Adopt Think–Pair–Share for HOTS Questions:

Pose questions such as, **"Why is the Hindu place-value system more efficient than Roman numerals?"** Students should first think individually, then discuss with a partner, and finally present their reasoning to the class.

(12) Use Exit Tickets for Formative Assessment:
At the end of the lesson, ask each student to answer three short prompts: **one new idea learned, one number system they can explain, and one reason zero/place value is important**. This gives the teacher quick feedback on conceptual understanding.

Recommended Best-Practice Focus
For this chapter, the most effective approach is **Activity → Observation → Pattern → Concept → Application**. Students should first *experience* counting and grouping, then identify patterns in landmark numbers and bases, understand **place value and zero**, and finally apply these ideas to unfamiliar number systems and problem-solving situations.

3.16 Innovations Opportunity List from the Chapter: (10 Ideas)

"Innovations" Opportunities in Teaching–Learning for 5th Standard Students:
Although the uploaded chapter is designed around the historical development of number representation, its ideas can be adapted into engaging, age-appropriate activities for **5th Standard students**.

(1) "Invent Your Own Number System" Challenge:
Divide students into small groups and ask each group to invent symbols for **1, 5, 10, 50 and 100**. They can then represent given numbers using their symbols and compare how efficient their system is with other groups. This helps children understand why **landmark numbers and standard number systems** became necessary.

(2) Ancient Number-System Learning Stations:
Create different classroom stations for **tally marks, Roman numerals, Egyptian numbers, base systems and Hindu numerals**. Students rotate through the stations and complete a small counting or number-representation challenge at each one. The activity turns the historical journey of numbers into an interactive learning experience.

(3) Human Place-Value Game:
Label classroom spaces as **Ones, Tens, Hundreds and Thousands** and give students digit cards. Students physically stand in different positions to form numbers such as 305, 350 and 503, demonstrating how the same digit changes value according to its position.

(4) "Zero the Superhero" Role Play:
Students can enact a short story in which **zero enters a number and protects an empty place**. For example, they can compare 35 with 305 and demonstrate how zero keeps 3 in the hundreds position. This makes the positional role of zero memorable and concrete.

(5) Build-a-Base with LEGO/Blocks:
Use blocks, beads or counters to introduce grouping into bases such as **2, 5 and 10**. For example, five individual counters can be exchanged for one "5-block," and five 5-blocks for one "25-block." Students thereby discover the idea behind landmark numbers such as 1,5,25,125, ...

(6) Number-System Escape Room:
Set up simple puzzles around the classroom: decode a Roman numeral, identify the base from 1,4,16,64, ..., find a missing landmark number, or explain the role of zero. Each correct answer gives students a clue leading to the final "Number Treasure."

(7) "Travel Through Time with Numbers" Digital Story:

Students can create a short digital presentation or comic showing how number representation developed from **grouping → landmark numbers → base systems → place value → zero**. Pictures, speech bubbles and simple animations can make the mathematical history accessible to younger learners.

(8) Number-System Detective Game:
Give students sequences such as **1, 3, 9, 27**, **1, 5, 25, 125**, and **1, 10, 100, 1000**. Students act as detectives, identify the multiplication pattern, predict the next landmark number and determine the base.

(9) Ancient Marketplace Simulation:
Create a pretend classroom marketplace where students "buy" and "sell" objects while recording quantities using different number representations. They can experience why a convenient and standard system of numerals is valuable for **counting and trade**, one of the needs for counting discussed in the chapter.

(10) 60-Second Mesopotamian Challenge
Give students exactly **60 seconds** to complete a simple counting challenge, then connect 60 seconds in a minute and 60 minutes in an hour to the chapter's discussion of the **base-60 Mesopotamian system**. This connects ancient mathematics with something children use every day.

(11) Math Museum Created by Students:
Convert one classroom wall into a **"Story of Numbers Museum."** Student teams can create exhibits on tally marks, grouping, Roman numerals, Egyptian numbers, place value, zero and Indian numerals, adding models and short explanations.

(12) AI-Assisted Number-System Quiz:
Under teacher supervision, students can use an age-appropriate digital or AI-based quiz prepared by the teacher to practise **matching symbols, identifying bases, finding place values and solving number-system puzzles**. The activity can provide immediate feedback and allow students to progress from simple recall to reasoning challenges.

Suggested Innovative Learning Sequence:
Explore → Create → Play → Discover → Explain → Apply
For 5th Standard students, the chapter can be transformed from a mainly historical topic into an **experiential mathematics journey**. Children first experience the difficulty of early counting, invent solutions through grouping and symbols, discover **bases and place value**, and finally appreciate why **zero and the Hindu number system** made number representation so powerful.

3.17: Practicing Questions/Question Bank :

A. Fill in the Blanks:

1. The written symbols used to represent numbers are called ________.
   Answer: Numerals

2. Marks used by early humans to keep count are called ________ **marks**.
   Answer: Tally

3. Counting objects by arranging them into fixed sets is called ________.
   Answer: Grouping

4. Easily recognisable reference numbers are called ________ **numbers**.
   Answer: Landmark

5. The Roman numeral **V** represents ________.
   Answer: 5

6. The Roman numeral **X** represents ________.
   Answer: 10

7. A number system having 10 as its base is called a ________ **system**.
   Answer: Decimal

8. The landmark numbers of a base-5 system begin with 1, 5, 25 and ________.
   Answer: 125

9. The later Mesopotamian number system used base ________.
   Answer: 60

10. The Hindu number system uses ________ basic digits.
    Answer: 10

11. The digits used in the Hindu number system are from ________ **to** ________.
    Answer: 0 to 9

12. In 375, the place value of 7 is ________.
    Answer: 70

13. The digit used to indicate an empty position in a place-value numeral is ________.
    Answer: Zero

14. An early use of zero as a dot is found in the ________ **manuscript**.
    Answer: Bakhshali

15. ________ codified arithmetic involving zero as a number.
    Answer: Brahmagupta

B. One-Mark Descriptive Questions:

1. **What is a number system?**
   Answer: A number system is a standard sequence of objects, names or written symbols arranged in a fixed order.

2. **What are numerals?**
   Answer: Numerals are written symbols used to represent numbers.

3. **What are tally marks?**
   Answer: Tally marks are marks or notches used to record quantities.

4. **What is grouping in counting?**
   Answer: Grouping means arranging objects into groups of a fixed size to make counting easier.

5. **What are landmark numbers?**
   Answer: Landmark numbers are recognisable reference numbers used as anchors for representing other numbers.

6. **Write 27 in Roman numerals.**
   Answer: 27 is written as **XXVII**.

7. **What is a decimal number system?**
   Answer: A decimal number system is a number system with base 10.

8. **What is place value?**
   Answer: Place value is the value associated with a digit because of its position in a numeral.

9. **What is the place value of 3 in 375?**
   Answer: The place value of 3 in 375 is **300**.

10. **Why is zero important in a place-value system?**
    Answer: Zero indicates an empty position and helps keep other digits in their correct places.

C. Two-Mark Descriptive Questions:

1. Why did early humans need counting?
Answer:

1. Humans needed counting to keep track of food, livestock and traded goods.
2. They also counted days and seasons for everyday activities.

2. How did grouping make counting easier?
Answer:

1. Grouping allowed several objects to be treated as one counting unit.
2. It made large collections easier and quicker to count.

3. What are landmark numbers? Why are they useful?
Answer:

1. Landmark numbers are easily recognised reference numbers.
2. They help represent and understand other numbers more conveniently.

4. Write any four Roman numeral symbols with their values.
Answer:

1. Four Roman symbols are **I = 1, V = 5, X = 10 and L = 50**.
2. These symbols represent important landmark values in the Roman system.

5. Explain a base-5 number system.
Answer:

1. In base 5, each landmark number is five times the previous one.
2. Its landmark numbers begin **1, 5, 25, 125, 625, ...**.

6. Explain the place values in 375.
Answer:

1. The digits 3, 7 and 5 represent 300, 70 and 5 respectively.
2. Therefore, $375 = 300 + 70 + 5$.

7. State two uses of zero.
Answer:

1. Zero acts as a positional digit or placeholder.
2. Zero is also treated as a number in its own right.

8. How is the ancient base-60 system connected with our daily life?
Answer:

1. We divide an hour into **60 minutes**.
2. We also divide a minute into **60 seconds**.

D. Three-Mark Descriptive Questions:

1. Describe three early methods of counting.
Answer:

1. People used physical objects such as **sticks** to keep count.
2. They used **body parts and tally marks** to represent quantities.
3. Later, they developed number names and written symbols.

2. Explain why grouping was an important development in counting.
Answer:

1. Counting every object separately becomes difficult for large collections.
2. Grouping allows several objects to be treated as one unit.
3. Common group sizes such as 2, 5, 10 and 20 made counting more efficient.

3. Explain the idea of a base using base 5 as an example.
Answer:

1. A base tells us how many units are grouped to form the next landmark number.
2. In base 5, each new landmark is five times the previous one.
3. Therefore, its landmarks are **1, 5, 25, 125, 625, ...**.

4. Explain how place value works in 375.
Answer:

1. The digit 3 is in the hundreds place and represents 300.
2. The digit 7 is in the tens place and represents 70.
3. The digit 5 is in the ones place and represents 5.

5. State three important features of the Hindu number system.
Answer:

1. It is a **base-10 place-value system**.
2. It uses the ten digits **0 to 9**.
3. It uses **zero** as an important positional digit and number.

6. Explain why zero is important using 305 as an example.
Answer:

1. In 305, 3 represents **300**.
2. Zero shows that there are **no tens**.
3. It keeps 3 and 5 in their correct positions and prevents ambiguity.

E. Five-Mark Descriptive Questions:

1. Explain the development of number representation from early counting to the modern number system.
Answer:

1. Early humans counted using objects, body parts and tally marks.
2. They began arranging quantities into **groups** to make counting easier.
3. **Landmark numbers** were introduced as convenient reference points.
4. Powers of fixed numbers led to **base systems and place value**.
5. The introduction of **zero** helped complete the efficient Hindu place-value system.

2. Describe the Hindu number system and explain why it is efficient.
Answer:

1. The Hindu number system is based on **10**.
2. It uses only ten digits from **0 to 9**.
3. The value of a digit changes according to its position.
4. Zero indicates an empty position and is also treated as a number.
5. These features allow very large numbers to be written clearly using only ten basic symbols.

3. Compare tally marks, Roman numerals and the Hindu number system.
Answer:

1. Tally marks generally represent objects individually and become inconvenient for large numbers.
2. Roman numerals improve representation by using landmark symbols.
3. However, multiplication and division are comparatively difficult using Roman numerals.
4. The Hindu system uses **base 10, place value and zero**.
5. Therefore, the Hindu system provides a more systematic method for both representing numbers and performing calculations.

4. Explain the importance of zero in five points.
Answer:

1. Zero can indicate the absence of a quantity in a particular place.
2. It acts as a **positional digit** in the Hindu number system.
3. It helps distinguish numbers such as **35 and 305**.
4. Indian mathematicians also developed zero as a **number in its own right**.

5. Its development made modern number representation and computation much more efficient.

5. Explain how the Indian numeral system became a worldwide number system.
Answer:

1. The system of ten digits developed in **India around 2000 years ago**.
2. Early evidence appears in sources such as the **Bakhshali manuscript**, and Aryabhata used the system for scientific computation.
3. The system reached the **Arab world around 800 CE**.
4. Scholars such as **Al-Khwārizmī and Al-Kindi** helped popularise it, and Fibonacci later promoted it in Europe.
5. Over time, the Hindu/Indian numeral system became widely used throughout the world.

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8th Standard Maths P1 Book Chapter 4

Chapter 4: QUADRILATERALS

4.1 Summary of the Chapter:

Chapter 4: Quadrilaterals — Summary:

(1) A **quadrilateral** is a closed figure having **four sides, four vertices, and four angles**. The chapter studies different special types of quadrilaterals and their properties.

(2) The **sum of the four interior angles of any quadrilateral is 360°**. This can be understood by dividing a quadrilateral into two triangles using a diagonal.

(3) A **rectangle** is a quadrilateral in which all four angles are **90°**. Its opposite sides are **equal and parallel**, while its diagonals are **equal and bisect each other**.

(4) A **square** has **four equal sides and four right angles**. Its diagonals are equal, **bisect each other at 90°**, and also **bisect the angles** of the square.

(5) A **parallelogram** is a quadrilateral whose opposite sides are parallel. Its opposite sides and opposite angles are **equal**, adjacent angles add up to **180°**, and its diagonals **bisect each other**.

(6) A **rhombus** is a quadrilateral in which **all four sides are equal**. Its opposite sides are parallel, opposite angles are equal, and adjacent angles add up to 180°.

(7) The diagonals of a **rhombus bisect each other at right angles** and also **bisect its angles**. Unlike the diagonals of a square, the diagonals of a rhombus need not be equal.

(8) A **kite** is a quadrilateral having **two non-overlapping pairs of adjacent sides of equal length**, while a **trapezium** is a quadrilateral having **at least one pair of parallel opposite sides**.

(9) The different quadrilaterals are related: **every square is both a rectangle and a rhombus**, and rectangles and rhombuses are special types of **parallelograms**. These relationships can be represented using **Venn diagrams**.

(10) The chapter develops these properties through **construction, measurement, geometric reasoning, congruence, parallel-line properties, and diagonal properties**, helping students distinguish between observation or conjecture and mathematical proof.

4.2 Important Formulas:

Important Formulas / Relations – Chapter 4: Quadrilaterals:

The following formulas and geometric relations are useful for solving the quantitative problems in the uploaded chapter.

1. **Angle Sum Property of a Quadrilateral**

$$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$$

Therefore, if three angles are known:

$$\boxed{\text{Fourth angle} = 360^\circ - (\text{sum of other three angles})}$$

2. **Linear Pair of Angles**

When two adjacent angles form a straight line:

$$\boxed{\angle 1 + \angle 2 = 180^\circ}$$

Hence,

$$\boxed{\angle 2 = 180^\circ - \angle 1}$$

3. **Vertically Opposite Angles**

When two lines intersect:

$$\boxed{\angle 1 = \angle 3,\ \angle 2 = \angle 4}$$

This relation is frequently used when the diagonals of quadrilaterals intersect.

4. **Rectangle – Angles**

Each angle of a rectangle is:

$$\boxed{90^\circ}$$

Therefore:

$$\boxed{\angle A = \angle B = \angle C = \angle D = 90^\circ}$$

5. **Rectangle – Opposite Sides**

For rectangle $ABCD$:

$$\boxed{AB = CD}$$
$$\boxed{AD = BC}$$

6. **Rectangle – Diagonals**

The diagonals are equal:

$$\boxed{AC = BD}$$

If diagonals AC and BD meet at O, they bisect each other:

$$\boxed{AO = OC}$$
$$\boxed{BO = OD}$$

7. **Angles Made by Two Intersecting Diagonals**

If one angle between two diagonals is x°, the four angles formed are:

$$\boxed{x^\circ, x^\circ, (180^\circ - x^\circ), (180^\circ - x^\circ)}$$

8. **Base Angles Formed by Equal Half-Diagonals**

If an isosceles triangle formed by the diagonals has vertex angle x°, each base angle is:

$$\boxed{\frac{180^\circ - x^\circ}{2}}$$

The chapter also derives:

$$\boxed{a = 90^\circ - \frac{x}{2}}$$

and

$$\boxed{b = \frac{x}{2}}$$

9. **Square – Sides**

For square $ABCD$:

$$\boxed{AB = BC = CD = DA}$$

10. **Square – Angles**

$$\boxed{\angle A = \angle B = \angle C = \angle D = 90^\circ}$$

11. **Square – Diagonals**

The diagonals are equal:

$$\boxed{AC = BD}$$

They bisect each other:

$$\boxed{AO = OC,\ BO = OD}$$

They intersect at right angles:

$$\boxed{\angle AOB = 90^\circ}$$

They also bisect the vertex angles. Hence each 90° angle is divided into:

$$\boxed{45^\circ + 45^\circ}$$

12. **Parallelogram – Opposite Sides**

For parallelogram $ABCD$:

$$\boxed{AB = CD}$$
$$\boxed{AD = BC}$$

13. **Parallelogram – Opposite Angles**

$$\boxed{\angle A = \angle C}$$

$$\boxed{\angle B = \angle D}$$

14. **Parallelogram – Adjacent Angles**

Any two adjacent angles are supplementary:

$$\boxed{\angle A + \angle B = 180^\circ}$$
$$\boxed{\angle B + \angle C = 180^\circ}$$
$$\boxed{\angle C + \angle D = 180^\circ}$$
$$\boxed{\angle D + \angle A = 180^\circ}$$

Therefore, if one angle is x°:

$$\boxed{\text{Adjacent angle} = 180^\circ - x^\circ}$$

and

$$\boxed{\text{Opposite angle} = x^\circ}$$

15. **Parallelogram – Diagonals**

If diagonals AC and BD meet at O:

$$\boxed{AO = OC}$$
$$\boxed{BO = OD}$$

Thus, the diagonals **bisect each other**.

16. **Rhombus – Sides**

For rhombus $ABCD$:

$$\boxed{AB = BC = CD = DA}$$

17. **Rhombus – Angles**

Opposite angles are equal:

$$\boxed{\angle A = \angle C}$$
$$\boxed{\angle B = \angle D}$$

Adjacent angles are supplementary:

$$\boxed{\angle A + \angle B = 180^\circ}$$

18. **Rhombus – Diagonals**

The diagonals bisect each other:

$$\boxed{AO = OC,\ BO = OD}$$

They intersect at right angles:

$$\boxed{\angle AOB = 90^\circ}$$

They also bisect the vertex angles.

19. **Trapezium – Angles**

If $PQ \parallel SR$, the interior angles on each non-parallel side are supplementary:

$$\boxed{\angle P + \angle S = 180^\circ}$$
$$\boxed{\angle Q + \angle R = 180^\circ}$$

Therefore:

$$\boxed{\angle S = 180^\circ - \angle P}$$
$$\boxed{\angle R = 180^\circ - \angle Q}$$

20. **Isosceles Trapezium**

If the non-parallel sides are equal, for example:

$$\boxed{UX = VW}$$

then the base angles are equal:

$$\boxed{\angle U = \angle V}$$

and similarly the other pair of base angles are equal.

21. **Kite – Equal Sides**

A kite has two non-overlapping pairs of adjacent equal sides. For example:

$$\boxed{AB = AD}$$

and

$$\boxed{CB = CD}$$

This equality of sides is important while solving problems involving congruent triangles in a kite.

Quick Formula Box

For examination problems, the most important relations to remember are:

$$\boxed{\text{Angles of quadrilateral} = 360^\circ}$$

$$\boxed{\text{Adjacent angles of parallelogram} = 180^\circ}$$

$$\boxed{\text{Opposite angles of parallelogram are equal}}$$

$$\boxed{\text{Diagonals of parallelogram bisect each other}}$$

$$\boxed{\text{Diagonals of rectangle are equal and bisect each other}}$$

$$\boxed{\text{Diagonals of square are equal, perpendicular and bisect each other}}$$

$$\boxed{\text{Diagonals of rhombus bisect each other at } 90^\circ}$$

$$\boxed{\text{Each angle of rectangle/square} = 90^\circ}$$

$$\boxed{\text{Angles on the same leg of a trapezium} = 180^\circ}$$

These are the principal relations required for solving the **angle, side-length, diagonal, construction, and reasoning-based quantitative problems** in Chapter 4.

4.3 Fill-Up the Blank LOTS type questions with Answers: (35 Questions)

Fill in the Blanks – LOTS Type:

1. A quadrilateral has ______ sides.
   Answer: four
2. The sum of the interior angles of a quadrilateral is ______.
   Answer: 360°
3. A rectangle has all its angles equal to ______.
   Answer: 90°
4. The opposite sides of a rectangle are ______ in length.
   Answer: equal
5. The opposite sides of a rectangle are ______ to each other.
   Answer: parallel
6. The diagonals of a rectangle are ______ in length.
   Answer: equal
7. The diagonals of a rectangle ______ each other.
   Answer: bisect
8. A square has all its sides ______ in length.
   Answer: equal
9. Each angle of a square measures ______.
   Answer: 90°
10. The diagonals of a square intersect each other at an angle of ______.
    Answer: 90°
11. The diagonals of a square ______ the angles of the square.
    Answer: bisect
12. A parallelogram is a quadrilateral whose opposite sides are ______.
    Answer: parallel
13. The opposite sides of a parallelogram are ______.
    Answer: equal

14. The opposite angles of a parallelogram are ______.
    Answer: equal
15. The adjacent angles of a parallelogram add up to ______.
    Answer: 180°
16. The diagonals of a parallelogram ______ each other.
    Answer: bisect
17. A quadrilateral in which all four sides are equal is called a ______.
    Answer: rhombus
18. The opposite sides of a rhombus are ______.
    Answer: parallel
19. The opposite angles of a rhombus are ______.
    Answer: equal
20. The diagonals of a rhombus intersect each other at ______ angles.
    Answer: right
21. The diagonals of a rhombus ______ the angles of the rhombus.
    Answer: bisect
22. A kite has two pairs of ______ equal sides.
    Answer: adjacent
23. A trapezium has at least one pair of ______ opposite sides.
    Answer: parallel
24. In a trapezium, the interior angles on the same non-parallel side add up to ______.
    Answer: 180°
25. A trapezium whose non-parallel sides are equal is called an ______ trapezium.
    Answer: isosceles
26. Every square is also a ______.
    Answer: rectangle
27. Every rectangle is a special type of ______.
    Answer: parallelogram
28. Every rhombus is also a ______.
    Answer: parallelogram
29. A square is both a rectangle and a ______.
    Answer: rhombus
30. When two diagonals cross at their midpoints, we say that they ______ each other.
    Answer: bisect

4.4 Fill-Up the Blank HOTS type questions with Answers: (30 Questions)

Fill in the Blanks – HOTS Type:

1. If three angles of a quadrilateral are 80°, 95°, and 105°, the fourth angle is ______.
   Answer: 80°
2. If one angle of a parallelogram is 70°, its adjacent angle is ______.
   Answer: 110°
3. If one angle of a parallelogram is 125°, the angle opposite to it is ______.
   Answer: 125°
4. If one angle of a rhombus is 60°, its adjacent angle is ______.
   Answer: 120°
5. If one angle of a rectangle is 90°, the sum of the remaining three angles is ______.
   Answer: 270°
6. If the diagonals of a rectangle intersect at O and $AO = 6$cm, then $OC =$ ______ cm.
   Answer: 6 cm

7. If one diagonal of a rectangle is 12 cm, the other diagonal is ______ cm.
   Answer: 12 cm
8. If the diagonals of a parallelogram bisect each other and $AO = 5$cm, then diagonal $AC =$______ cm.
   Answer: 10 cm
9. If the diagonals of a square intersect at $O$and $AO = 4$cm, then $OC =$______ cm.
   Answer: 4 cm
10. If one angle formed at the intersection of the diagonals of a rhombus is 90°, the adjacent angle there is ______.
    Answer: 90°
11. If a quadrilateral has four equal sides and one angle is 90°, the quadrilateral is a ______.
    Answer: square
12. If a quadrilateral has four right angles and all sides equal, it is both a rectangle and a ______.
    Answer: rhombus
13. If the opposite sides of a quadrilateral are parallel and one angle is 65°, the opposite angle is ______.
    Answer: 65°
14. If one angle of a parallelogram is $x°$, its adjacent angle is ______.
    Answer: $(180 - x)°$
15. If one angle between the diagonals of a rectangle is $x°$, the vertically opposite angle is ______.
    Answer: $x°$
16. If one angle between two intersecting diagonals is 40°, its adjacent angle is ______.
    Answer: 140°
17. If one angle of an isosceles trapezium is 75°, the angle on the same non-parallel side is ______.
    Answer: 105°
18. If a trapezium has parallel sides $PQ \parallel SR$and $\angle P = 110°$, then $\angle S =$______.
    Answer: 70°
19. If the diagonals of a square bisect its 90°vertex angle, each part measures ______.
    Answer: 45°
20. If the diagonals of a rhombus bisect a 120°angle, each half is ______.
    Answer: 60°
21. If two adjacent angles of a parallelogram are $3x°$and $2x°$, then $x =$______.
    Answer: 36°
22. If opposite angles of a parallelogram are $2x + 10°$and $4x - 30°$, then $x =$______.
    Answer: 20°
23. If three angles of a quadrilateral are 90°, 90°, and 90°, the fourth angle must be ______.
    Answer: 90°
24. A quadrilateral whose diagonals are equal and bisect each other is a ______.
    Answer: rectangle
25. A quadrilateral whose diagonals bisect each other at right angles and whose sides are all equal is a ______.
    Answer: rhombus

26. If a rhombus has one angle equal to 50°, its opposite angle is ______ and each adjacent angle is ______.
    Answer: 50°, 130°
27. If one pair of opposite sides of a quadrilateral is parallel, the quadrilateral may be classified as a ______.
    Answer: trapezium
28. If the non-parallel sides of a trapezium are equal, it becomes an ______ trapezium.
    Answer: isosceles
29. A square belongs to the sets of rectangles, rhombuses, and ______.
    Answer: parallelograms
30. If the sum of two adjacent angles of a quadrilateral is 180°because they lie on the same side of a transversal between parallel lines, the angles are called ______.
    Answer: supplementary angles

4.5 One-mark questions with Answers of LOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – LOTS Type:

1. **What is a rectangle?**
   Answer: A rectangle is a quadrilateral in which all four angles are 90°.
2. **State the angle-sum property of a quadrilateral.**
   Answer: The sum of the measures of all four interior angles of a quadrilateral is 360°.
3. **What can you say about the opposite sides of a rectangle?**
   Answer: The opposite sides of a rectangle are equal and parallel.
4. **State one property of the diagonals of a rectangle.**
   Answer: The diagonals of a rectangle are equal in length and bisect each other.
5. **What is a square?**
   Answer: A square is a quadrilateral having four equal sides and four right angles.
6. **At what angle do the diagonals of a square intersect?**
   Answer: The diagonals of a square intersect each other at 90°.
7. **What do the diagonals of a square do to its angles?**
   Answer: The diagonals of a square bisect its angles.
8. **What is a parallelogram?**
   Answer: A parallelogram is a quadrilateral in which the opposite sides are parallel.
9. **What is the relation between the opposite sides of a parallelogram?**
   Answer: The opposite sides of a parallelogram are equal.
10. **What is the relation between the opposite angles of a parallelogram?**
    Answer: The opposite angles of a parallelogram are equal.
11. **What is the sum of two adjacent angles of a parallelogram?**
    Answer: The sum of two adjacent angles of a parallelogram is 180°.
12. **What happens to the diagonals of a parallelogram at their point of intersection?**
    Answer: The diagonals of a parallelogram bisect each other.
13. **What is a rhombus?**
    Answer: A rhombus is a quadrilateral in which all four sides have the same length.
14. **What is the relation between the opposite angles of a rhombus?**
    Answer: The opposite angles of a rhombus are equal.
15. **At what angle do the diagonals of a rhombus intersect?**
    Answer: The diagonals of a rhombus intersect each other at 90°.
16. **State one property of the diagonals of a rhombus.**
    Answer: The diagonals of a rhombus bisect each other at right angles.
17. **What do the diagonals of a rhombus do to its vertex angles?**
    Answer: The diagonals of a rhombus bisect its vertex angles.

18. **What is a kite?**
    Answer: A kite is a quadrilateral having two non-overlapping pairs of adjacent sides of equal length.
19. **What is a trapezium?**
    Answer: A trapezium is a quadrilateral having at least one pair of parallel opposite sides.
20. **What is an isosceles trapezium?**
    Answer: An isosceles trapezium is a trapezium whose non-parallel sides are equal in length.
21. **What is the sum of the interior angles on the same non-parallel side of a trapezium?**
    Answer: The interior angles on the same non-parallel side of a trapezium add up to 180°.
22. **What is the relation between the base angles of an isosceles trapezium?**
    Answer: The angles opposite the equal non-parallel sides of an isosceles trapezium are equal.
23. **Is every square a rectangle?**
    Answer: Yes, every square is a rectangle because it has four right angles.
24. **Is every square a rhombus?**
    Answer: Yes, every square is a rhombus because all its sides have equal length.
25. **Is every rhombus a parallelogram?**
    Answer: Yes, every rhombus is a parallelogram because its opposite sides are parallel.
26. **Is every rectangle a parallelogram?**
    Answer: Yes, every rectangle is a parallelogram because its opposite sides are parallel.
27. **What does it mean when two diagonals bisect each other?**
    Answer: It means that each diagonal divides the other diagonal into two equal parts.
28. **What is the measure of each angle of a square?**
    Answer: Each angle of a square measures 90°.
29. **What is the relationship between a square, rectangle and rhombus?**
    Answer: A square is both a rectangle and a rhombus.
30. **Which special quadrilateral has equal diagonals that bisect each other at right angles?**
    Answer: A square has equal diagonals that bisect each other at right angles.

4.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – HOTS Type:

1. **A quadrilateral has three angles measuring 90°each; what is its fourth angle?**
   Answer: The fourth angle is 90°, because the sum of the angles of a quadrilateral is 360°.
2. **One angle of a parallelogram is 65°; what is its opposite angle?**
   Answer: The opposite angle is 65°, because opposite angles of a parallelogram are equal.
3. **One angle of a parallelogram is 70°; find its adjacent angle.**
   Answer: The adjacent angle is 110°, because adjacent angles of a parallelogram add up to 180°.
4. **Can a parallelogram have angles 80°and 80°as adjacent angles? Give a reason.**
   Answer: No, because adjacent angles of a parallelogram must add up to 180°.

5. **If one angle of a parallelogram is $120°$, what are the measures of the other three angles?**
   Answer: The other three angles are $60°$, $120°$, and $60°$.
6. **The diagonals of a parallelogram meet at O, and $AO = 7$cm; what is OC?**
   Answer: $OC = 7$cm because the diagonals of a parallelogram bisect each other.
7. **A rectangle has one diagonal of length 10 cm; what is the length of its other diagonal?**
   Answer: The other diagonal is 10 cm because the diagonals of a rectangle are equal.
8. **The diagonals of a rectangle intersect at O, and $AC = 16$cm; what is AO?**
   Answer: $AO = 8$cm because the diagonals of a rectangle bisect each other.
9. **A quadrilateral has four right angles and all four sides equal; identify it.**
   Answer: The quadrilateral is a square because it has four equal sides and four $90°$angles.
10. **Why can every square be considered a rectangle?**
    Answer: Every square is a rectangle because all four of its angles are $90°$.
11. **Why can every square also be considered a rhombus?**
    Answer: Every square is a rhombus because all four of its sides are equal.
12. **Can every rectangle be called a square? Give a reason.**
    Answer: No, because a rectangle need not have all four sides equal.
13. **Can every rhombus be called a square? Give a reason.**
    Answer: No, because the angles of a rhombus need not all be $90°$.
14. **One angle of a rhombus is $50°$; what is its opposite angle?**
    Answer: Its opposite angle is $50°$, because opposite angles of a rhombus are equal.
15. **One angle of a rhombus is $110°$; find an adjacent angle.**
    Answer: An adjacent angle is $70°$, because adjacent angles of a rhombus are supplementary.
16. **The diagonals of a rhombus intersect at O; what is $\angle AOB$?**
    Answer: $\angle AOB = 90°$, because the diagonals of a rhombus bisect each other at right angles.
17. **A diagonal of a rhombus bisects a $100°$vertex angle; what is each resulting angle?**
    Answer: Each resulting angle is $50°$, because a diagonal of a rhombus bisects its vertex angle.
18. **A diagonal of a square divides one vertex angle into two equal parts; what is each part?**
    Answer: Each part is $45°$, because each $90°$angle of a square is bisected by its diagonal.
19. **If two diagonals are equal and bisect each other, which quadrilateral discussed in the chapter can be formed?**
    Answer: A rectangle can be formed when the diagonals are equal and bisect each other.
20. **If the diagonals of a quadrilateral are perpendicular, can you always conclude that it is a rhombus?**
    Answer: No, because a kite can also have perpendicular diagonals.
21. **In trapezium $PQRS$, $PQ \parallel SR$and $\angle P = 125°$; find $\angle S$.**
    Answer: $\angle S = 55°$, because the interior angles on the same non-parallel side add up to $180°$.
22. **In trapezium $PQRS$, $PQ \parallel SR$and $\angle Q = 105°$; what is $\angle R$?**
    Answer: $\angle R = 75°$, because $\angle Q + \angle R = 180°$.

23. **A trapezium has equal non-parallel sides; what special type of trapezium is it?**
    Answer: It is an isosceles trapezium because its non-parallel sides have equal lengths.
24. **If one base angle of an isosceles trapezium is $80°$, what is the other base angle adjacent to the same base?**
    Answer: The other base angle is $80°$, because the corresponding base angles of an isosceles trapezium are equal.
25. **A quadrilateral has two non-overlapping pairs of adjacent equal sides; what can you infer about it?**
    Answer: The quadrilateral is a kite because this is the defining side property of a kite.
26. **If the diagonals of a square intersect at $O$and $AO = 5$cm, what is OC?**
    Answer: $OC = 5$cm because the diagonals of a square bisect each other.
27. **If one angle between two intersecting diagonals is $60°$, what is its adjacent angle?**
    Answer: The adjacent angle is $120°$, because adjacent angles forming a linear pair add up to $180°$.
28. **If one angle between two intersecting diagonals is $75°$, what is its vertically opposite angle?**
    Answer: Its vertically opposite angle is $75°$, because vertically opposite angles are equal.
29. **A parallelogram has one right angle; what special quadrilateral does it become?**
    Answer: It becomes a rectangle because its remaining angles also become $90°$.
30. **A rhombus has one right angle; what special quadrilateral does it become?**
    Answer: It becomes a square because it then has four equal sides and four right angles.

4.7 Two-mark questions with Answers of LOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – LOTS Type:

1. What is a rectangle? State one property of its sides.

Answer:

1. A rectangle is a quadrilateral in which all four angles are $90°$.
2. Its opposite sides are equal and parallel to each other.

2. State any two properties of the diagonals of a rectangle.

Answer:

1. The diagonals of a rectangle are equal in length.
2. The diagonals bisect each other at their point of intersection.

3. What is a square? State one property of its sides.

Answer:

1. A square is a quadrilateral in which all four angles are $90°$.
2. All four sides of a square are equal in length.

4. State any two properties of the diagonals of a square.

Answer:

1. The diagonals of a square are equal in length and bisect each other.
2. They intersect at $90°$and also bisect the angles of the square.

5. Define a parallelogram and state one property of its sides.

Answer:

1. A parallelogram is a quadrilateral in which the opposite sides are parallel.
2. The opposite sides of a parallelogram are equal in length.

6. State two properties of the angles of a parallelogram.

Answer:

1. The opposite angles of a parallelogram are equal.

2. Every pair of adjacent angles of a parallelogram adds up to 180°.

7. What happens to the diagonals of a parallelogram?
Answer:
1. The diagonals of a parallelogram bisect each other.
2. Hence, their point of intersection is the midpoint of both diagonals.

8. What is a rhombus? State one property of its sides.
Answer:
1. A rhombus is a quadrilateral in which all four sides have the same length.
2. The opposite sides of a rhombus are parallel to each other.

9. State any two properties of the angles of a rhombus.
Answer:
1. The opposite angles of a rhombus are equal.
2. The adjacent angles of a rhombus add up to 180°.

10. State two important properties of the diagonals of a rhombus.
Answer:
1. The diagonals of a rhombus bisect each other at right angles.
2. The diagonals also bisect the angles of the rhombus.

11. What is a kite? Describe its equal sides.
Answer:
1. A kite is a quadrilateral having two non-overlapping pairs of adjacent sides of equal length.
2. Thus, the equal sides occur next to each other rather than as opposite pairs.

12. What is a trapezium? State its basic property.
Answer:
1. A trapezium is a quadrilateral having at least one pair of parallel opposite sides.
2. If $PQ \parallel SR$, the angles on each non-parallel side are supplementary.

13. What is an isosceles trapezium? State one angle property.
Answer:
1. An isosceles trapezium is a trapezium whose non-parallel sides have equal lengths.
2. The angles opposite the equal sides are equal.

14. State the angle-sum property of a quadrilateral and explain how it is useful.
Answer:
1. The sum of the four interior angles of a quadrilateral is 360°.
2. Therefore, an unknown angle can be found by subtracting the sum of the other three angles from 360°.

15. What does it mean when the diagonals of a quadrilateral bisect each other?
Answer:
1. Bisecting means dividing a quantity into two equal parts.
2. Hence, when diagonals bisect each other, their point of intersection divides each diagonal into two equal segments.

16. State two differences between a rectangle and a square.
Answer:
1. A rectangle has equal opposite sides, whereas a square has all four sides equal.
2. The diagonals of a square intersect at 90°, while this is not a required property of a rectangle.

17. State two similarities between a square and a rectangle.
Answer:
1. Both a square and a rectangle have four angles of 90°.
2. In both, the diagonals are equal in length and bisect each other.

18. State two similarities between a rhombus and a parallelogram.

Answer:

1. In both figures, the opposite sides are parallel and the opposite angles are equal.
2. In both figures, the diagonals bisect each other.

19. What can be said about the adjacent and opposite angles of a parallelogram?

Answer:

1. Adjacent angles of a parallelogram are supplementary and add up to $180°$.
2. Opposite angles of a parallelogram are equal.

20. Explain the relationship between a square, rectangle, and rhombus.

Answer:

1. A square is a rectangle because all its angles are $90°$.
2. A square is also a rhombus because all its sides are equal.

21. State two angle relations in a trapezium $PQRS$, where $PQ \parallel SR$.

Answer:

1. $\angle S + \angle P = 180°$.
2. $\angle R + \angle Q = 180°$.

22. How are the diagonals of a rectangle divided at their point of intersection?

Answer:

1. If diagonals AC and BD intersect at O, then $AO = OC$.
2. Similarly, $BO = OD$, because the diagonals bisect each other.

23. State two properties that distinguish the diagonals of a rhombus.

Answer:

1. The diagonals of a rhombus bisect each other at $90°$.
2. Each diagonal also bisects a pair of opposite angles of the rhombus.

24. What happens when two lines intersect and form vertically opposite angles and linear pairs?

Answer:

1. The vertically opposite angles formed by the intersecting lines are equal.
2. Each pair of adjacent angles forming a straight line adds up to $180°$.

25. How can you identify a square using its sides and angles?

Answer:

1. Check whether all four sides of the quadrilateral are equal.
2. Check whether all four interior angles are $90°$; if both conditions hold, it is a square.

4.8. Two-mark questions with Answers of HOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – HOTS Type:

1. One angle of a parallelogram is $65°$. Find its adjacent and opposite angles.

Answer:

1. The adjacent angle is $180° - 65° = 115°$.
2. The opposite angle is $65°$, because opposite angles of a parallelogram are equal.

2. Three angles of a quadrilateral are $75°$, $95°$, and $110°$. Find the fourth angle.

Answer:

1. The sum of the given angles is $75° + 95° + 110° = 280°$.
2. The fourth angle is $360° - 280° = 80°$.

3. A parallelogram has one angle equal to $120°$. Determine the measures of its remaining angles.

Answer:

1. The opposite angle is $120°$, while each adjacent angle is $180° - 120° = 60°$.
2. Therefore, its four angles are $120°, 60°, 120°$, and $60°$.

4. The diagonals of parallelogram $ABCD$ intersect at O. If $AO = 6$cm and $BO = 4$cm, find AC and BD.
Answer:
1. Since the diagonals bisect each other, $AC = 2 \times AO = 12$cm.
2. Similarly, $BD = 2 \times BO = 8$cm.

5. A rectangle has a diagonal $AC = 14$cm. If its diagonals meet at O, find AO and BD.
Answer:
1. Since the diagonals bisect each other, $AO = \frac{14}{2} = 7$cm.
2. Since the diagonals of a rectangle are equal, $BD = AC = 14$cm.

6. A quadrilateral has four equal sides and one angle of 90°. What special quadrilateral is it? Give a reason.
Answer:
1. The quadrilateral is a **square** because its four equal sides make it a rhombus and the right angle gives it the square condition.
2. A square has all sides equal and all four angles equal to 90°.

7. Why is every square a rectangle but every rectangle is not necessarily a square?
Answer:
1. Every square is a rectangle because all four of its angles are 90°.
2. Every rectangle is not necessarily a square because a rectangle need not have all four sides equal.

8. Why is every square a rhombus but every rhombus is not necessarily a square?
Answer:
1. Every square is a rhombus because all four sides of a square have equal length.
2. Every rhombus is not necessarily a square because its angles need not all be 90°.

9. One angle of a rhombus is 70°. Find its opposite angle and one adjacent angle.
Answer:
1. Its opposite angle is 70°, because opposite angles of a rhombus are equal.
2. Its adjacent angle is $180^\circ - 70^\circ = 110^\circ$.

10. A diagonal of a rhombus bisects a 120° vertex angle. Find the two angles formed.
Answer:
1. A diagonal of a rhombus bisects its vertex angle into two equal parts.
2. Therefore, each angle is $120^\circ \div 2 = 60^\circ$.

11. Can you conclude that a quadrilateral is a rhombus merely because its diagonals are perpendicular? Explain.
Answer:
1. No, perpendicular diagonals alone are not sufficient to prove that a quadrilateral is a rhombus.
2. A kite can also have perpendicular diagonals, so additional properties must be checked.

12. One angle between two intersecting diagonals is 60°. Find its vertically opposite and adjacent angles.
Answer:
1. The vertically opposite angle is 60°.
2. Each adjacent angle is $180^\circ - 60^\circ = 120^\circ$.

13. In trapezium $PQRS$, $PQ \parallel SR$ and $\angle P = 130^\circ$. Find $\angle S$ and explain.
Answer:
1. Since $PQ \parallel SR$, $\angle P + \angle S = 180^\circ$.
2. Therefore, $\angle S = 180^\circ - 130^\circ = 50^\circ$.

14. In trapezium $PQRS$, $PQ \parallel SR$, $\angle Q = 105^\circ$, and $\angle P = 120^\circ$. Find $\angle R$ and $\angle S$.

Answer:
1. $\angle R = 180° - 105° = 75°$.
2. $\angle S = 180° - 120° = 60°$.

15. A trapezium has equal non-parallel sides. What additional conclusion can you make about its angles?

Answer:
1. The trapezium is an **isosceles trapezium** because its non-parallel sides are equal.
2. Its angles opposite the equal sides are equal.

16. A parallelogram has one right angle. What type of special quadrilateral does it become? Explain.

Answer:
1. If one angle is $90°$, its adjacent angle is also $180° - 90° = 90°$.
2. Thus, all four angles become $90°$, making the parallelogram a rectangle.

17. A rhombus has one angle measuring $90°$. What special quadrilateral does it become?

Answer:
1. Since adjacent angles are supplementary, all four angles of the rhombus become $90°$.
2. As it already has four equal sides, the rhombus becomes a square.

18. A square has a vertex angle of $90°$. What angles are formed when a diagonal passes through that vertex?

Answer:
1. A diagonal of a square bisects its vertex angle into two equal parts.
2. Therefore, the two angles formed are $45°$and $45°$.

19. A student says, "If the diagonals of a quadrilateral bisect each other, it can be identified as a parallelogram." Is the statement correct?

Answer:
1. Yes, the statement is correct according to the relationship established in the chapter.
2. A quadrilateral whose diagonals bisect each other must be a parallelogram.

20. A student claims that every isosceles trapezium is a parallelogram. Is the claim correct? Why?

Answer:
1. No, the claim is not correct because an isosceles trapezium need not have both pairs of opposite sides parallel.
2. Its other pair of opposite sides may not be parallel, so it need not be a parallelogram.

21. A quadrilateral has opposite angles equal. What can you infer about the quadrilateral?

Answer:
1. According to the chapter, a quadrilateral in which both pairs of opposite angles are equal is a parallelogram.
2. Thus, its opposite sides will also be parallel and equal.

22. A quadrilateral has three right angles. What can you deduce about the fourth angle and the quadrilateral?

Answer:
1. The fourth angle is $360° - 270° = 90°$.
2. Since all four angles are right angles, the quadrilateral is a rectangle.

23. The diagonals of a rectangle intersect at an angle of $60°$. Find the other three angles at the point of intersection.

Answer:
1. The vertically opposite angle is also $60°$.
2. The other two angles are $120°$each because each forms a linear pair with $60°$.

24. Two adjacent angles of a parallelogram are $x°$and $2x°$. Find x.
Answer:

1. Since adjacent angles are supplementary, $x + 2x = 180°$.
2. Therefore, $3x = 180°$and $\boxed{x = 60°}$.

25. The opposite angles of a parallelogram are $(2x + 10)°$and $(3x - 20)°$. Find x.
Answer:

1. Since opposite angles are equal, $2x + 10 = 3x - 20$.
2. Therefore, $\boxed{x = 30°}$.

4.9. Three-mark questions with Answers of LOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – LOTS Type:
1. Define a rectangle and state any two of its properties.
Answer:

1. A **rectangle** is a quadrilateral in which all four angles are 90°.
2. Its opposite sides are equal and parallel to each other.
3. Its diagonals are equal in length and bisect each other.

2. Define a square and state any two properties of its diagonals.
Answer:

1. A **square** is a quadrilateral having four equal sides and four angles of 90°.
2. Its diagonals are equal in length and bisect each other at 90°.
3. Its diagonals also bisect the angles of the square.

3. Define a parallelogram and state two properties of its angles.
Answer:

1. A **parallelogram** is a quadrilateral in which opposite sides are parallel.
2. The opposite angles of a parallelogram are equal.
3. The adjacent angles of a parallelogram add up to 180°.

4. State three important properties of a parallelogram.
Answer:

1. The opposite sides of a parallelogram are equal.
2. The opposite angles are equal, while adjacent angles add up to 180°.
3. The diagonals of a parallelogram bisect each other.

5. Define a rhombus and state two of its properties.
Answer:

1. A **rhombus** is a quadrilateral in which all four sides have the same length.
2. Its opposite sides are parallel, and its opposite angles are equal.
3. Its adjacent angles add up to 180°.

6. State three properties of the diagonals of a rhombus.
Answer:

1. The diagonals of a rhombus bisect each other.
2. They intersect each other at right angles, that is, at 90°.
3. The diagonals also bisect the angles of the rhombus.

7. Explain the angle properties of a parallelogram.
Answer:

1. The opposite angles of a parallelogram are equal.
2. Each pair of adjacent angles is supplementary and adds up to 180°.
3. Thus, if one angle is $x°$, its opposite angle is $x°$and each adjacent angle is $(180 - x)°$.

8. What is a trapezium? State two of its angle properties.
Answer:

1. A **trapezium** is a quadrilateral having at least one pair of parallel opposite sides.

2. If $PQ \parallel SR$, then $\angle P + \angle S = 180°$.
3. Similarly, $\angle Q + \angle R = 180°$.

9. What is an isosceles trapezium? State two of its properties.

Answer:

1. An **isosceles trapezium** is a trapezium whose non-parallel sides are equal in length.
2. The angles opposite the equal sides are equal.
3. The angles on each non-parallel side are supplementary because the other pair of sides is parallel.

10. Define a kite and describe its side arrangement.

Answer:

1. A **kite** is a quadrilateral with two non-overlapping pairs of adjacent sides having the same length.
2. In a kite $ABCD$, for example, one pair may be $AB = AD$.
3. The other pair may be $CB = CD$, so the equal sides occur in adjacent pairs.

11. State the angle-sum property of a quadrilateral and explain how an unknown angle can be found.

Answer:

1. The sum of the four interior angles of a quadrilateral is $360°$.
2. First, add the measures of the three known angles.
3. Subtract this sum from $360°$to obtain the fourth angle.

12. Explain the properties of the sides and diagonals of a rectangle.

Answer:

1. The opposite sides of a rectangle are equal in length.
2. The opposite sides are also parallel to each other.
3. Its diagonals are equal in length and bisect each other.

13. Explain how the diagonals of a parallelogram divide each other.

Answer:

1. The diagonals of a parallelogram intersect at a common point.
2. The point of intersection is the midpoint of both diagonals.
3. Therefore, each diagonal divides the other into two equal parts.

14. State three similarities between a square and a rectangle.

Answer:

1. Both a square and a rectangle have four right angles.
2. In both figures, opposite sides are parallel.
3. Their diagonals are equal in length and bisect each other.

15. State three similarities between a rhombus and a parallelogram.

Answer:

1. The opposite sides of both a rhombus and a parallelogram are parallel.
2. Their opposite angles are equal and their adjacent angles add up to $180°$.
3. The diagonals of both figures bisect each other.

16. Explain the relationship among a square, rectangle, and rhombus.

Answer:

1. A square is a **rectangle** because all four of its angles are $90°$.
2. A square is also a **rhombus** because all four of its sides are equal.
3. Thus, a square possesses important properties of both a rectangle and a rhombus.

17. Explain what it means when the diagonals of a quadrilateral bisect each other.

Answer:

1. To **bisect** means to divide something into two equal parts.
2. If diagonals $AC$and BDintersect at O, then $AO = OC$and $BO = OD$.
3. Thus, $O$is the midpoint of both diagonals.

18. Describe three properties that distinguish a square from a general parallelogram.
Answer:
1. All four sides of a square are equal in length.
2. All four angles of a square are 90°.
3. Its diagonals are equal, intersect at 90°, and bisect its angles.

19. What angle relationships arise when two diagonals intersect?
Answer:
1. The vertically opposite angles formed by the intersecting diagonals are equal.
2. Each angle and its adjacent angle form a linear pair whose sum is 180°.
3. Hence, if one angle is $x°$, the four angles are $x°, x°, (180-x)°, (180-x)°$.

20. Explain the main differences between a rectangle and a rhombus.
Answer:
1. A rectangle has four right angles, whereas a rhombus need not have four right angles.
2. A rhombus has all four sides equal, whereas a rectangle generally has only opposite sides equal.
3. The diagonals of a rectangle are equal, while the diagonals of a rhombus bisect each other at right angles.

4.10. Three-mark questions with Answers of HOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – HOTS Type:

1. Three angles of a quadrilateral are $75°$, $105°$, and $90°$. Find the fourth angle and justify your answer.
Answer:
1. The sum of the three given angles is $75° + 105° + 90° = 270°$.
2. The sum of the interior angles of a quadrilateral is $360°$.
3. Therefore, the fourth angle is $360° - 270° = \boxed{90°}$.

2. One angle of a parallelogram is $70°$. Find the other three angles.
Answer:
1. The angle opposite $70°$is also $70°$, because opposite angles are equal.
2. Each adjacent angle is $180° - 70° = 110°$.
3. Therefore, the four angles are $70°, 110°, 70°$,and $110°$.

3. Two adjacent angles of a parallelogram are $2x°$and $3x°$. Find $x$and the two angles.
Answer:
1. Adjacent angles of a parallelogram are supplementary, so $2x + 3x = 180°$.
2. Thus, $5x = 180°$, giving $x = 36°$.
3. Therefore, the two angles are $72°$and $108°$.

4. The diagonals of parallelogram $ABCD$intersect at O, where $AO = 8$cm and $BO = 6$cm. Find the lengths of both diagonals.
Answer:
1. Since the diagonals of a parallelogram bisect each other, $AO = OC = 8$cm.
2. Similarly, $BO = OD = 6$cm.
3. Hence, $AC = 16$cm and $BD = 12$cm.

5. A rectangle has a diagonal of 18 cm, and its diagonals intersect at O. Find the length of each half of both diagonals.
Answer:
1. The diagonals of a rectangle are equal, so both diagonals measure 18cm.
2. The diagonals bisect each other at their point of intersection.
3. Therefore, each half of both diagonals measures $\boxed{9\text{ cm}}$.

6. A quadrilateral has four equal sides and one angle equal to 90°. Identify the quadrilateral and justify your answer.

Answer:

1. Four equal sides give the quadrilateral the side property of a rhombus.
2. With one right angle, its angle properties make all four angles 90°.
3. Therefore, the quadrilateral is a **square**, having four equal sides and four right angles.

7. A rhombus has one interior angle of 120°. Find the remaining three angles.

Answer:

1. The angle opposite 120°is also 120°.
2. Each adjacent angle is $180° - 120° = 60°$.
3. Hence, the remaining angles are $60°, 120°$,and $60°$.

8. A diagonal of a rhombus bisects a vertex angle measuring 100°. Find the two angles formed and explain the result.

Answer:

1. A diagonal of a rhombus bisects the vertex angle into two equal parts.
2. Therefore, each part measures $100° \div 2 = 50°$.
3. Hence, the diagonal forms two angles of $\boxed{50°}$each.

9. A square has a diagonal drawn from one vertex. Determine the two angles formed at that vertex.

Answer:

1. Each interior angle of a square measures 90°.
2. A diagonal of a square bisects the angle through which it passes.
3. Therefore, the diagonal divides 90°into two angles of $\boxed{45°}$each.

10. In trapezium $PQRS$, $PQ \parallel SR$, $\angle P = 115°$, and $\angle Q = 70°$. Find $\angle S$and $\angle R$.

Answer:

1. Since $PQ \parallel SR$, $\angle P + \angle S = 180°$, so $\angle S = 65°$.
2. Similarly, $\angle Q + \angle R = 180°$, so $\angle R = 110°$.
3. Hence, $\boxed{\angle S = 65°}$and $\boxed{\angle R = 110°}$.

11. A student says that every rectangle is a square because both have four right angles. Analyse the statement.

Answer:

1. Both a rectangle and a square have four angles of 90°.
2. However, a square has all four sides equal, while a rectangle need not have all four sides equal.
3. Therefore, every square is a rectangle, but every rectangle is **not necessarily a square**.

12. A student claims that every rhombus is a square because both have four equal sides. Is the claim correct?

Answer:

1. A rhombus and a square both have four equal sides.
2. However, a square has four right angles, whereas a rhombus need not have four right angles.
3. Therefore, every square is a rhombus, but every rhombus is **not necessarily a square**.

13. The diagonals of a rhombus intersect at O. If one angle at $O$is 90°, determine the other three angles.

Answer:

1. The diagonals of a rhombus intersect each other at right angles.
2. Thus, each of the four angles formed at Omeasures 90°.
3. Therefore, the other three angles are $\boxed{90°, 90°, \text{ and } 90°}$.

14. Two diagonals intersect such that one angle formed is 55°. Determine the other three angles at the intersection.
Answer:
1. The vertically opposite angle is also 55°.
2. Each adjacent angle is $180^\circ - 55^\circ = 125^\circ$.
3. Therefore, the other three angles are $\boxed{55^\circ, 125^\circ, \text{and } 125^\circ}$.

15. A parallelogram has one angle equal to 90°. Explain why it can be classified as a rectangle.
Answer:
1. The angle opposite the 90°angle is also 90°.
2. Each adjacent angle is $180^\circ - 90^\circ = 90^\circ$.
3. Hence, all four angles are right angles, so the parallelogram is a **rectangle**.

16. A rhombus has one right angle. Explain why it must be a square.
Answer:
1. If one angle of a rhombus is 90°, its opposite angle is also $90^\circ$and its adjacent angles are 90°.
2. A rhombus already has all four sides equal.
3. Hence, it has four equal sides and four right angles, so it is a **square**.

17. A quadrilateral has angles $80^\circ, 100^\circ, 80^\circ$,and 100°. What special quadrilateral can it be, based on its angle pattern?
Answer:
1. The opposite angles are equal: $80^\circ = 80^\circ$and $100^\circ = 100^\circ$.
2. Each pair of adjacent angles adds up to 180°.
3. These angle properties are consistent with a **parallelogram**.

18. In an isosceles trapezium, one base angle is 75°. Determine all four angles.
Answer:
1. The other angle on the same base is 75°, because the base angles of an isosceles trapezium are equal.
2. Each angle on the other base is $180^\circ - 75^\circ = 105^\circ$.
3. Therefore, the four angles are $\boxed{75^\circ, 75^\circ, 105^\circ, \text{and } 105^\circ}$.

19. A student observes that a quadrilateral has two non-overlapping pairs of adjacent equal sides. Identify the quadrilateral and explain how you know.
Answer:
1. The equal sides occur in two pairs, with the equal sides in each pair adjacent to one another.
2. This is the defining side property of a **kite**.
3. Therefore, the quadrilateral can be identified as a kite.

20. A quadrilateral has four right angles and its diagonals are equal and bisect each other. Identify the quadrilateral and justify your answer.
Answer:
1. Four right angles satisfy the defining angle property of a rectangle.
2. Equal diagonals that bisect each other are also properties of a rectangle.
3. Therefore, the quadrilateral can be identified as a **rectangle**.

4.11 Five-mark questions with Answers of LOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – LOTS Type:
1. Define a rectangle and explain its important properties.
Answer:

1. A **rectangle** is a quadrilateral in which all four angles are 90°.
2. The opposite sides of a rectangle are equal in length.
3. The opposite sides are also parallel to each other.
4. The diagonals of a rectangle are equal in length.
5. The diagonals bisect each other at their point of intersection.

2. Define a square and describe its important properties.
Answer:

1. A **square** is a quadrilateral having four equal sides and four right angles.
2. Each interior angle of a square measures 90°.
3. The diagonals of a square are equal and bisect each other.
4. The diagonals intersect each other at 90°.
5. The diagonals also bisect the angles of the square.

3. Define a parallelogram and explain its main properties.
Answer:

1. A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.
2. The opposite sides of a parallelogram are equal in length.
3. The opposite angles of a parallelogram are equal.
4. Each pair of adjacent angles is supplementary and adds up to 180°.
5. The diagonals of a parallelogram bisect each other.

4. Define a rhombus and describe its important properties.
Answer:

1. A **rhombus** is a quadrilateral in which all four sides have the same length.
2. The opposite sides of a rhombus are parallel.
3. The opposite angles are equal, while adjacent angles add up to 180°.
4. The diagonals of a rhombus bisect each other at right angles.
5. The diagonals also bisect the angles of the rhombus.

5. Explain the important properties of a trapezium.
Answer:

1. A **trapezium** is a quadrilateral having at least one pair of parallel opposite sides.
2. If $PQ \parallel SR$, then $PQ$and SRform the pair of parallel opposite sides.
3. The angles on one non-parallel side satisfy $\angle P + \angle S = 180^\circ$.
4. The angles on the other non-parallel side satisfy $\angle Q + \angle R = 180^\circ$.
5. Thus, the interior angles on each non-parallel side of the trapezium are supplementary.

6. Define an isosceles trapezium and explain its properties.
Answer:

1. An **isosceles trapezium** is a trapezium whose non-parallel sides are equal in length.
2. It has at least one pair of opposite sides that are parallel.
3. The other pair of opposite sides has equal lengths.
4. The angles opposite the equal sides are equal.
5. The angles on each non-parallel side are supplementary because the other pair of sides is parallel.

7. Explain the angle properties of a quadrilateral and a parallelogram.
Answer:

1. The sum of the four interior angles of any quadrilateral is 360°.
2. The opposite angles of a parallelogram are equal.
3. The adjacent angles of a parallelogram are supplementary.
4. Therefore, if one angle of a parallelogram is x°, its opposite angle is also x°.
5. Each angle adjacent to it is $(180 - x)^\circ$.

8. Describe the properties of the diagonals of a rectangle, square, parallelogram, and rhombus.

Answer:

1. The diagonals of a **rectangle** are equal and bisect each other.
2. The diagonals of a **square** are equal and bisect each other at right angles.
3. The diagonals of a square also bisect its vertex angles.
4. The diagonals of a **parallelogram** bisect each other.
5. The diagonals of a **rhombus** bisect each other at right angles and also bisect its angles.

9. Explain the relationship among a parallelogram, rectangle, rhombus, and square.

Answer:

1. A rectangle is a special parallelogram with four right angles.
2. A rhombus is a special parallelogram with all four sides equal.
3. A square has all the properties of a rectangle because it has four right angles.
4. A square also has all the properties of a rhombus because all four of its sides are equal.
5. Hence, a square is both a rectangle and a rhombus, and all three are parallelograms.

10. Explain how the sum of the interior angles of a quadrilateral is obtained.

Answer:

1. Draw a diagonal joining two opposite vertices of a quadrilateral.
2. The diagonal divides the quadrilateral into two triangles.
3. The sum of the interior angles of each triangle is 180°.
4. Therefore, the total angle sum of the two triangles is $180^\circ + 180^\circ = 360^\circ$.
5. Hence, the sum of the four interior angles of a quadrilateral is $\boxed{360^\circ}$.

11. Explain what happens when the diagonals of a parallelogram intersect.

Answer:

1. The two diagonals of a parallelogram meet at a point inside the figure.
2. Each diagonal divides the other into two equal parts.
3. If $AC$and BDmeet at O, then $AO = OC$.
4. Similarly, $BO = OD$.
5. Therefore, the diagonals of a parallelogram **bisect each other**.

12. Define a kite and compare it with a rhombus using their side properties.

Answer:

1. A **kite** is a quadrilateral having two non-overlapping pairs of adjacent sides of equal length.
2. In a kite, the equal sides occur next to each other in two pairs.
3. A **rhombus** has all four sides equal in length.
4. Therefore, a kite requires two pairs of adjacent equal sides, while a rhombus has equality among all four sides.
5. These side properties help us recognise and distinguish the two types of quadrilaterals.

4.12 Five-mark questions with Answers of HOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – HOTS Type

1. Three angles of a quadrilateral are 75°, 85°, and 110°. Find the fourth angle and explain your reasoning.

Answer:

1. The sum of the interior angles of a quadrilateral is 360°.
2. The sum of the three given angles is $75^\circ + 85^\circ + 110^\circ = 270^\circ$.
3. Let the fourth angle be x°, so $270^\circ + x^\circ = 360^\circ$.
4. Therefore, $x = 360^\circ - 270^\circ = 90^\circ$.
5. Hence, the fourth angle of the quadrilateral is 90°.

2. One angle of a parallelogram is 65°. Determine all its angles using its properties.

Answer:

1. Opposite angles of a parallelogram are equal.
2. Therefore, the angle opposite $65°$ is also $65°$.
3. Adjacent angles of a parallelogram are supplementary.
4. Hence, each adjacent angle is $180° - 65° = 115°$.
5. Therefore, the four angles are $65°, 115°, 65°$, **and** $115°$.

3. Two adjacent angles of a parallelogram are $2x°$ and $3x°$. Find x and all four angles.

Answer:

1. Adjacent angles of a parallelogram add up to $180°$, so $2x + 3x = 180°$.
2. Thus, $5x = 180°$, giving $x = 36°$.
3. The two adjacent angles are $2(36°) = 72°$ and $3(36°) = 108°$.
4. Opposite angles are equal, so the other two angles are $72°$ and $108°$.
5. Hence, the four angles are $72°, 108°, 72°$, **and** $108°$.

4. The diagonals of parallelogram $ABCD$ intersect at O. If $AO = 7$cm and $BO = 5$cm, find AC and BD with justification.

Answer:

1. The diagonals of a parallelogram bisect each other.
2. Therefore, $AO = OC = 7$cm.
3. Hence, $AC = AO + OC = 7 + 7 = 14$cm.
4. Similarly, $BO = OD = 5$cm, so $BD = 5 + 5 = 10$cm.
5. Thus, the lengths of the diagonals are $AC = 14$**cm and** $BD = 10$**cm**.

5. A rectangle has a diagonal of 20 cm, and its diagonals intersect at O. Find the lengths of both diagonals and all four half-diagonals.

Answer:

1. The diagonals of a rectangle are equal, so both diagonals measure 20cm.
2. Thus, if the diagonals are AC and BD, then $AC = BD = 20$cm.
3. The diagonals of a rectangle bisect each other at O.
4. Therefore, $AO = OC = BO = OD = 20 \div 2 = 10$cm.
5. Hence, **both diagonals are 20 cm and each half-diagonal is 10 cm**.

6. A rhombus has one interior angle of $120°$. Find the remaining angles and explain how its diagonal divides the $120°$ angle.

Answer:

1. Opposite angles of a rhombus are equal, so the angle opposite $120°$ is also $120°$.
2. Adjacent angles are supplementary, so each adjacent angle is $180° - 120° = 60°$.
3. Therefore, the four interior angles are $120°, 60°, 120°$, and $60°$.
4. A diagonal of a rhombus bisects the vertex angles through which it passes.
5. Hence, a diagonal through a $120°$ vertex divides it into **two angles of $60°$ each**.

7. A student says, "Every rectangle is a square because both have equal diagonals and four right angles." Analyse the statement.

Answer:

1. A rectangle and a square both have four right angles.
2. Their diagonals are also equal in length and bisect each other.
3. However, a square has all four sides equal, while a rectangle need not have all four sides equal.
4. Therefore, the given properties alone do not make every rectangle a square.
5. Hence, **every square is a rectangle, but every rectangle is not necessarily a square**.

8. A student says, "Every rhombus must be a square because all four sides are equal." Evaluate the statement.

Answer:

1. A rhombus has all four sides equal in length.

2. A square also has all four sides equal, so the two figures share this property.
3. However, a square has four $90°$angles, while a rhombus need not have four right angles.
4. A rhombus becomes a square when it also has the required right-angle property.
5. Therefore, **every square is a rhombus, but every rhombus is not necessarily a square**.

9. In trapezium $PQRS$, $PQ \parallel SR$, $\angle P = 125°$, and $\angle Q = 70°$. Find $\angle S$and $\angle R$, and verify the angle sum.

Answer:

1. Since $PQ \parallel SR$, $\angle P + \angle S = 180°$, so $\angle S = 55°$.
2. Similarly, $\angle Q + \angle R = 180°$, so $\angle R = 110°$.
3. Therefore, the four angles are $125°, 70°, 110°$,and $55°$.
4. Their sum is $125° + 70° + 110° + 55° = 360°$.
5. Hence, the calculated angles also satisfy the **angle-sum property of a quadrilateral**.

10. One angle formed by two intersecting diagonals is $50°$. Determine all four angles formed at their intersection.

Answer:

1. Vertically opposite angles formed by two intersecting lines are equal.
2. Therefore, the angle vertically opposite $50°$is also $50°$.
3. Each adjacent angle forms a linear pair with $50°$.
4. Hence, each adjacent angle is $180° - 50° = 130°$.
5. Therefore, the four angles are $50°, 130°, 50°$,**and** $130°$.

11. A parallelogram has one angle equal to $90°$. Show that it satisfies the angle condition of a rectangle.

Answer:

1. The angle opposite the $90°$angle is also $90°$, because opposite angles are equal.
2. Each adjacent angle is supplementary to $90°$.
3. Therefore, each adjacent angle is $180° - 90° = 90°$.
4. Thus, all four angles of the parallelogram are right angles.
5. Hence, the parallelogram satisfies the defining angle condition of a **rectangle**.

12. A rhombus has one right angle. Use its properties to show that it is a square.

Answer:

1. A rhombus already has all four sides equal.
2. If one angle is $90°$, its opposite angle is also $90°$.
3. Its adjacent angles are supplementary to $90°$, so they are also $90°$.
4. Thus, the rhombus has four equal sides and four right angles.
5. Therefore, the rhombus satisfies the properties of a **square**.

4.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – Applied Type:

1. A carpenter is making a rectangular window frame. Three of its corner angles are $90°$. Find the fourth angle and explain which properties should be checked to confirm that the frame is rectangular.

Answer:

1. The sum of the interior angles of a quadrilateral is $360°$.
2. Therefore, the fourth angle is $360° - (90° + 90° + 90°) = 90°$.
3. The carpenter should check that the opposite sides are equal and parallel.
4. He can also check whether the two diagonals are equal and bisect each other.
5. Thus, four right angles together with these side and diagonal properties confirm the rectangular shape.

2. A designer makes a square floor tile of side 30 cm and draws both diagonals. Explain five geometric facts that can be used to check whether the tile is a perfect square.

Answer:

1. All four sides of the tile should be equal, so each side should measure 30cm.
2. Each of the four corner angles should measure $90°$.
3. The two diagonals should be equal in length.
4. The diagonals should bisect each other and intersect at $90°$.
5. Each diagonal should also bisect the angles through which it passes.

3. An engineer designs a parallelogram-shaped panel in which one interior angle is $70°$. Find all its angles and explain the properties used.

Answer:

1. The angle opposite $70°$is also $70°$, because opposite angles of a parallelogram are equal.
2. An adjacent angle is $180° - 70° = 110°$, because adjacent angles are supplementary.
3. The angle opposite $110°$is also $110°$.
4. Therefore, the four angles are $70°, 110°, 70°$,and $110°$.
5. Their sum is $360°$, which also satisfies the angle-sum property of a quadrilateral.

4. A decorative rhombus-shaped wall frame has one corner angle of $120°$. Find the remaining angles and determine the angles formed when a diagonal bisects the $120°$corner.

Answer:

1. The angle opposite $120°$is also $120°$, because opposite angles of a rhombus are equal.
2. Each adjacent angle is $180° - 120° = 60°$.
3. Hence, the four interior angles are $120°, 60°, 120°$,and $60°$.
4. A diagonal of a rhombus bisects the vertex angles through which it passes.
5. Therefore, the $120°$corner is divided into two angles of $60°$each.

5. A park has a parallelogram-shaped section whose diagonals intersect at O. If $AO = 12$m and $BO = 9$m, calculate the lengths of both diagonals.

Answer:

1. The diagonals of a parallelogram bisect each other at their point of intersection.
2. Therefore, $AO = OC = 12$m.
3. Hence, $AC = AO + OC = 12 + 12 = 24$m.
4. Similarly, $BO = OD = 9$m, so $BD = 9 + 9 = 18$m.
5. Thus, the two complete diagonals measure **24 m and 18 m**.

6. A rectangular notice board has a diagonal of 160 cm. The diagonals intersect at O. Find the length of the other diagonal and the four segments formed at O.

Answer:

1. The diagonals of a rectangle are equal, so the other diagonal is also 160cm.
2. The diagonals of a rectangle bisect each other.
3. Therefore, each diagonal is divided into two equal parts at O.
4. Each segment measures $160 \div 2 = 80$cm.
5. Hence, both diagonals measure **160 cm**, and the four half-diagonal segments measure **80 cm each**.

7. A craftsperson makes a four-sided decoration with all four sides equal and one corner measuring $90°$. Identify the shape and justify your answer using its properties.

Answer:

1. Since all four sides are equal, the decoration has the side property of a rhombus.
2. One of its angles is $90°$, so the opposite angle is also $90°$.
3. The adjacent angles are supplementary to $90°$, so they also measure $90°$.
4. Thus, the decoration has four equal sides and four right angles.

5. Therefore, the decoration is a **square**.

8. A trapezium-shaped garden $PQRS$has $PQ \parallel SR$. If $\angle P = 110^\circ$and $\angle Q = 75^\circ$, find the other two angles.

Answer:

1. Since $PQ \parallel SR$, the angles on each non-parallel side are supplementary.
2. Therefore, $\angle S = 180^\circ - 110^\circ = 70^\circ$.
3. Similarly, $\angle R = 180^\circ - 75^\circ = 105^\circ$.
4. The four angles are therefore $110^\circ, 75^\circ, 105^\circ$,and 70°.
5. Their total is 360°, confirming the angle-sum property of the quadrilateral.

9. A student makes a quadrilateral model with two non-overlapping pairs of adjacent equal sides. Identify the model and explain how its side arrangement differs from a rhombus.

Answer:

1. A quadrilateral with two non-overlapping pairs of adjacent equal sides is a **kite**.
2. In a kite, the equal sides occur as adjacent pairs.
3. Thus, one pair of neighbouring sides may have one equal length and the other neighbouring pair another equal length.
4. In a rhombus, however, all four sides have the same length.
5. Therefore, examining how the equal sides are arranged helps distinguish a kite from a rhombus.

10. An architect wants a four-sided design that has the properties of both a rectangle and a rhombus. Which quadrilateral should be chosen? Explain.

Answer:

1. The architect should choose a **square**.
2. Like a rectangle, a square has four right angles and equal diagonals that bisect each other.
3. Like a rhombus, a square has all four sides equal.
4. Its diagonals also intersect at right angles and bisect its vertex angles.
5. Hence, a square combines the important properties of both a rectangle and a rhombus.

4.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams:

Olympiad-Type Multiple Choice Questions:

1. The angles of a quadrilateral are $75^\circ, 85^\circ, 95^\circ$, and x°. What is x?

A) 95°
B) 100°
C) 105°
D) 110°

Answer: C) 105°

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2. One angle of a parallelogram is 68°. What is the measure of its adjacent angle?

A) 68°
B) 102°
C) 112°
D) 122°

Answer: C) 112°

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3. The angles of a parallelogram are in the ratio $2:3$. What is the smaller angle?

A) 60°
B) 72°

C) 75°
D) 90°
Answer: B) 72°
Since adjacent angles are supplementary, $2x + 3x = 180°$, *so* $x = 36°$ *and* $2x = 72°$.

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4. In parallelogram $ABCD$, $\angle A = (3x + 10)°$ and $\angle C = (5x - 30)°$. Find x.
A) 10
B) 15
C) 20
D) 25
Answer: C) 20

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5. The diagonals of parallelogram $ABCD$ intersect at O. If $AO = 8$cm, what is AC?
A) 4 cm
B) 8 cm
C) 12 cm
D) 16 cm
Answer: D) 16 cm

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6. Which property is always true for the diagonals of a rectangle?
A) They are perpendicular only
B) They are unequal
C) They are equal and bisect each other
D) They bisect the vertex angles
Answer: C) They are equal and bisect each other.

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7. The diagonals of a rectangle intersect at O. If $AC = 18$cm, what is AO?
A) 6 cm
B) 9 cm
C) 12 cm
D) 18 cm
Answer: B) 9 cm

---

8. Which of the following properties distinguishes a square from a general rectangle?
A) Opposite sides are parallel
B) All angles are 90°
C) Diagonals are equal
D) All four sides are equal
Answer: D) All four sides are equal.

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9. A diagonal of a square bisects one of its vertex angles. What is the measure of each resulting angle?
A) 30°
B) 40°
C) 45°
D) 60°
Answer: C) 45°

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10. Which statement about the diagonals of a square is correct?

A) They are unequal and perpendicular
B) They are equal but do not bisect each other
C) They are equal, perpendicular, and bisect each other
D) They are parallel
Answer: C) They are equal, perpendicular, and bisect each other.

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11. One interior angle of a rhombus is 124°. What is the measure of an adjacent angle?
A) 46°
B) 56°
C) 66°
D) 124°
Answer: B) 56°

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12. A diagonal of a rhombus bisects an angle of 116°. What is the measure of each part?
A) 48°
B) 52°
C) 58°
D) 62°
Answer: C) 58°

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13. At what angle do the diagonals of a rhombus intersect?
A) 45°
B) 60°
C) 90°
D) 180°
Answer: C) 90°

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14. A rhombus has one angle equal to 90°. What must the rhombus be?
A) Kite
B) Rectangle only
C) Trapezium only
D) Square
Answer: D) Square

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15. Which of the following is both a rectangle and a rhombus?
A) Kite
B) Square
C) General parallelogram
D) Isosceles trapezium
Answer: B) Square

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16. In trapezium $PQRS$, $PQ \parallel SR$. If $\angle P = 108^\circ$, what is $\angle S$?
A) 62°
B) 72°
C) 82°
D) 108°
Answer: B) 72°

---

17. In trapezium $PQRS$, $PQ \parallel SR$. If $\angle Q = 65^\circ$, find $\angle R$.

A) 65°
B) 105°
C) 115°
D) 125°
Answer: C) 115°

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18. A trapezium whose non-parallel sides are equal is called:
A) Rhombus
B) Kite
C) Isosceles trapezium
D) Rectangle
Answer: C) Isosceles trapezium.

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19. Which description correctly identifies a kite?
A) A quadrilateral with four right angles
B) A quadrilateral with two non-overlapping pairs of adjacent equal sides
C) A quadrilateral with four equal sides only
D) A quadrilateral whose diagonals are always equal
Answer: B) A quadrilateral with two non-overlapping pairs of adjacent equal sides.

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20. If one angle formed by two intersecting diagonals is 48°, what is its vertically opposite angle?
A) 42°
B) 48°
C) 132°
D) 180°
Answer: B) 48°

---

21. If one angle formed by two intersecting lines is 48°, what is an adjacent angle?
A) 48°
B) 90°
C) 122°
D) 132°
Answer: D) 132°

---

22. Which set of angles can represent the angles of a parallelogram?
A) $70^\circ, 110^\circ, 70^\circ, 110^\circ$
B) $70^\circ, 100^\circ, 80^\circ, 110^\circ$
C) $60^\circ, 100^\circ, 60^\circ, 140^\circ$
D) $80^\circ, 80^\circ, 100^\circ, 100^\circ$
Answer: A) $70^\circ, 110^\circ, 70^\circ, 110^\circ$

---

23. A quadrilateral has four equal sides, but its angles are not all 90°. Which shape best describes it?
A) Rectangle
B) Rhombus
C) Isosceles trapezium
D) Square
Answer: B) Rhombus.

24. Which statement is NOT always true for a parallelogram?
A) Opposite sides are equal
B) Opposite angles are equal
C) Diagonals bisect each other
D) Diagonals are equal
Answer: D) Diagonals are equal.

25. A quadrilateral has four right angles, but its four sides are not all equal. Which is the most specific classification?
A) Square
B) Rhombus
C) Rectangle
D) Kite
Answer: C) Rectangle

26. Which statement is NOT always true for a rhombus?
A) All four sides are equal
B) Diagonals intersect at right angles
C) Opposite angles are equal
D) Diagonals are equal in length
Answer: D) Diagonals are equal in length.

27. In a parallelogram, one angle is twice its adjacent angle. Find the smaller angle.
A) $45°$
B) $60°$
C) $75°$
D) $90°$
Answer: B) $60°$
If the smaller angle is x, *then* $x + 2x = 180°$, *giving* $x = 60°$.

28. Three angles of a quadrilateral are equal, and the fourth angle is $120°$. What is each of the equal angles?
A) $60°$
B) $70°$
C) $80°$
D) $90°$
Answer: C) $80°$
The three equal angles total $360° - 120° = 240°$, *so each is* $80°$.

29. The diagonals of a parallelogram are 24cm and 18cm. If they intersect at O, which statement is correct?
A) The four half-diagonals are all 12 cm
B) $AO = OC = 12$cm and $BO = OD = 9$cm
C) $AO = OC = 9$cm and $BO = OD = 12$cm
D) Each diagonal is divided in the ratio $1:2$
Answer: B) $AO = OC = 12$cm and $BO = OD = 9$cm.

30. Which chain of classification is correct?

A) Square → Rectangle → Parallelogram
B) Rectangle → Square → Rhombus
C) Parallelogram → Rectangle → Square in every case
D) Rhombus → Square in every case
Answer: A) Square → Rectangle → Parallelogram

---

31. A quadrilateral has equal diagonals that bisect each other. Which special quadrilateral discussed in the chapter is indicated by these properties?
A) Kite
B) Rectangle
C) Rhombus only
D) General trapezium
Answer: B) Rectangle.

---

32. Which pair of properties is sufficient to distinguish a square from both a general rectangle and a general rhombus?
A) Opposite sides parallel and opposite angles equal
B) Diagonals bisect each other and opposite sides equal
C) Four equal sides and four right angles
D) Adjacent angles supplementary and opposite sides parallel
Answer: C) Four equal sides and four right angles.

---

33. In a parallelogram, two adjacent angles are $(x + 20)^\circ$and $(2x + 10)^\circ$. Find x.
A) 40°
B) 45°
C) 50°
D) 60°
Answer: C) 50°
Since adjacent angles are supplementary, $x + 20 + 2x + 10 = 180$, *so* $3x = 150$*and* $x = 50$.

---

34. A square has diagonals $AC$and BDintersecting at O. If $AC = 16$cm, which statement is correct?
A) $AO = 16$cm and $BD = 8$cm
B) $AO = 8$cm and $BD = 16$cm
C) $AO = 4$cm and $BD = 8$cm
D) $AO = 8$cm and $BD = 8$cm
Answer: B) $AO = 8$cm and $BD = 16$cm.

---

35. Which statement gives the best reason why a square belongs to both the rectangle and rhombus families?
A) It has at least one pair of parallel sides
B) It has equal diagonals only
C) It has four right angles as well as four equal sides
D) Its adjacent angles add up to 180°
Answer: C) It has four right angles as well as four equal sides.

4.15 Best Practices Opportunity List from the Chapter: (10 Ideas)

"Best Practices" Opportunities While Teaching Chapter 4: Quadrilaterals:

(1) **Use a Property-Comparison Chart:**
Prepare a classroom chart comparing the **rectangle, square, parallelogram, rhombus, kite, and trapezium** based on sides, angles, parallel sides, and diagonals. This helps students recognise both similarities and differences among the quadrilaterals discussed in the chapter.

(2) **Learning by Drawing and Measuring:**
Ask students to draw different quadrilaterals using a ruler and protractor and then measure their **sides, angles, and diagonals**. Students can verify properties such as equal opposite sides, supplementary adjacent angles, and diagonals bisecting each other.

(3) **Paper-Folding Investigation of Diagonals:**
Use paper models of rectangles, squares, parallelograms, and rhombuses to explore their diagonals through folding. Students can investigate whether the diagonals are **equal, perpendicular, bisect each other, or bisect vertex angles**.

(4) **Angle-Sum Discovery Activity:**
Let students cut out the four corners of a paper quadrilateral and arrange them around a point. This visually reinforces that the four interior angles together make 360°, supporting the chapter's geometric reasoning about the angle sum.

(5) **Think–Pair–Share for Classification:**
Show students a quadrilateral and ask, "Could this also belong to another quadrilateral family?" Students first think individually, then discuss with a partner, and finally justify their classification to the class—for example, why a **square is both a rectangle and a rhombus**.

(6) **Property Cards and Shape-Matching Game:**
Prepare cards such as "opposite angles are equal," "diagonals intersect at 90°," and "two pairs of adjacent sides are equal." Students match each property card with the appropriate quadrilateral, strengthening recall and classification skills.

(7) **Use Progressive Questioning: LOTS → HOTS:**
Begin with questions such as "What is a rhombus?" and gradually progress to "A rhombus has one 90°angle; what special quadrilateral does it become?" This develops students from **remembering properties to applying and reasoning with them**.

(8) **Encourage Property-Based Justification:**
Instead of accepting answers such as "It is a square," require students to add a reason: "It has four equal sides and four right angles." This develops mathematical communication and prevents students from identifying figures only by their appearance.

(9) **Real-Life Quadrilateral Hunt:**
Ask students to identify quadrilateral shapes in **windows, floor tiles, notice boards, designs, frames, and classroom objects** and state which properties support their classification. This connects the chapter concepts with practical observation.

(10) **Maintain a Quadrilateral Concept Map:**
Develop a concept map showing relationships among the shapes—for example, **square → rectangle → parallelogram** and **square → rhombus → parallelogram**. Students can update the map as each new quadrilateral and its properties are introduced.

(11) **Peer Explanation and Error Correction:**
Give students statements such as "Every rectangle is a square" or "The diagonals of every parallelogram are equal" and ask groups to decide whether they are correct. Students should correct false statements by referring to the properties established in the chapter.

(12) **End with a Quick Formative Assessment:**

Use a short exit ticket containing one **property question, one angle calculation, and one classification/reasoning question**. This allows the teacher to identify misconceptions before moving to the next concept.

4.16 Innovations Opportunity List from the Chapter: (10 Ideas)

"Innovations" Opportunities While Teaching–Learning Chapter 4: Quadrilaterals:
Since the chapter focuses on recognising and comparing **rectangles, squares, parallelograms, rhombuses, kites and trapeziums**, the following child-friendly innovations can make the concepts more visual and activity-oriented.

(1) Quadrilateral Detective Challenge: Create a "Who Am I?" game in which students receive clues such as "I have four equal sides," "My diagonals meet at right angles," or "I have at least one pair of parallel opposite sides." Students identify the mystery quadrilateral and explain which clue helped them.

(2) Geo-Board Shape Laboratory: Let students construct different quadrilaterals using rubber bands on geo-boards or pin boards. They can change one vertex at a time and observe how a rectangle can change into a parallelogram or how different properties distinguish a square and rhombus.

(3) Digital Drag-and-Discover Activity: Using an interactive geometry application or classroom smartboard, allow students to drag the vertices of quadrilaterals while observing changes in **side lengths, angles and diagonals**. Students can discover which properties remain unchanged even when the figure changes its appearance.

(4) Quadrilateral AR/Camera Hunt: Students can use a tablet or classroom camera to photograph quadrilateral-shaped objects around the school, such as windows, boards, tiles and decorative patterns. They then classify the objects and create a digital "Quadrilaterals Around Us" gallery.

(5) Paper-Folding Diagonal Lab: Give students paper squares, rectangles and rhombuses and ask them to fold along their diagonals. They can investigate whether diagonals are **equal, perpendicular, bisect each other, or bisect angles**, turning abstract properties into visible discoveries.

(6) Quadrilateral Family Tree: Students create an interactive family tree showing relationships such as **Square** → **Rectangle** → **Parallelogram** and **Square** → **Rhombus** → **Parallelogram**. Flaps, movable arrows or magnetic cards can reveal the properties shared by different members of the family.

(7) Build-a-City with Quadrilaterals: Groups design a miniature "Geometry City" using quadrilateral shapes for buildings, parks, windows, roads and signboards. Each group labels the shapes used and explains at least one mathematical property of each.

(8) 360° Angle Puzzle: Students cut off the four corners of a paper quadrilateral and arrange them around a single point. Seeing the four angles form one complete turn helps them discover visually that the interior angles of a quadrilateral total 360°.

(9) Human Quadrilateral Activity: Four students become vertices and hold ropes to form different quadrilaterals. The class gives instructions such as "make opposite sides parallel," "make all sides equal," or "make four right angles," transforming properties into a collaborative physical activity.

(10) Create-Your-Own Quadrilateral Puzzle: Students design a shape puzzle for classmates by giving three or four property clues without revealing the shape's name. Their classmates identify whether it is a **square, rectangle, parallelogram, rhombus, kite, or trapezium**.

(11) **Error Detective Game:** Display statements such as "Every rectangle is a square" or "Every square is a rhombus." Students become "Math Detectives," decide whether each statement is correct, and use properties from the chapter to justify their decisions.

(12) **Student-Made QR Learning Gallery:** Groups prepare mini-posters for different quadrilaterals containing drawings, properties, examples and one challenge question; these can be linked through QR codes to short student-recorded explanations. The completed gallery becomes a student-created revision resource for the chapter.

4.17: Practicing Questions/Question Bank

A. Fill in the Blanks:

1. The sum of the interior angles of a quadrilateral is ________.
   Answer: 360°

2. All four angles of a rectangle are ________.
   Answer: 90°

3. The diagonals of a rectangle are ________ in length.
   Answer: Equal

4. All four sides of a ________ are equal.
   Answer: Rhombus

5. The opposite angles of a parallelogram are ________.
   Answer: Equal

6. Adjacent angles of a parallelogram add up to ________.
   Answer: 180°

7. The diagonals of a parallelogram ________ each other.
   Answer: Bisect

8. The diagonals of a rhombus intersect at an angle of ________.
   Answer: 90°

9. A quadrilateral with two non-overlapping pairs of adjacent equal sides is called a ________.
   Answer: Kite

10. A trapezium with equal non-parallel sides is called an ________ trapezium.
    Answer: Isosceles

B. One-Mark Descriptive Questions:

1. **What is a quadrilateral?**
   Answer: A quadrilateral is a closed polygon having four sides.

2. **What is a rectangle?**
   Answer: A rectangle is a quadrilateral having four right angles.

3. **What is a square?**
   Answer: A square is a quadrilateral having four equal sides and four right angles.

4. **What is a parallelogram?**
   Answer: A parallelogram is a quadrilateral whose opposite sides are parallel.

5. **What is a rhombus?**
   Answer: A rhombus is a quadrilateral having all four sides equal.

6. **What is the sum of the angles of a quadrilateral?**
   Answer: The sum of the interior angles of a quadrilateral is $360°$.

7. **What happens to the diagonals of a parallelogram?**
   Answer: The diagonals of a parallelogram bisect each other.

8. **What is a kite?**
   Answer: A kite has two non-overlapping pairs of adjacent sides of equal length.

9. **What is a trapezium?**
   Answer: A trapezium has at least one pair of parallel opposite sides.

10. **Which quadrilateral is both a rectangle and a rhombus?**
    Answer: A square is both a rectangle and a rhombus.

C. Two-Mark Descriptive Questions:

1. **State two properties of a rectangle.**
   Answer:
   1. All four angles are $90°$.
   2. Its diagonals are equal and bisect each other.

2. **State two properties of a square.**
   Answer:
   1. All four sides are equal.
   2. All four angles measure $90°$.

3. **State two angle properties of a parallelogram.**
   Answer:
   1. Opposite angles are equal.
   2. Adjacent angles add up to $180°$.

4. **How are the diagonals of a rhombus special?**
   Answer:
   1. They bisect each other at right angles.
   2. They also bisect the vertex angles.

5. **One angle of a parallelogram is $70°$. Find its adjacent angle.**
   Answer:
   1. Adjacent angles add up to $180°$.
   2. Therefore, the adjacent angle is $180° - 70° = 110°$.

6. **State two similarities between a square and a rectangle.**
   Answer:
   1. Both have four right angles.

2. Both have equal diagonals that bisect each other.

D. Three-Mark Descriptive Questions:

1. **Explain three important properties of a parallelogram.**
   Answer:
   1. Opposite sides are equal and parallel.
   2. Opposite angles are equal and adjacent angles are supplementary.
   3. Its diagonals bisect each other.
2. **Explain three properties of a rhombus.**
   Answer:
   1. All four sides are equal.
   2. Opposite angles are equal.
   3. Its diagonals bisect each other at right angles.
3. **Three angles of a quadrilateral are $80°, 90°$,and $100°$; find the fourth angle.**
   Answer:
   1. The sum of the three angles is $270°$.
   2. The angle sum of a quadrilateral is $360°$.
   3. Therefore, the fourth angle is $360° - 270° = 90°$.
4. **One angle of a parallelogram is $60°$. Find its remaining angles.**
   Answer:
   1. Its opposite angle is also $60°$.
   2. Each adjacent angle is $180° - 60° = 120°$.
   3. Therefore, the remaining angles are $120°, 60°$,and $120°$.
5. **Explain why a square is both a rectangle and a rhombus.**
   Answer:
   1. A square has four right angles, like a rectangle.
   2. It has four equal sides, like a rhombus.
   3. Therefore, a square satisfies the important properties of both figures.

E. Five-Mark Descriptive Questions:

1. **Explain the important properties of a square.**
   Answer:
   1. All four sides of a square are equal.
   2. Each interior angle measures $90°$.
   3. Its opposite sides are parallel.
   4. Its diagonals are equal and bisect each other at right angles.

5. Its diagonals also bisect its vertex angles.

2. **Explain the important properties of a parallelogram.**
   Answer:

   1. Both pairs of opposite sides are parallel.
   2. Opposite sides are equal.
   3. Opposite angles are equal.
   4. Adjacent angles add up to $180°$.
   5. Its diagonals bisect each other.

3. **A parallelogram has one angle measuring $65°$; determine all its angles and explain.**
   Answer:

   1. The opposite angle is also $65°$.
   2. An adjacent angle is $180° - 65° = 115°$.
   3. The angle opposite $115°$is also $115°$.
   4. Thus, the angles are $65°, 115°, 65°$,and $115°$.
   5. Their sum is $360°$, satisfying the angle-sum property.

4. **Compare a rectangle and a rhombus using five properties.**
   Answer:

   1. A rectangle has four right angles, while a rhombus need not have four right angles.
   2. A rhombus has all four sides equal, while a rectangle generally has equal opposite sides.
   3. Both have opposite sides parallel.
   4. The diagonals of a rectangle are equal, whereas those of a rhombus intersect at right angles.
   5. The diagonals of both figures bisect each other.

5. **Explain how the angle-sum property of a quadrilateral can be demonstrated using a diagonal.**
   Answer:

   1. Draw a diagonal between two opposite vertices of a quadrilateral.
   2. The diagonal divides the quadrilateral into two triangles.
   3. The angles of each triangle add up to $180°$.
   4. Therefore, the two triangles together have an angle sum of $180° + 180° = 360°$.
   5. Hence, the sum of the four interior angles of a quadrilateral is $360°$.

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8th Standard Maths Part-1 Book Chapter 5

Chapter 5: NUMBER PLAY

5.1 Summary of the Chapter:

Chapter 5 – Number Play: Summary:

(1) **Consecutive Numbers:** The chapter explores patterns in consecutive numbers, including how natural numbers can be expressed as sums of consecutive numbers and how algebra can be used to study such patterns.

(2) **Parity of Numbers:** It develops understanding of **even and odd numbers** and shows how parity behaves under addition and subtraction. Expressions formed by changing + and – signs can often be analysed without calculating every value.

(3) **Using Algebra to Generalise:** Letter-numbers such as $a, b, p, q,$and $k$are used to represent numbers and prove general mathematical properties instead of relying only on individual examples.

(4) **Multiples of 4:** Even numbers can be classified according to their remainder on division by 4. Adding two multiples of 4 gives a multiple of 4, and adding two even numbers that each leave remainder 2 on division by 4 also gives a multiple of 4.

(5) **Factors and Multiples:** Important relationships between factors and multiples are developed. For example, if $a$divides both $M$and N, then $a$also divides $M + N$and $M - N$.

(6) **Always, Sometimes or Never:** Students examine mathematical statements and decide whether they are **Always True, Sometimes True, or Never True**, supporting their conclusions with algebra, examples, counterexamples and visualisation.

(7) **LCM and Divisibility:** If a number is divisible by two numbers $k$and m, it is divisible by their **LCM**. This idea connects divisibility with factors, multiples and prime factorisation.

(8) **Remainders and Algebraic Forms:** Numbers having a particular remainder can be represented algebraically. For example, numbers leaving remainder 3 when divided by 5 can be written as $5k + 3$, or equivalently as $5k - 2$for suitable values of k.

(9) **Divisibility Shortcuts:** The chapter revisits divisibility tests for **2, 4, 5, 8 and 10** and explains them through place value and algebra rather than merely asking students to memorise rules.

(10) **Divisibility by 3 and 9:** A number is divisible by **9** when its digit sum is divisible by 9; similarly, a number is divisible by **3** when its digit sum is divisible by 3. The chapter explains *why* these rules work using place values and remainders.

(11) **Divisibility by 11:** The rule for divisibility by 11 is developed from the alternating behaviour of place values relative to multiples of 11, leading to the method involving the **alternating sum of digits**.

(12) **Mathematical Reasoning and Problem Solving:** Throughout the chapter, students solve number puzzles, investigate consecutive numbers, divisibility and remainders, test conjectures, and justify results using **algebra, visualisation, examples and counterexamples**. The central aim is to develop mathematical thinking rather than simply memorising procedures.

5.2 Important Formulas:

Important Formulas and Algebraic Forms – Chapter 5: Number Play:

1. **Even Number**

$$\boxed{2n}$$

where $n$is any integer.

2. **Odd Number**

$$\boxed{2n+1}$$

Any odd number can be represented in this form. The chapter uses these forms to reason about parity.

3. **Consecutive Natural Numbers**

$$\boxed{n,\ n+1,\ n+2,\ n+3, \ldots}$$

For four consecutive numbers:

$$\boxed{n,\ n+1,\ n+2,\ n+3}$$

4. **Consecutive Even Numbers**

$$\boxed{2n,\ 2n+2,\ 2n+4,\ 2n+6, \ldots}$$

5. **Consecutive Odd Numbers**

$$\boxed{2n+1,\ 2n+3,\ 2n+5,\ 2n+7, \ldots}$$

6. **Multiple of a Number**

A multiple of $a$can be represented as

$$\boxed{an}$$

where $n$is an integer.
For example, multiples of 8 are

$$\boxed{8n}$$

and multiples of 7 are

$$\boxed{7n}$$

The chapter repeatedly uses this representation to prove divisibility properties.

7. **Number Divisible by k**

If $A$is divisible by k,

$$\boxed{A = kn}$$

for some integer n.

8. **Sum of Two Multiples of the Same Number**

If

$$M = ka,\ N = kb$$

then

$$M + N = ka + kb$$
$$\boxed{M + N = k(a+b)}$$

Hence, the sum is also divisible by k.

9. **Difference of Two Multiples of the Same Number**

$$M - N = ka - kb$$
$$\boxed{M - N = k(a-b)}$$

Therefore, the difference of two multiples of $k$is also a multiple of k.

10. **Even Numbers that are Multiples of 4**

$$\boxed{4n}$$

Such numbers leave remainder **0** when divided by 4.

11. **Even Numbers that are Not Multiples of 4**

$$\boxed{4n+2}$$

Such numbers leave remainder **2** when divided by 4.

12. **Sum of Two Multiples of 4**

$$4p + 4q$$
$$\boxed{4p + 4q = 4(p+q)}$$

Hence, their sum is always a multiple of 4.

13. **Sum of Two Even Numbers that are Not Multiples of 4**

$$(4p + 2) + (4q + 2)$$
$$= 4p + 4q + 4$$
$$\boxed{= 4(p + q + 1)}$$

Hence, their sum is also a multiple of 4.

14. **Sum of a Multiple of 4 and an Even Non-Multiple of 4**

$$4p + (4q + 2)$$
$$\boxed{= 4(p + q) + 2}$$

Therefore, the result is even but **not divisible by 4**.

15. **General Remainder Form**

If a number leaves remainder r when divided by d, it can be written as

$$\boxed{N = dq + r}$$

where

$$\boxed{0 \leq r < d}$$

This is one of the most useful forms for solving remainder problems.

16. **Numbers Leaving Remainder 3 when Divided by 5**

$$\boxed{5k + 3}$$

They can also be written as

$$\boxed{5k - 2}$$

for suitable positive integer values of k.

17. **Numbers Leaving Remainder 2 when Divided by 6**

$$\boxed{6k + 2}$$

This form is useful in problems involving sums of several numbers having the same remainder.

18. **Divisibility by Two Numbers**

If A is divisible by both m and n, then A must be divisible by

$$\boxed{\mathrm{LCM}(m, n)}$$

Thus,

$$\boxed{m \mid A,\ n \mid A \Rightarrow \mathrm{LCM}(m, n) \mid A}$$

19. **Divisibility by 9**

For a number with digits $a, b, c, d, \ldots$,

$$\boxed{\text{Digit Sum} = a + b + c + d + \cdots}$$

A number is divisible by 9 if

$$\boxed{\text{Sum of digits is divisible by 9}}$$

The remainder on division by 9 can also be determined from the digit sum.

20. **Divisibility by 3**

$$\boxed{\text{Number divisible by 3} \quad \Leftrightarrow \quad \text{sum of its digits divisible by 3}}$$

21. **Divisibility by 11 – Alternating Sum Rule**

For a number with digits arranged from right to left,

$$\boxed{a - b + c - d + e - f + \cdots}$$

If this alternating sum is **0 or a multiple of 11**, the original number is divisible by 11. The chapter develops this rule using the alternating behaviour of powers of 10 with respect to multiples of 11.

22. **Place-Value Representation of a Number**

A four-digit number $abcd$ can be written algebraically as

$$\boxed{1000a + 100b + 10c + d}$$

More generally,

$$\boxed{\cdots + 1000d + 100c + 10b + a}$$

This form is important for proving divisibility shortcuts.

Quick Formula Box

$\boxed{\text{Even} = 2n}$

$\boxed{\text{Odd} = 2n + 1}$

$\boxed{\text{Multiple of } k = kn}$

$\boxed{\text{Remainder form} = kq + r}$

$\boxed{\text{Multiple of } 4 = 4n}$

$\boxed{\text{Even non-multiple of } 4 = 4n + 2}$

$\boxed{ka + kb = k(a + b)}$

$\boxed{ka - kb = k(a - b)}$

$\boxed{\text{Divisibility by 3: digit sum divisible by 3}}$

$\boxed{\text{Divisibility by 9: digit sum divisible by 9}}$

$\boxed{\text{Divisibility by 11: alternating digit sum is 0 or a multiple of 11}}$

These are the main algebraic forms and rules required for solving the **quantitative, reasoning, divisibility, remainder, consecutive-number and number-pattern problems** in Chapter 5.

5.3 Fill-Up the Blank LOTS type questions with Answers: (36 Questions)

LOTS Type – Fill in the Blanks with Answers:

1. A number that is divisible by 2 is called an ______ number.
   Answer: Even
2. An even number can be represented algebraically as ______.
   Answer: $2n$
3. An odd number can be represented algebraically as ______.
   Answer: $2n + 1$
4. The sum of two odd numbers is always an ______ number.
   Answer: Even
5. The sum of an odd number and an even number is always an ______ number.
   Answer: Odd
6. Even numbers that are multiples of 4 leave a remainder of ______ when divided by 4.
   Answer: 0
7. Even numbers that are not multiples of 4 leave a remainder of ______ when divided by 4.
   Answer: 2
8. The sum of two multiples of 4 is always a multiple of ______.
   Answer: 4
9. If $a$divides both $M$and N, then $a$also divides $M + N$and ______.
   Answer: $M - N$
10. If a number is divisible by 7, all ______ of that number are also divisible by 7.
    Answer: Multiples
11. If a number is divisible by both $k$and m, then it is divisible by the ______ of $k$and m.
    Answer: LCM
12. Numbers that leave a remainder of 3 when divided by 5 can be written in the form ______.
    Answer: $5k + 3$
13. The numbers 3, 8, 13, 18 and 23 all leave a remainder of ______ when divided by 5.
    Answer: 3

14. A number whose units digit is 0 is divisible by ______.
    Answer: 10
15. In the number 4075, the digit in the tens place is ______.
    Answer: 7
16. A number is divisible by 9 if the ______ of its digits is divisible by 9.
    Answer: Sum
17. The sum of the digits of 7309 is ______.
    Answer: 19
18. When 7309 is divided by 9, the remainder is ______.
    Answer: 1
19. A number is divisible by 3 if the sum of its digits is divisible by ______.
    Answer: 3
20. Every multiple of 9 is also a multiple of ______.
    Answer: 3
21. The numbers 15, 33 and 87 are multiples of ______ but not multiples of 9.
    Answer: 3
22. The divisibility shortcut for ______ uses an alternating pattern of place values.
    Answer: 11
23. In the divisibility test for 11, the units place is one ______ than a multiple of 11.
    Answer: More
24. In the divisibility test for 11, the tens place is one ______ than a multiple of 11.
    Answer: Less
25. The place value of the digit $b$in the tens place is represented as ______.
    Answer: $10b$
26. The place value of the digit $c$in the hundreds place is represented as ______.
    Answer: $100c$
27. If two numbers are multiples of 8, their sum will also be a multiple of ______.
    Answer: 8
28. $8a + 8b$can be factorised as ______.
    Answer: $8(a + b)$
29. The expression $4p + 4q$can be written as ______.
    Answer: $4(p + q)$
30. The expression $(4p + 2) + (4q + 2)$simplifies to ______.
    Answer: $4(p + q + 1)$
31. A mathematical statement may be classified as **Always True, Sometimes True, or** ______.
    Answer: Never True
32. If a number is divisible by 12, it is also divisible by all the ______ of 12.
    Answer: Factors
33. The sum of any two multiples of 2 is also a multiple of ______.
    Answer: 2
34. The general algebraic form of a multiple of 5 is ______.
    Answer: $5k$
35. Numbers that are 3 more than multiples of 5 have the algebraic form ______.
    Answer: $5k + 3$
36. Mathematical properties in this chapter are explored using algebra, visualisation, examples and ______.
    Answer: Counterexamples

5.4 Fill-Up the Blank HOTS type questions with Answers: (36 Questions)

HOTS Type – Fill in the Blanks with Answers:

1. If four consecutive integers are $n, n+1, n+2, n+3$, their sum is ______.
   Answer: $4n+6$
2. If changing a '+' sign to a '–' sign in an expression changes its value by an even number, the **parity** of the result remains ______.
   Answer: Same
3. All expressions of the form $a \pm b \pm c \pm d$, for fixed integers a, b, c, d, have the ______ parity.
   Answer: Same
4. If $4p$and $4q+2$are added, the result has the form ______ and therefore is not divisible by 4.
   Answer: $4(p+q)+2$
5. Two even numbers, neither divisible by 4, can be represented as $4p+2$and $4q+2$. Their sum is ______.
   Answer: $4(p+q+1)$
6. If $M = 8a$and $N = 8b$, then $M - N = 8(a-b)$; hence, $M - N$is always divisible by ______.
   Answer: 8
7. If a number $A$is divisible by both 9 and 4, then it must be divisible by $\mathrm{LCM}(9,4) =$______.
   Answer: 36
8. If a number is divisible by both 6 and 4, it is necessarily divisible by their LCM, which is ______.
   Answer: 12
9. The statement "If a number is divisible by both 6 and 4, it must be divisible by 24" is ______ true.
   Answer: Sometimes
10. If a number leaves remainder 3 when divided by 5, subtracting 3 from it produces a number divisible by ______.
    Answer: 5
11. If $N = 5k+3$, then $N+2 = 5(k+1)$. Therefore, $N+2$is always divisible by ______.
    Answer: 5
12. Three numbers, each leaving remainder 2 when divided by 6, have a sum of the form $18k+6$, which is always divisible by ______.
    Answer: 6
13. If a number leaves remainder 2 when divided by both 3 and 4, then subtracting 2 from it gives a common multiple of 3 and 4; therefore, it has the general form ______.
    Answer: $12k+2$
14. If 661 leaves remainder 3 and 4779 leaves remainder 5 when divided by 7, then $4779 + 661$leaves remainder ______ when divided by 7.
    Answer: 1
15. Using the same information, $4779 - 661$leaves remainder ______ when divided by 7.
    Answer: 2
16. If the sum of the digits of a number is 27, the number must be divisible by both 9 and ______.
    Answer: 3
17. Without performing division, 358095 is divisible by 9 because the sum of its digits is ______.

Answer: 30?
Correction/Answer: 30, so it is not divisible by 9.

18. The number 405 is divisible by 9 because $4 + 0 + 5 =$______.
Answer: 9
19. If a number has digit sum 19, its remainder when divided by 9 is ______.
Answer: 1
20. Since $10 = 9 + 1$, $100 = 99 + 1$, and $1000 = 999 + 1$, every power of 10 leaves remainder ______ when divided by 9.
Answer: 1
21. If the digit sum of a number is itself a multiple of 9, the original number is ______ by 9.
Answer: Divisible
22. If the sum of the digits of a number is not divisible by 3, the number itself is ______ divisible by 3.
Answer: Not
23. For divisibility by 11, the place values $1,100,10000$, ...behave as ______ more than multiples of 11.
Answer: 1
24. For divisibility by 11, the place values $10,1000,100000$, ...behave as ______ less than multiples of 11.
Answer: 1
25. If the alternating sum of the digits of a number is 0, the number is divisible by ______.
Answer: 11
26. The number 462 is divisible by 11 because the alternating digit calculation $4 - 6 + 2$gives ______.
Answer: 0
27. If $A$is divisible by k, every multiple $mA$is also divisible by ______.
Answer: k
28. If $A$is divisible by 12, then Amust also be divisible by 2, 3, 4 and ______.
Answer: 6
29. The statement "The sum of an odd number and an even number is a multiple of 6" is ______ true because such a sum is odd whereas every multiple of 6 is even.
Answer: Never
30. If $8a + 8b = 8(a + b)$, the factorisation proves that the sum of any two multiples of 8 is also a multiple of ______.
Answer: 8
31. If three consecutive even numbers are $2n, 2n + 2, 2n + 4$, their sum simplifies to ______.
Answer: $6n + 6$
32. Since $6n + 6 = 6(n + 1)$, the sum of three consecutive even numbers is always divisible by ______.
Answer: 6
33. The product of two consecutive integers $n(n + 1)$is always divisible by ______ because one of the two integers must be even.
Answer: 2
34. If a number is divisible by 36, it must also be divisible by both 4 and ______.
Answer: 9

35. If a number is divisible by 8, all its multiples remain divisible by 8 because multiplying it by another integer does not remove the factor ______.
    Answer: 8
36. To disprove an "always true" mathematical statement, finding just one suitable ______ is sufficient.
    Answer: Counterexample

These HOTS fill-ups require students to **analyse patterns, apply divisibility rules, manipulate algebraic forms, reason with remainders, make generalisations, and draw logical conclusions**, rather than merely recall definitions.

5.5 One-mark questions with Answers of LOTS Type: (35 Questions)

One-Mark Descriptive Questions with Answers – LOTS Type:

1. **What is an even number?**
   Answer: An even number is an integer that is exactly divisible by 2.
2. **How can an even number be represented algebraically?**
   Answer: An even number can be represented as $2n$, where $n$is an integer.
3. **How can an odd number be represented algebraically?**
   Answer: An odd number can be represented as $2n + 1$, where $n$is an integer.
4. **What is the parity of the sum of two odd numbers?**
   Answer: The sum of two odd numbers is always even.
5. **What is the parity of the sum of an odd number and an even number?**
   Answer: The sum of an odd number and an even number is always odd.
6. **What happens to the parity of expressions $a \pm b \pm c \pm d$when the signs are changed?**
   Answer: All such expressions formed from the same four numbers have the same parity.
7. **What remainder does a multiple of 4 leave when divided by 4?**
   Answer: A multiple of 4 leaves a remainder of 0 when divided by 4.
8. **What remainder does an even number that is not a multiple of 4 leave when divided by 4?**
   Answer: It leaves a remainder of 2 when divided by 4.
9. **What is the sum of two numbers that are multiples of 4?**
   Answer: The sum of two multiples of 4 is also a multiple of 4.
10. **If $a$divides both $M$and N, what can be said about $M + N$?**
    Answer: $M + N$is also divisible by a.
11. **If $a$divides both $M$and N, what can be said about $M - N$?**
    Answer: $M - N$is also divisible by a.
12. **What happens to the multiples of a number that is divisible by 7?**
    Answer: All multiples of that number are also divisible by 7.
13. **How are mathematical statements about factors and multiples classified in the chapter?**
    Answer: They are classified as **Always True, Sometimes True, or Never True**.
14. **What is the general form of numbers leaving remainder 3 when divided by 5?**
    Answer: Such numbers can be represented as $5k + 3$.
15. **What is another form for numbers leaving remainder 3 when divided by 5?**
    Answer: They can also be represented as $5k - 2$for suitable positive integer values of k.
16. **What is the divisibility rule for 10?**
    Answer: A number is divisible by 10 if its units digit is 0.

17. **What is the divisibility rule for 9?**
Answer: A number is divisible by 9 if the sum of its digits is divisible by 9.
18. **What is the divisibility rule for 3?**
Answer: A number is divisible by 3 if the sum of its digits is divisible by 3.
19. **Is every multiple of 9 also a multiple of 3?**
Answer: Yes, every multiple of 9 is also a multiple of 3.
20. **Give one example of a number divisible by 3 but not by 9.**
Answer: 15 is divisible by 3 but not by 9.
21. **What is the basic divisibility test for 11?**
Answer: A number is divisible by 11 when the alternating sum of its digits is 0 or a multiple of 11.
22. **What pattern do place values follow in the divisibility rule for 11?**
Answer: Successive place values alternate between one more and one less than a multiple of 11.
23. **What is the place-value expansion of a four-digit number $dcba$?**
Answer: It is $1000d + 100c + 10b + a$.
24. **What is the sum of two multiples of 8?**
Answer: The sum of two multiples of 8 is also a multiple of 8.
25. **What does $8a + 8b$simplify to?**
Answer: $8a + 8b$simplifies to $8(a + b)$.
26. **If a number is divisible by both $k$and m, by which important number must it also be divisible?**
Answer: It must be divisible by the LCM of $k$and m.
27. **What is the sum of the digits of 7309?**
Answer: The sum of the digits of 7309 is $7 + 3 + 0 + 9 = 19$.
28. **What remainder does 7309 leave when divided by 9?**
Answer: 7309 leaves a remainder of 1 when divided by 9.
29. **What is a counterexample in mathematical reasoning?**
Answer: A counterexample is an example that shows that a proposed general statement is not always true.
30. **Which methods are emphasised in the chapter for mathematical reasoning?**
Answer: The chapter emphasises algebra, visualisation, examples and counterexamples for mathematical reasoning.
31. **What is meant by consecutive numbers?**
Answer: Consecutive numbers are numbers that follow one another in order without any gaps.
32. **How can four consecutive numbers starting from $n$be represented?**
Answer: They can be represented as n, $n + 1$, $n + 2$, $n + 3$.
33. **What happens when two even numbers that are not multiples of 4 are added?**
Answer: Their sum is always a multiple of 4.
34. **What is the algebraic form of a multiple of 8?**
Answer: A multiple of 8 can be represented as $8n$, where $n$is an integer.
35. **What is the main purpose of divisibility shortcuts discussed in the chapter?**
Answer: Divisibility shortcuts help determine whether a number is divisible by another number without performing full division.

5.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – HOTS Type:

1. **Why do $a + b - c - d$ and $a - b - c - d$ have the same parity?**
   Answer: Their difference is $2b$, which is even, so both expressions have the same parity.
2. **Why do all expressions of the form $a \pm b \pm c \pm d$ have the same parity for fixed integers a, b, c, d?**
   Answer: Changing any sign changes the value by an even number and therefore does not change its parity.
3. **If $4p$ and $4q$ are added, why is the result divisible by 4?**
   Answer: Since $4p + 4q = 4(p + q)$, the sum contains 4 as a factor.
4. **Why does the sum of two even numbers that are not multiples of 4 become a multiple of 4?**
   Answer: Each leaves remainder 2 on division by 4, so their remainders add to 4, producing a multiple of 4.
5. **What can you conclude about $4p + (4q + 2)$?**
   Answer: It has the form $4(p + q) + 2$, so it is even but not divisible by 4.
6. **If 8 divides both M and N, why must 8 divide $M + N$?**
   Answer: Writing $M = 8a$ and $N = 8b$ gives $M + N = 8(a + b)$, which is divisible by 8.
7. **If 8 divides both M and N, what can you infer about $M - N$?**
   Answer: $M - N$ is also divisible by 8 because it can be written as $8(a - b)$.
8. **If a number is divisible by 7, why are all its multiples also divisible by 7?**
   Answer: Multiplying a number containing 7 as a factor by any integer still keeps 7 as a factor.
9. **Is every number divisible by 7 also divisible by 14? Give a reason.**
   Answer: No, because a multiple of 7 is divisible by 14 only when the remaining factor makes it a multiple of 14.
10. **If a number is divisible by 12, why must it also be divisible by 3?**
    Answer: Since 3 is a factor of 12, every multiple of 12 is also divisible by 3.
11. **A number is divisible by both 9 and 4; what further divisibility can you conclude?**
    Answer: The number is divisible by 36 because $\text{LCM}(9, 4) = 36$.
12. **Is a number divisible by both 6 and 4 necessarily divisible by 24?**
    Answer: No, because such a number must be divisible by $\text{LCM}(6, 4) = 12$, but need not always be divisible by 24.
13. **Why can the sum of an odd number and an even number never be a multiple of 6?**
    Answer: Their sum is odd, whereas every multiple of 6 is even.
14. **If $N = 5k + 3$, what remainder does N leave when divided by 5?**
    Answer: N leaves remainder 3 because it is exactly 3 more than a multiple of 5.
15. **Why do $5k + 3$ and $5(k + 1) - 2$ describe the same type of numbers?**
    Answer: They are algebraically equal because $5(k + 1) - 2 = 5k + 3$.
16. **If three numbers each leave remainder 2 when divided by 6, what can you conclude about their sum?**
    Answer: Their sum is divisible by 6 because the three remainders add to $2 + 2 + 2 = 6$.
17. **If 661 leaves remainder 3 and 4779 leaves remainder 5 on division by 7, what remainder does their sum leave?**
    Answer: Their sum leaves remainder 1 because $3 + 5 = 8$, which is 1 more than a multiple of 7.

18. **Using the same information, what remainder does $4779 - 661$leave when divided by 7?**
    Answer: It leaves remainder 2 because the difference of the remainders is $5 - 3 = 2$.
19. **Without dividing, how can you decide whether 405 is divisible by 9?**
    Answer: Since $4 + 0 + 5 = 9$, which is divisible by 9, 405 is divisible by 9.
20. **Without dividing, determine whether 8888 is divisible by 9.**
    Answer: It is not divisible by 9 because its digit sum $8 + 8 + 8 + 8 = 32$is not divisible by 9.
21. **Is 358095 divisible by 9? Justify without division.**
    Answer: No, because its digit sum is $3 + 5 + 8 + 0 + 9 + 5 = 30$, which is not divisible by 9.
22. **If a number has a digit sum of 27, what can you infer about its divisibility?**
    Answer: The number is divisible by both 9 and 3 because 27 is divisible by both 9 and 3.
23. **Why does the digit-sum test for divisibility by 9 work?**
    Answer: Every power of 10 leaves remainder 1 when divided by 9, so a number and its digit sum have the same remainder modulo 9.
24. **If the sum of the digits of a number is not divisible by 3, what can you conclude?**
    Answer: The original number is not divisible by 3.
25. **Why does the divisibility test for 11 involve alternating signs?**
    Answer: Successive place values alternate between one more and one less than multiples of 11.
26. **Is 462 divisible by 11? Give a reason without division.**
    Answer: Yes, because the alternating sum $4 - 6 + 2 = 0$, showing that 462 is divisible by 11.
27. **If the alternating sum of a number's digits is 22, what can you conclude?**
    Answer: The number is divisible by 11 because 22 is a multiple of 11.
28. **Why is $8a + 8b$always divisible by 8?**
    Answer: Factoring gives $8(a + b)$, which shows that 8 is a factor of the expression.
29. **How can one counterexample disprove an "Always True" statement?**
    Answer: A single counterexample demonstrates that the statement fails in at least one case and therefore cannot be always true.
30. **Why is algebra useful for proving number patterns?**
    Answer: Algebra represents whole classes of numbers at once and therefore helps establish whether a pattern holds generally.
31. **If the middle number of five consecutive integers is p, what is the first number?**
    Answer: The first number is $p - 2$.
32. **If three consecutive even numbers are $2n, 2n + 2, 2n + 4$, why is their sum divisible by 6?**
    Answer: Their sum is $6n + 6 = 6(n + 1)$, so it is divisible by 6.
33. **Why is the product of two consecutive integers always even?**
    Answer: One of any two consecutive integers must be even, so their product contains 2 as a factor.
34. **If a number is divisible by 36, why must it be divisible by 4 and 9?**
    Answer: Since both 4 and 9 are factors of 36, every multiple of 36 is divisible by both.
35. **What mathematical approach should be used to test whether a divisibility conjecture is always true?**
    Answer: The conjecture should be tested with examples and counterexamples and then justified generally using algebra or mathematical reasoning.

5.7 Two-mark questions with Answers of LOTS Type: (28 Questions)

Two-Mark Descriptive Questions with Answers – LOTS Type:

1. What are even and odd numbers? Give their algebraic forms.

Answer:

1. An even number is divisible by 2 and can be written as $2n$.
2. An odd number is not divisible by 2 and can be written as $2n + 1$.

2. State the parity rules for adding two odd numbers and an odd and an even number.

Answer:

1. The sum of two odd numbers is always even.
2. The sum of an odd number and an even number is always odd.

3. How can four consecutive numbers be represented algebraically?

Answer:

1. If the first number is n, the four consecutive numbers are $n, n + 1, n + 2, n + 3$.
2. Each number after the first is obtained by adding 1 to the preceding number.

4. What are the two possible forms of an even number with respect to division by 4?

Answer:

1. An even number divisible by 4 can be written as $4p$.
2. An even number not divisible by 4 can be written as $4p + 2$.

5. Show that the sum of two multiples of 4 is a multiple of 4.

Answer:

1. Let the two numbers be $4p$and $4q$; their sum is $4p + 4q$.
2. Since $4p + 4q = 4(p + q)$, the sum is a multiple of 4.

6. What happens when two even numbers that are not multiples of 4 are added?

Answer:

1. Such numbers can be written as $4p + 2$and $4q + 2$.
2. Their sum is $4(p + q + 1)$, so it is a multiple of 4.

7. State two divisibility properties involving the sum and difference of numbers.

Answer:

1. If $a$divides both $M$and N, then $a$divides $M + N$.
2. If $a$divides both $M$and N, then $a$also divides $M - N$.

8. What happens to the multiples of a number that is divisible by another number?

Answer:

1. If $A$is divisible by k, every multiple of $A$is also divisible by k.
2. For example, if $A = 7m$, then $nA = 7(mn)$, which is also divisible by 7.

9. What is the relation between divisibility and the factors of a divisor?

Answer:

1. If a number is divisible by k, it is also divisible by every factor of k.
2. Thus, a number divisible by 12 is also divisible by factors such as 2, 3, 4 and 6.

10. State the LCM property of divisibility.

Answer:

1. If a number is divisible by both $k$and m, it is divisible by $\text{LCM}(k, m)$.
2. For example, a number divisible by both 4 and 6 must be divisible by $\text{LCM}(4, 6) = 12$.

11. What are the three classifications used in the "Always, Sometimes, or Never" activity?

Answer:

1. A mathematical statement may be classified as **Always True, Sometimes True, or Never True**.

2. Examples and counterexamples can be used to decide which classification applies.

12. How can numbers that leave remainder 3 when divided by 5 be represented?

Answer:

1. Such numbers can be written in the form $5k + 3$.
2. They can also be expressed as $5k - 2$for suitable positive integer values of k.

13. State the divisibility rule for 10 and explain it briefly.

Answer:

1. A number is divisible by 10 when its units digit is 0.
2. This works because all the other place-value terms are multiples of 10.

14. State the divisibility rule for 9.

Answer:

1. Find the sum of all the digits of the given number.
2. If this digit sum is divisible by 9, the original number is also divisible by 9.

15. State the divisibility rule for 3.

Answer:

1. Add all the digits of the given number.
2. If the resulting sum is divisible by 3, the original number is divisible by 3.

16. How can the remainder of a number on division by 9 be found using its digits?

Answer:

1. Find the sum of the digits of the number and reduce it using multiples of 9.
2. The resulting remainder is the same as the remainder obtained when the original number is divided by 9.

17. Explain why 7309 leaves remainder 1 when divided by 9.

Answer:

1. The digit sum of 7309 is $7 + 3 + 0 + 9 = 19$.
2. Since $19 = 18 + 1$, 7309 leaves remainder 1 when divided by 9.

18. State the divisibility rule for 11.

Answer:

1. Find the alternating sum of the digits of the given number.
2. If this result is 0 or a multiple of 11, the original number is divisible by 11.

19. What place-value pattern is used to understand divisibility by 11?

Answer:

1. The place values $1,100,10000$, ...are one more than suitable multiples of 11.
2. The place values $10,1000,100000$, ...are one less than suitable multiples of 11.

20. Write the place-value expansion of a four-digit number $dcba$.

Answer:

1. The four-digit number can be written as $1000d + 100c + 10b + a$.
2. This algebraic expansion helps explain several divisibility shortcuts.

21. Show that the sum of two multiples of 8 is also a multiple of 8.

Answer:

1. If the two multiples are $8a$and $8b$, their sum is $8a + 8b$.
2. Since $8a + 8b = 8(a + b)$, their sum is divisible by 8.

22. Why can the sum of an odd number and an even number never be a multiple of 6?

Answer:

1. The sum of an odd number and an even number is always odd.
2. Every multiple of 6 is even, so such a sum cannot be a multiple of 6.

23. What is a counterexample, and why is it useful?

Answer:

1. A counterexample is a specific example for which a proposed general statement is false.

2. It is useful for showing that a mathematical statement is not **Always True**.

24. What mathematical methods are used in the chapter to establish number properties?

Answer:

1. The chapter uses **algebra and visualisation** to identify and explain number patterns.
2. It also uses **examples and counterexamples** to develop mathematical reasoning.

25. Why are divisibility shortcuts useful in Number Play?

Answer:

1. Divisibility shortcuts help determine whether a number is divisible by another number without carrying out complete division.
2. The chapter develops and explains shortcuts for numbers such as **3, 9 and 11**.

5.8. Two-mark questions with Answers of HOTS Type: (15 Questions)

Two-Mark Descriptive Questions with Answers – HOTS Type:

1. Why do all expressions formed by changing the $+$and $-$signs between four fixed numbers have the same parity?

Answer:

1. Changing one sign changes the value of the expression by twice one of the numbers, which is always even.
2. Adding or subtracting an even number does not change parity, so all such expressions have the same parity.

2. Two even numbers are not divisible by 4; explain why their sum must be divisible by 4.

Answer:

1. The numbers can be written as $4p + 2$and $4q + 2$.
2. Their sum is $4p + 4q + 4 = 4(p + q + 1)$, which is divisible by 4.

3. One even number is divisible by 4 and another is not; what can you conclude about their sum?

Answer:

1. Writing them as $4p$and $4q + 2$, their sum becomes $4(p + q) + 2$.
2. Hence, their sum is even but is not divisible by 4.

4. If $a$divides both $M$and N, explain algebraically why $a$divides $M + N$.

Answer:

1. Write $M = ap$and $N = aq$, where $p$and $q$are integers.
2. Then $M + N = a(p + q)$, showing that $M + N$is divisible by a.

5. If $a$divides both $M$and N, why is their difference also divisible by a?

Answer:

1. If $M = ap$and $N = aq$, then $M - N = ap - aq$.
2. Since $M - N = a(p - q)$, their difference is also divisible by a.

6. A number is divisible by both 8 and 12; what other divisibility can you definitely conclude?

Answer:

1. The number must be divisible by the LCM of 8 and 12.
2. Since $\text{LCM}(8, 12) = 24$, the number is definitely divisible by 24.

7. A student says, "Every number divisible by both 4 and 6 must be divisible by 24." Is the student correct?

Answer:

1. No, because a number divisible by both 4 and 6 is guaranteed to be divisible only by $\text{LCM}(4, 6) = 12$.
2. For example, 12 is divisible by both 4 and 6 but is not divisible by 24.

8. Why can an odd number plus an even number never be a multiple of 6?
Answer:
1. The sum of an odd number and an even number is always odd.
2. Every multiple of 6 is even, so an odd sum cannot be divisible by 6.

9. A number leaves remainder 3 when divided by 5; what happens when 2 is added to it? Explain.
Answer:
1. The number can be represented as $5k + 3$, so adding 2 gives $5k + 5$.
2. Since $5k + 5 = 5(k + 1)$, the new number is divisible by 5.

10. Explain why $5k + 3$and $5(k + 1) - 2$represent the same set of numbers.
Answer:
1. Expanding the second expression gives $5(k + 1) - 2 = 5k + 5 - 2$.
2. This simplifies to $5k + 3$, so both forms describe the same numbers.

11. Three numbers each leave remainder 2 when divided by 6; what can you say about their sum?
Answer:
1. The sum of their remainders is $2 + 2 + 2 = 6$.
2. Since 6 itself is divisible by 6, the sum of the three numbers is divisible by 6.

12. Without performing division, determine whether 738 is divisible by 9.
Answer:
1. The sum of its digits is $7 + 3 + 8 = 18$.
2. Since 18 is divisible by 9, 738 is also divisible by 9.

13. A number has digit sum 28; what remainder will it leave when divided by 9?
Answer:
1. The number and its digit sum have the same remainder when divided by 9.
2. Since $28 = 27 + 1$, the number leaves remainder 1 when divided by 9.

14. Why does the digit-sum test for divisibility by 9 work?
Answer:
1. Each power of 10 leaves remainder 1 when divided by 9.
2. Therefore, a number and the sum of its digits leave the same remainder when divided by 9.

15. Without division, decide whether 12345 is divisible by 3.
Answer:
1. Its digit sum is $1 + 2 + 3 + 4 + 5 = 15$.
2. Since 15 is divisible by 3, 12345 is also divisible by 3.

16. If a number is divisible by 9, why must it also be divisible by 3?
Answer:
1. Every multiple of 9 has a digit sum that is divisible by 9 and hence also by 3.
2. Therefore, every number divisible by 9 is necessarily divisible by 3.

17. Without performing division, determine whether 462 is divisible by 11.
Answer:
1. The alternating sum of its digits is $4 - 6 + 2 = 0$.
2. Since the alternating sum is 0, 462 is divisible by 11.

18. Why are alternating signs used in the divisibility test for 11?
Answer:
1. Successive powers of 10 behave alternately as one more and one less than multiples of 11.
2. Hence, the digits contribute alternately with positive and negative signs when testing divisibility by 11.

19. If the alternating sum of the digits of a number is 22, what can you conclude?
Answer:
1. Since 22 is a multiple of 11, the alternating sum is divisible by 11.
2. Therefore, the original number is also divisible by 11.

20. Explain why $8a + 8b$is always a multiple of 8.
Answer:
1. Taking 8 as a common factor gives $8a + 8b = 8(a + b)$.
2. Since $a + b$is an integer, the expression is always divisible by 8.

21. A number is divisible by 36; explain why it must also be divisible by both 4 and 9.
Answer:
1. Both 4 and 9 are factors of 36.
2. Therefore, every multiple of 36 is necessarily divisible by both 4 and 9.

22. How can a counterexample help decide whether a mathematical statement is "Always True"?
Answer:
1. Test the statement with different cases and look for a case in which it fails.
2. If even one counterexample is found, the statement cannot be classified as "Always True."

23. Why is algebra more useful than checking only a few examples when proving a number pattern?
Answer:
1. A few examples show that a pattern works only for the particular numbers tested.
2. An algebraic representation can establish that the pattern holds for all numbers of the required form.

24. If $A$is divisible by 12, why are all multiples of Aalso divisible by 12?
Answer:
1. Write $A = 12k$; then any multiple mAequals $12km$.
2. Since $mA = 12(km)$, every multiple of $A$is also divisible by 12.

25. A number has a digit sum of 45; what can you conclude about its divisibility by 3 and 9?
Answer:
1. Since 45 is divisible by both 3 and 9, the original number is divisible by both numbers.
2. Thus, the digit-sum rules establish its divisibility without performing long division.

5.9. Three-mark questions with Answers of LOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – LOTS Type:
1. Explain the parity rules for addition and subtraction of odd and even numbers.
Answer:
1. The sum or difference of two even numbers is **even**.
2. The sum or difference of two odd numbers is also **even**.
3. The sum or difference of an odd number and an even number is **odd**.

2. How can four consecutive integers be represented algebraically? Explain with an example.
Answer:
1. If the first integer is n, four consecutive integers are n, $n + 1$, $n + 2$, $n + 3$.
2. Consecutive integers differ from one another by exactly 1.
3. For example, if $n = 5$, the four consecutive integers are 5,6,7,8.

3. Explain the two forms of even numbers based on their remainders when divided by 4.
Answer:
1. An even number divisible by 4 can be represented as $4p$.
2. An even number not divisible by 4 can be represented as $4p + 2$.
3. Thus, an even number leaves either remainder **0 or 2** when divided by 4.

4. Show that the sum of two multiples of 4 is always a multiple of 4.
Answer:
1. Let the two multiples of 4 be $4p$and $4q$.
2. Their sum is $4p + 4q = 4(p + q)$.
3. Since 4 is a factor of the sum, the result is always a multiple of 4.

5. Explain why the sum of two even numbers that are not multiples of 4 is a multiple of 4.
Answer:
1. Such numbers can be represented as $4p + 2$and $4q + 2$.
2. Their sum is $(4p + 2) + (4q + 2) = 4p + 4q + 4$.
3. This equals $4(p + q + 1)$, so the sum is divisible by 4.

6. State and explain the divisibility property involving the sum and difference of two numbers.
Answer:
1. If $a$divides both $M$and N, then $a$also divides $M + N$.
2. The same divisor $a$also divides the difference $M - N$.
3. Thus, a common divisor of two numbers also divides their sum and difference.

7. Explain the relationship between the divisibility of a number and its multiples.
Answer:
1. Suppose a number $A$is divisible by k.
2. Then Acan be written as $A = kn$for some integer n.
3. Every multiple of Awill therefore also contain $k$as a factor and be divisible by k.

8. Explain the relationship between divisibility and the factors of the divisor.
Answer:
1. If a number is divisible by k, it is also divisible by every factor of k.
2. For example, if a number is divisible by 12, it is also divisible by 2, 3, 4 and 6.
3. This is because each of these numbers is a factor of 12.

9. Explain the LCM property used in divisibility problems.
Answer:
1. If a number is divisible by two numbers $k$and m, it is divisible by their LCM.
2. For example, a number divisible by both 4 and 6 must be divisible by $\text{LCM}(4, 6)$.
3. Since $\text{LCM}(4, 6) = 12$, the number must be divisible by 12.

10. Explain the “Always, Sometimes, or Never” classification used in the chapter.
Answer:
1. An **Always True** statement is true for every possible case.
2. A **Sometimes True** statement is true only for certain cases.
3. A **Never True** statement does not hold for any of the cases under consideration.

11. How can numbers leaving remainder 3 when divided by 5 be represented algebraically?
Answer:
1. A multiple of 5 can be represented as $5k$.
2. A number that is 3 more than a multiple of 5 can therefore be written as $5k + 3$.
3. The same type of numbers can also be represented as $5k - 2$for suitable positive integer values of k.

12. Explain the divisibility rule for 9 with an example.
Answer:
1. Add all the digits of the given number.
2. If the digit sum is divisible by 9, the original number is also divisible by 9.
3. For example, $405$is divisible by 9 because $4 + 0 + 5 = 9$.

13. How can the remainder on division by 9 be determined using the digit sum?
Answer:
1. Find the sum of all the digits of the given number.
2. Determine the remainder when this digit sum is divided by 9.
3. The original number leaves the same remainder when divided by 9.

14. Explain why 7309 leaves remainder 1 when divided by 9.
Answer:
1. The sum of the digits of 7309 is $7 + 3 + 0 + 9 = 19$.
2. When 19 is divided by 9, it leaves remainder 1 because $19 = 2(9) + 1$.
3. Therefore, 7309 also leaves remainder 1 when divided by 9.

15. Explain the divisibility rule for 3.
Answer:
1. Add all the digits of the given number.
2. Check whether the resulting digit sum is divisible by 3.
3. If the digit sum is divisible by 3, the original number is also divisible by 3.

16. Explain the basic divisibility rule for 11.
Answer:
1. Find the alternating sum of the digits of the given number.
2. Check whether the alternating sum is 0 or a multiple of 11.
3. If it is, the original number is divisible by 11.

17. Explain the place-value idea behind the divisibility test for 11.
Answer:
1. Place values $1,100,10000$, ...behave as one more than suitable multiples of 11.
2. Place values $10,1000,100000$, ...behave as one less than suitable multiples of 11.
3. This alternating pattern leads to the alternating-sum divisibility test for 11.

18. Write and explain the place-value expansion of a four-digit number $dcba$.
Answer:
1. The digit $d$contributes $1000d$and $c$contributes $100c$.
2. The digit $b$contributes $10b$, while $a$contributes a.
3. Hence, the number is represented as $1000d + 100c + 10b + a$.

19. Show that the sum of two multiples of 8 is also a multiple of 8.
Answer:
1. Let the two numbers be $8a$and $8b$.
2. Their sum is $8a + 8b = 8(a + b)$.
3. Therefore, the sum contains 8 as a factor and is divisible by 8.

20. What mathematical reasoning methods are emphasised in the chapter?
Answer:
1. **Algebra** is used to express and establish general number patterns.
2. **Visualisation and examples** are used to understand and test mathematical ideas.
3. **Counterexamples** are used to show when proposed statements are not always true.

5.10. Three-mark questions with Answers of HOTS Type: (25 Questions)

Three-Mark Descriptive Questions with Answers – HOTS Type:

1. Two even numbers are not divisible by 4. Prove that their sum is divisible by 4.
Answer:
1. Any even number not divisible by 4 can be written as $4p + 2$, so let the numbers be $4p + 2$ and $4q + 2$.
2. Their sum is $(4p + 2) + (4q + 2) = 4p + 4q + 4 = 4(p + q + 1)$.
3. Since the sum contains 4 as a factor, it is always divisible by 4.

2. One even number is a multiple of 4 and another is not a multiple of 4. Analyse their sum.
Answer:
1. Represent the numbers as $4p$ and $4q + 2$.
2. Their sum is $4p + 4q + 2 = 4(p + q) + 2$.
3. Therefore, the sum is even but leaves remainder 2 when divided by 4.

3. Explain why changing the signs in $a \pm b \pm c \pm d$ does not change the parity of the result.
Answer:
1. Changing the sign of a term from $+b$ to $-b$ changes the expression by $2b$.
2. Since $2b$ is always even, the new result differs from the old result by an even number.
3. Hence, all such expressions formed from the same four integers have the same parity.

4. If a divides both M and N, prove that it also divides $M + N$ and $M - N$.
Answer:
1. Since $a \mid M$ and $a \mid N$, write $M = ap$ and $N = aq$.
2. Then $M + N = a(p + q)$ and $M - N = a(p - q)$.
3. Therefore, both the sum and difference are divisible by a.

5. A student claims that every number divisible by both 4 and 6 is divisible by 24. Examine the claim.
Answer:
1. A number divisible by both 4 and 6 must be divisible by $\text{LCM}(4, 6) = 12$.
2. The number 12 itself is divisible by both 4 and 6 but is not divisible by 24.
3. Therefore, the student's claim is not always true; 12 provides a counterexample.

6. A number is divisible by both 8 and 12. What divisibility can you definitely establish? Explain.
Answer:
1. A number divisible by two numbers must be divisible by their LCM.
2. Since $\text{LCM}(8, 12) = 24$, the given number must be a multiple of 24.
3. Hence, divisibility by both 8 and 12 guarantees divisibility by 24.

7. Explain why an odd number plus an even number can never be a multiple of 6.
Answer:
1. The sum of an odd number and an even number is always odd.
2. Every multiple of 6 is divisible by 2 and is therefore even.
3. Hence, an odd sum cannot be a multiple of 6.

8. A number leaves remainder 3 when divided by 5. Show that adding 2 makes it divisible by 5.
Answer:
1. Represent the number as $5k + 3$.
2. Adding 2 gives $5k + 3 + 2 = 5k + 5 = 5(k + 1)$.
3. Therefore, the new number is exactly divisible by 5.

9. Explain why $5k + 3$ and $5(k + 1) - 2$ represent the same type of numbers.
Answer:
1. Expand the second expression as $5(k + 1) - 2 = 5k + 5 - 2$.
2. Simplifying it gives $5k + 3$.

3. Hence, both expressions represent numbers that leave remainder 3 when divided by 5.

10. Three numbers each leave remainder 2 when divided by 6. Determine whether their sum is divisible by 6.

Answer:

1. Represent the three numbers as $6a + 2$, $6b + 2$, $6c + 2$.
2. Their sum is $6a + 6b + 6c + 6 = 6(a + b + c + 1)$.
3. Therefore, the sum is always divisible by 6.

11. Given that 661 leaves remainder 3 and 4779 leaves remainder 5 when divided by 7, find the remainder when their sum is divided by 7.

Answer:

1. Add the two remainders to get $3 + 5 = 8$.
2. Since $8 = 7 + 1$, 8 leaves remainder 1 when divided by 7.
3. Therefore, $661 + 4779$leaves remainder **1** when divided by 7.

12. Using the same information, find the remainder when $4779 - 661$is divided by 7.

Answer:

1. The respective remainders are 5 and 3.
2. Subtracting them gives $5 - 3 = 2$.
3. Therefore, $4779 - 661$leaves remainder **2** when divided by 7.

13. Without performing division, determine whether 738 is divisible by 9 and justify your answer.

Answer:

1. The sum of the digits of 738 is $7 + 3 + 8 = 18$.
2. Since 18 is divisible by 9, the divisibility rule for 9 applies.
3. Therefore, 738 is divisible by 9.

14. A number has digit sum 28. Determine its remainder when divided by 9 and explain.

Answer:

1. A number and its digit sum leave the same remainder when divided by 9.
2. Since $28 = 3 \times 9 + 1$, the digit sum leaves remainder 1.
3. Therefore, the original number also leaves remainder **1** when divided by 9.

15. Explain mathematically why the digit-sum test for divisibility by 9 works.

Answer:

1. The place values $10, 100, 1000, \ldots$are each 1 more than a multiple of 9.
2. Hence, each digit contributes its own value to the remainder when the number is divided by 9.
3. Therefore, the number and the sum of its digits have the same remainder modulo 9.

16. Without division, determine whether 12345 is divisible by 3 and explain your reasoning.

Answer:

1. Its digit sum is $1 + 2 + 3 + 4 + 5 = 15$.
2. Since 15 is divisible by 3, the divisibility test for 3 is satisfied.
3. Therefore, 12345 is divisible by 3.

17. A number has digit sum 45. What can you conclude about its divisibility by 3 and 9?

Answer:

1. Since $45$is divisible by 9, the original number is divisible by 9.
2. Since $45$is also divisible by 3, the original number is divisible by 3.
3. Therefore, the number is divisible by **both 3 and 9**.

18. Determine whether 462 is divisible by 11 without carrying out long division.

Answer:

1. Using alternating signs, calculate $4 - 6 + 2 = 0$.
2. An alternating digit sum of 0 satisfies the divisibility condition for 11.

3. Therefore, 462 is divisible by 11.

19. Explain why the divisibility test for 11 uses an alternating sum of digits.

Answer:

1. Successive powers of 10 behave alternately as one more and one less than suitable multiples of 11.
2. Thus, successive digits contribute alternately with positive and negative signs when considering divisibility by 11.
3. This produces the alternating-sum rule used to test divisibility by 11.

20. A number has an alternating digit sum of 22. What can you conclude about the number?

Answer:

1. The alternating digit sum 22 is a multiple of 11.
2. Therefore, it satisfies the divisibility criterion for 11.
3. Hence, the original number is divisible by 11.

21. Prove that the sum of any two multiples of 8 is also a multiple of 8.

Answer:

1. Let the two numbers be $8a$and $8b$.
2. Their sum is $8a + 8b = 8(a + b)$.
3. Since $a + b$is an integer, the sum is a multiple of 8.

22. A number is divisible by 36. Explain why it must also be divisible by both 4 and 9.

Answer:

1. The number can be written as $36k$for some integer k.
2. Since $36 = 4 \times 9$, we have $36k = 4(9k) = 9(4k)$.
3. Therefore, the number is divisible by both 4 and 9.

23. How can a counterexample be used to test an "Always True" statement?

Answer:

1. First, test the statement using different values that satisfy its conditions.
2. If even one valid example makes the statement false, that example is a counterexample.
3. Therefore, the existence of one counterexample proves that the statement is not "Always True."

24. Why is algebraic reasoning stronger than checking only a few numerical examples when studying number patterns?

Answer:

1. Numerical examples show that a pattern works only for the particular cases tested.
2. Algebra represents an entire class of numbers using variables and establishes the underlying relationship.
3. Therefore, algebra can justify a pattern generally rather than only suggesting it from examples.

25. If $A$is divisible by k, prove that every multiple of $A$is also divisible by k.

Answer:

1. Since $A$is divisible by k, write $A = kn$.
2. Any multiple mAcan then be written as $m(kn) = k(mn)$.
3. Therefore, every multiple of $A$is also divisible by k.

5.11 Five-mark questions with Answers of LOTS Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – LOTS Type:

1. Explain the parity rules for odd and even numbers with suitable examples.

Answer:

1. An **even number** is divisible by 2 and can be represented as $2n$.
2. An **odd number** can be represented as $2n + 1$.
3. The sum or difference of two even numbers is always even; for example, $8 + 6 = 14$.
4. The sum or difference of two odd numbers is also even; for example, $9 + 7 = 16$.
5. The sum or difference of an odd and an even number is odd; for example, $9 + 6 = 15$.

---

2. Explain the two types of even numbers based on division by 4 and their addition properties.

Answer:

1. An even number divisible by 4 can be represented in the form $4p$.
2. An even number not divisible by 4 can be represented in the form $4p + 2$.
3. The sum of two multiples of 4 is $4p + 4q = 4(p + q)$, which is divisible by 4.
4. The sum of two even non-multiples of 4 is $(4p + 2) + (4q + 2) = 4(p + q + 1)$, which is also divisible by 4.
5. Adding a multiple of 4 and an even non-multiple of 4 gives a number of the form $4k + 2$, which is not divisible by 4.

---

3. Explain five important properties of factors and multiples discussed in the chapter.

Answer:

1. If $a$divides both $M$and N, then $a$also divides $M + N$.
2. If $a$divides both $M$and N, then $a$also divides $M - N$.
3. If a number $A$is divisible by k, every multiple of $A$is also divisible by k.
4. If $A$is divisible by k, then $A$is also divisible by every factor of k.
5. If a number is divisible by both $k$and m, it is divisible by LCM(k, m).

---

4. Explain how remainders can be represented algebraically, using numbers that leave remainder 3 when divided by 5.

Answer:

1. A multiple of 5 can be represented as $5k$, where $k$is an integer.
2. A number leaving remainder 3 on division by 5 can therefore be represented as $5k + 3$.
3. Examples of such numbers are 3,8,13,18,23, ...
4. These numbers can also be expressed in the form $5k - 2$for suitable positive values of k.
5. Algebraic forms such as $5k + 3$help us identify and reason about all numbers having the same remainder.

---

5. Explain the divisibility test for 9 and the mathematical idea behind it.

Answer:

1. To test divisibility by 9, first find the sum of all the digits of the given number.
2. If this digit sum is divisible by 9, then the original number is also divisible by 9.
3. The place values 10,100,1000, ...are each 1 more than a suitable multiple of 9.
4. Therefore, a number and the sum of its digits leave the same remainder when divided by 9.
5. For example, 7309has digit sum $7 + 3 + 0 + 9 = 19$, so both 19 and 7309 leave remainder 1 when divided by 9.

---

6. Explain the divisibility test for 3 with suitable examples.

Answer:

1. Add all the digits of the given number to test whether it is divisible by 3.

2. If the digit sum is divisible by 3, the original number is also divisible by 3.
3. For example, for 12345, the digit sum is $1 + 2 + 3 + 4 + 5 = 15$.
4. Since 15 is divisible by 3, 12345 is also divisible by 3.
5. A number may be divisible by 3 without being divisible by 9, as seen in numbers such as 15 and 33.

---

7. Explain the divisibility test for 11 and the place-value idea behind it.
Answer:
1. The divisibility test for 11 uses the **alternating sum of the digits** of a number.
2. The place values $1,100,10000$, ...behave as one more than suitable multiples of 11.
3. The place values $10,1000,100000$, ...behave as one less than suitable multiples of 11.
4. Hence, the digits contribute alternately with positive and negative signs when checking divisibility by 11.
5. If the resulting alternating sum is 0 or a multiple of 11, the original number is divisible by 11.

---

8. Explain how place-value expansion helps in understanding divisibility rules.
Answer:
1. A number can be expressed as the sum of the place values of its digits.
2. For example, a four-digit number $dcba$can be written as $1000d + 100c + 10b + a$.
3. For divisibility by 10, all terms except the units term are multiples of 10.
4. Therefore, the units digit determines whether the whole number is divisible by 10.
5. Similar place-value reasoning is used in the chapter to understand divisibility shortcuts for numbers such as 3, 9 and 11.

---

9. Explain the "Always, Sometimes, or Never" approach used for studying number properties.
Answer:
1. An **Always True** statement is one that holds for every case satisfying the given conditions.
2. A **Sometimes True** statement holds for some cases but fails for others.
3. A **Never True** statement does not hold for any case satisfying the conditions.
4. Numerical examples can be used to examine whether a statement appears to be true.
5. Counterexamples and mathematical reasoning help determine whether a proposed statement can be accepted as a general property.

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10. Explain how algebra is used to establish divisibility properties with multiples of 8.
Answer:
1. A multiple of 8 can be represented algebraically as $8a$.
2. Another multiple of 8 can similarly be represented as $8b$.
3. Their sum is $8a + 8b$, which can be factorised as $8(a + b)$.
4. Since $a + b$is an integer, $8(a + b)$is again a multiple of 8.
5. Thus, algebra shows generally that the sum of any two multiples of 8 is divisible by 8.

5.12 Five-mark questions with Answers of HOTS Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – HOTS Type:
1. Two even numbers are not multiples of 4. Prove algebraically that their sum is always a multiple of 4.
Answer:

1. Any even number that is not a multiple of 4 can be represented in the form $4p + 2$.
2. Let the two given numbers be $4p + 2$and $4q + 2$.
3. Their sum is $(4p + 2) + (4q + 2) = 4p + 4q + 4$.
4. Factoring out 4 gives $4(p + q + 1)$.
5. Therefore, the sum of two even numbers that are not multiples of 4 is always divisible by 4.

---

2. A student claims, “If a number is divisible by both 6 and 8, it must be divisible by 48.” Analyse the claim and justify your answer.
Answer:

1. A number divisible by both 6 and 8 must be divisible by their least common multiple.
2. We have $\text{LCM}(6, 8) = 24$.
3. Therefore, divisibility by both 6 and 8 guarantees divisibility by 24, not necessarily by 48.
4. For example, 24 is divisible by both 6 and 8 but is not divisible by 48.
5. Hence, the student's claim is **not always true**, and 24 serves as a counterexample.

---

3. A number leaves remainder 3 when divided by 5. Use algebra to analyse what happens when 2, 7, and 12 are separately added to it.
Answer:

1. Represent the original number as $5k + 3$.
2. Adding 2 gives $5k + 5 = 5(k + 1)$, which is divisible by 5.
3. Adding 7 gives $5k + 10 = 5(k + 2)$, which is also divisible by 5.
4. Adding 12 gives $5k + 15 = 5(k + 3)$, which is again divisible by 5.
5. Therefore, adding any number of the form $5m + 2$to $5k + 3$produces a multiple of 5.

---

4. Explain why changing any of the signs in an expression $a \pm b \pm c \pm d$does not change the parity of its value.
Answer:

1. Consider changing one term from $+b$to $-b$while keeping the other terms unchanged.
2. This changes the value of the expression by $2b$.
3. Since $2b$is always even, the two resulting values differ by an even number.
4. Two integers differing by an even number must have the same parity.
5. Hence, changing any combination of signs in $a \pm b \pm c \pm d$does not change whether the result is odd or even.

---

5. A number has digit sum 37. Without performing long division, determine its remainder when divided by 9 and explain why the method works.
Answer:

1. A number and the sum of its digits leave the same remainder when divided by 9.
2. The given digit sum is 37, and $37 = 4 \times 9 + 1$.
3. Therefore, 37 leaves remainder 1 when divided by 9.
4. This works because 10,100,1000, ...are each 1 more than suitable multiples of 9.
5. Hence, the original number also leaves **remainder 1** when divided by 9.

---

6. A number has digit sum 54. Analyse its divisibility by 3 and 9 without knowing the number itself.
Answer:

1. A number is divisible by 3 if the sum of its digits is divisible by 3.
2. Since $54 \div 3 = 18$, the given number must be divisible by 3.

3. A number is divisible by 9 if the sum of its digits is divisible by 9.
4. Since $54 \div 9 = 6$, the given number must also be divisible by 9.
5. Therefore, any number having digit sum 54 is divisible by **both 3 and 9**.

---

7. Without performing division, investigate whether 91,784 is divisible by 11 and justify your conclusion.

Answer:

1. To test divisibility by 11, find the alternating sum of the digits.
2. For 91,784, the alternating calculation is $9 - 1 + 7 - 8 + 4$.
3. Simplifying gives $9 - 1 + 7 - 8 + 4 = 11$.
4. Since 11 is a multiple of 11, the divisibility condition is satisfied.
5. Therefore, **91,784 is divisible by 11**.

---

8. Prove that if two numbers are divisible by 12, then their sum and difference are also divisible by 12.

Answer:

1. Let the two numbers be $M = 12p$and $N = 12q$.
2. Their sum is $M + N = 12p + 12q = 12(p + q)$.
3. Therefore, $M + N$is divisible by 12.
4. Their difference is $M - N = 12p - 12q = 12(p - q)$.
5. Hence, both the sum and difference of two multiples of 12 are divisible by 12.

---

9. A student says, "Every number divisible by 9 is divisible by 3, and every number divisible by 3 is divisible by 9." Evaluate both statements.

Answer:

1. Since 3 is a factor of 9, every number divisible by 9 is also divisible by 3.
2. Thus, the first statement is **always true**.
3. However, a number divisible by 3 need not necessarily be divisible by 9.
4. For example, 15 is divisible by 3, but its digit sum $1 + 5 = 6$shows that it is not divisible by 9.
5. Therefore, the second statement is **not always true**, and 15 provides a counterexample.

---

10. Explain how examples, counterexamples, and algebra can be used together to investigate a number pattern.

Answer:

1. First, several numerical examples can be tested to observe whether a proposed number pattern appears to hold.
2. Different cases should then be examined to search for a counterexample to the proposed statement.
3. If a counterexample is found, the statement cannot be classified as **Always True**.
4. If the pattern continues to hold, algebra can represent the numbers generally and provide mathematical justification.
5. Thus, examples, counterexamples, and algebra together help develop reliable mathematical reasoning about number patterns.

5.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – Applied Type:

1. A school arranges students in groups of both 6 and 8 without leaving anyone out. What is the smallest possible number of students? Explain using divisibility.

Answer:
1. The required number must be divisible by both **6 and 8**.
2. Therefore, we find the LCM of 6 and 8.
3. $\mathrm{LCM}(6,8) = 24$.
4. Hence, **24 students** can be arranged exactly in groups of either 6 or 8.
5. This applies the chapter property that a number divisible by two given numbers is divisible by their LCM.

---

2. Two boxes contain numbers of chocolates that are even but not multiples of 4. Show that the total number of chocolates in the two boxes must be divisible by 4.
Answer:
1. An even number that is not a multiple of 4 can be represented as $4p + 2$.
2. Let the numbers of chocolates in the boxes be $4p + 2$and $4q + 2$.
3. Their total is $(4p + 2) + (4q + 2) = 4p + 4q + 4$.
4. This simplifies to $4(p + q + 1)$, which is a multiple of 4.
5. Therefore, the total number of chocolates in the two boxes will always be divisible by **4**.

---

3. A teacher writes a number on the board that leaves remainder 3 when divided by 5. How many counters should be added to make the total divisible by 5? Explain algebraically.
Answer:
1. A number leaving remainder 3 when divided by 5 can be written as $5k + 3$.
2. To reach the next multiple of 5, we need to add $5 - 3 = 2$.
3. Adding 2 gives $5k + 3 + 2 = 5k + 5$.
4. This can be written as $5(k + 1)$, which is divisible by 5.
5. Therefore, the teacher should add **2 counters** to obtain a multiple of 5.

---

4. A library has a five-digit book-code number whose digit sum is 36. Without performing division, determine whether the code is divisible by 3 and 9.
Answer:
1. According to the digit-sum rule, divisibility by 3 and 9 can be checked using the sum of the digits.
2. The given digit sum is **36**, which is divisible by 3.
3. Therefore, the book-code number is divisible by **3**.
4. Since $36 = 4 \times 9$, the digit sum is also divisible by 9.
5. Hence, the book-code number is divisible by **both 3 and 9**.

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5. A ticket number is 53,284. Find the remainder when it is divided by 9 without performing long division.
Answer:
1. A number and the sum of its digits leave the same remainder when divided by 9.
2. The digit sum of 53,284 is $5 + 3 + 2 + 8 + 4 = 22$.
3. Dividing 22 by 9 gives $22 = 2 \times 9 + 4$.
4. Therefore, the digit sum leaves remainder **4** when divided by 9.
5. Hence, the ticket number **53,284 also leaves remainder 4** when divided by 9.

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6. A student identification number is 47,619. Determine whether it is divisible by 9 without performing division.
Answer:

1. Add the digits of the identification number: $4 + 7 + 6 + 1 + 9 = 27$.
2. The number 27 is divisible by 9 because $27 = 3 \times 9$.
3. According to the divisibility rule for 9, a number is divisible by 9 when its digit sum is divisible by 9.
4. Therefore, 47,619 satisfies the divisibility condition for 9.
5. Hence, the student identification number **47,619 is divisible by 9**.

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7. A locker has the number 72,864. Check whether it is divisible by 3 and explain the method.
Answer:
1. To test divisibility by 3, add the digits of the locker number.
2. We get $7 + 2 + 8 + 6 + 4 = 27$.
3. Since 27 is divisible by 3, the digit-sum test is satisfied.
4. Therefore, 72,864 is divisible by 3 without carrying out complete division.
5. Hence, the locker number **72,864 is divisible by 3**.

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8. A competition participant receives the number 81,675. Use the divisibility rule for 11 to determine whether this number is divisible by 11.
Answer:
1. Apply the alternating-sum rule to the digits of 81,675.
2. The alternating sum is $8 - 1 + 6 - 7 + 5$.
3. Simplifying gives $8 - 1 + 6 - 7 + 5 = 11$.
4. Since 11 is itself a multiple of 11, the divisibility condition is satisfied.
5. Therefore, the participant number **81,675 is divisible by 11**.

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9. Two machines produce 336 and 192 items respectively. Without dividing each number, show that their combined production is divisible by 8.
Answer:
1. Both 336 and 192 are multiples of 8, since $336 = 8 \times 42$and $192 = 8 \times 24$.
2. Their combined production is $336 + 192$.
3. Algebraically, this is $8(42) + 8(24) = 8(42 + 24)$.
4. Thus, the total has 8 as a factor and is divisible by 8.
5. Therefore, the combined production of **528 items is divisible by 8**.

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10. A student says, "If a bus carries a number of passengers divisible by both 4 and 6, then the number must be divisible by 24." Test the statement with a suitable example.
Answer:
1. A number divisible by both 4 and 6 must be divisible by $\text{LCM}(4, 6) = 12$.
2. Consider a bus carrying **36 passengers**.
3. The number 36 is divisible by both 4 and 6.
4. However, 36 is not divisible by 24, so it provides a counterexample to the student's statement.
5. Therefore, being divisible by both 4 and 6 does **not necessarily** mean that the number is divisible by 24.

5.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams:

Olympiad-Type Multiple Choice Questions:
1. Two even numbers are not divisible by 4. Which statement about their sum is always true?

A) It is odd
B) It is divisible by 4
C) It leaves remainder 2 when divided by 4
D) It is divisible by 8
Answer: B) It is divisible by 4
2. If a and b are integers, which expression is always even?
A) $2a + 2b$
B) $2a + b$
C) $a + b$
D) $ab + 1$
Answer: A) $2a + 2b$
3. If a number is divisible by both 8 and 12, it must necessarily be divisible by:
A) 16
B) 20
C) 24
D) 48
Answer: C) 24
4. Which number is a counterexample to the statement "Every number divisible by both 4 and 6 is divisible by 24"?
A) 24
B) 48
C) 72
D) 12
Answer: D) 12
5. A number leaves remainder 3 when divided by 5. Which expression represents such a number?
A) $5k$
B) $5k + 1$
C) $5k + 3$
D) $5k + 4$
Answer: C) $5k + 3$
6. If $N = 5k + 3$, which number must be added to N to obtain the next multiple of 5?
A) 1
B) 2
C) 3
D) 5
Answer: B) 2
7. Three numbers each leave remainder 2 when divided by 6. What remainder does their sum leave when divided by 6?
A) 0
B) 1
C) 2
D) 4
Answer: A) 0
8. If M and N are both divisible by 7, which expression must be divisible by 7?
A) $M + N$ only
B) $M - N$ only
C) Both $M + N$ and $M - N$
D) Neither
Answer: C) Both $M + N$ and $M - N$

9. If 661leaves remainder 3 and 4779leaves remainder 5 when divided by 7, what remainder does their sum leave?
A) 0
B) 1
C) 2
D) 3
Answer: B) 1
10. Using the information in Question 9, what remainder does $4779 - 661$leave when divided by 7?
A) 1
B) 2
C) 3
D) 5
Answer: B) 2
11. A number has digit sum 37. What remainder will it leave when divided by 9?
A) 0
B) 1
C) 2
D) 4
Answer: B) 1
12. Which of the following numbers is divisible by 9?
A) 724
B) 738
C) 742
D) 754
Answer: B) 738
13. The digit sum of a number is 45. Which conclusion is necessarily correct?
A) It is divisible only by 3
B) It is divisible only by 9
C) It is divisible by both 3 and 9
D) It is not divisible by 3
Answer: C) It is divisible by both 3 and 9
14. Which number is divisible by 3 but not by 9?
A) 27
B) 45
C) 54
D) 33
Answer: D) 33
15. The sum of the digits of a number is 28. Which statement is correct?
A) The number is divisible by 9
B) It leaves remainder 1 when divided by 9
C) It leaves remainder 2 when divided by 9
D) It is necessarily divisible by 3
Answer: B) It leaves remainder 1 when divided by 9
16. Why does the digit-sum test for divisibility by 9 work?
A) Every power of 10 is divisible by 9
B) Every power of 10 leaves remainder 1 when divided by 9
C) Every digit is divisible by 9
D) 10 is a factor of 9
Answer: B) Every power of 10 leaves remainder 1 when divided by 9

17. Which of the following is divisible by 11?
A) 451
B) 462
C) 473
D) 485
Answer: B) 462
18. For the number 462, the alternating digit sum used for divisibility by 11 is:
A) $4 + 6 + 2 = 12$
B) $4 - 6 + 2 = 0$
C) $4 + 6 - 2 = 8$
D) $4 - 6 - 2 = -4$
Answer: B) $4 - 6 + 2 = 0$
19. If the alternating sum of the digits of a number is 22, then the number is:
A) Always divisible by 2
B) Divisible by 11
C) Not divisible by 11
D) Always divisible by 9
Answer: B) Divisible by 11
20. Which expression correctly represents the place-value expansion of the four-digit number $dcba$**?**
A) $d + c + b + a$
B) $100d + 10c + b + a$
C) $1000d + 100c + 10b + a$
D) $1000a + 100b + 10c + d$
Answer: C) $1000d + 100c + 10b + a$
21. If A**is divisible by 12, which of the following must be true?**
A) $A$is divisible by 5
B) $A$is divisible by 8
C) $A$is divisible by 6
D) $A$is divisible by 7
Answer: C) A**is divisible by 6**
22. Which statement is Never True?
A) Sum of two odd numbers is even
B) Sum of two even numbers is even
C) Sum of an odd and an even number is a multiple of 6
D) Sum of two multiples of 8 is a multiple of 8
Answer: C) Sum of an odd and an even number is a multiple of 6
23. If $8a$**and** $8b$**are two multiples of 8, their sum can be expressed as:**
A) $8(a + b)$
B) $16ab$
C) $8(a - b)$
D) $a + 8b$
Answer: A) $8(a + b)$
24. Two even numbers leave remainder 2 each when divided by 4. What remainder does their sum leave when divided by 4?
A) 0
B) 1
C) 2

D) 3
Answer: A) 0
25. If one even number is a multiple of 4 and another even number is not a multiple of 4, their sum leaves which remainder on division by 4?
A) 0
B) 1
C) 2
D) 3
Answer: C) 2
26. Which expression is equivalent to $5k+3$?
A) $5(k+1)-2$
B) $5(k+1)+3$
C) $5(k-1)-3$
D) $5(k-1)+2$
Answer: A) $5(k+1)-2$
27. The smallest positive number divisible by both 9 and 12 is:
A) 18
B) 24
C) 36
D) 108
Answer: C) 36
28. Which statement is always true if $A$is divisible by k?
A) Every factor of $A$is divisible by k
B) Every multiple of $A$is divisible by k
C) $A+1$is divisible by k
D) $A-1$is divisible by k
Answer: B) Every multiple of $A$is divisible by k
29. A number is divisible by both 4 and 9. Which number must divide it?
A) 13
B) 18
C) 27
D) 36
Answer: D) 36
30. Which is the best method to disprove the statement "All numbers having property Palso have property Q"?
A) Check only one supporting example
B) Find one valid counterexample
C) Calculate the average of the numbers
D) Check only even numbers
Answer: B) Find one valid counterexample

Answer Key:
1–10: 1-B, 2-A, 3-C, 4-D, 5-C, 6-B, 7-A, 8-C, 9-B, 10-B
11–20: 11-B, 12-B, 13-C, 14-D, 15-B, 16-B, 17-B, 18-B, 19-B, 20-C
21–30: 21-C, 22-C, 23-A, 24-A, 25-C, 26-A, 27-C, 28-B, 29-D, 30-B

5.15 Best Practices Opportunity List from the Chapter: (10 Ideas)

"Best Practices" Opportunities for Teaching:

Based on the chapter's focus on **number patterns, parity, factors and multiples, remainders, divisibility tests, algebraic reasoning, examples and counterexamples**, the following practices can make teaching more effective.

(1) **Begin with Pattern Discovery:** Give students sets of consecutive numbers and let them experiment with $+$and $-$signs before introducing the general rule. Encourage them to observe whether the results are odd or even and formulate their own conjectures.

(2) **Use "Always–Sometimes–Never" Discussions:** Present statements about factors, multiples and divisibility and ask students to classify them as **Always True, Sometimes True, or Never True**. Require students to support their classification with examples, counterexamples or reasoning.

(3) **Move from Numerical Examples to Algebra:** First establish patterns using actual numbers and then represent them algebraically—for example, use $4p$and $4p+2$to distinguish the two kinds of even numbers. This helps students understand why a pattern works rather than merely memorising it.

(4) **Teach Divisibility Rules through Place Value:** Instead of asking students simply to memorise tests for **3, 9 and 11**, demonstrate why the tests work using place-value expansion. This reflects the chapter's emphasis on understanding the mathematical reasoning behind divisibility shortcuts.

(5) **Encourage Counterexample Hunting:** Give students statements such as "A number divisible by both 4 and 6 must be divisible by 24" and challenge them to find a counterexample. This develops logical reasoning and helps students distinguish an observed pattern from a valid generalisation.

(6) **Use Remainder-Based Number Games:** Ask students to generate numbers of forms such as $5k+3$, identify their common remainder, and predict what happens when particular numbers are added or subtracted. This makes the chapter's remainder concepts more exploratory.

(7) **Promote Multiple Solution Strategies:** For a divisibility problem, invite students to solve it using **direct calculation, divisibility rules, factorisation, LCM, or algebraic representation** wherever applicable. Comparing methods helps students recognise connections among different number properties.

(8) **Use Think–Pair–Share for Conjectures:** Give a number pattern individually, allow students to develop a conjecture, then discuss it with a partner before presenting a justification to the class. The final explanation should include the mathematical reason, not merely the answer.

(9) **Maintain a "Number Play Reasoning Journal":** Students can record each pattern under four headings—**Observation, Conjecture, Example/Counterexample, and Reason**. This reinforces the chapter's emphasis on moving from experimentation to mathematical justification.

(10) **Include Short Exit Challenges:** End each lesson with one reasoning problem—for example, finding a remainder without division, checking divisibility by 9 or 11, or deciding whether a statement is always/sometimes/never true. These quick assessments help the teacher identify whether students have understood the reasoning behind the rules rather than only memorised them.

5.16 Innovations Opportunity List from the Chapter: (10 Ideas)

"Innovations" Opportunities in Teaching–Learning for 8th Standard:
The chapter provides excellent opportunities for innovative learning through **number patterns, parity, factors and multiples, remainders, divisibility tests, algebraic reasoning, conjectures and counterexamples**.

(1) **Number Detective Challenge:** Create a classroom "Number Detective" game in which students receive clues such as "I am divisible by 3 and 9," "I leave remainder 3 when divided by 5," or "My alternating digit sum is a multiple of 11." Students identify possible numbers and explain the reasoning behind each answer.

(2) **Divisibility Escape Room:** Organise teams and provide a sequence of puzzles based on divisibility by **3, 9 and 11**, factors, multiples and LCM. Each correct solution provides a code or clue for unlocking the next challenge, turning practice into collaborative problem-solving.

(3) **Digital Divisibility Lab:** Using a spreadsheet or simple coding activity, students can enter different numbers and automatically calculate **digit sums, alternating digit sums, remainders and divisibility results**. They then compare the computer-generated results with the rules developed in the chapter.

(4) **Conjecture–Test–Prove Laboratory:** Give students patterns such as "the sum of two even non-multiples of 4 is divisible by 4." Students first **conjecture**, then test several examples, search for counterexamples, and finally establish the result algebraically using $4p + 2$and $4q + 2$.

(5) **Remainder Wheel Activity:** Design a circular "remainder wheel" for divisors such as 4, 5, 6 or 9. Students place numbers into remainder categories and predict how addition or subtraction changes the remainder, connecting visual patterns with expressions such as $5k + 3$.

(6) **Always–Sometimes–Never Digital Poll:** Display number statements and let students vote instantly for **Always, Sometimes, or Never**. After voting, groups must defend their choices with an example, counterexample or algebraic argument, making assessment interactive and reasoning-focused.

(7) **Student-Created Olympiad Puzzle Studio:** After learning the concepts, groups design their own challenging MCQs involving **parity, divisibility, LCM, remainders and number patterns**. Groups exchange questions, solve them, identify ambiguities, and improve the questions before creating a class Olympiad question bank.

(8) **Human Number Line and Remainder Walk:** Mark numbers on the classroom floor and ask students to physically move according to instructions such as "jump through multiples of 4" or "stand on numbers leaving remainder 2 when divided by 5." This converts abstract remainder and multiple concepts into a kinaesthetic learning experience.

(9) **Divisibility Rule Discovery Workshop:** Instead of directly giving the rules for 3, 9 and 11, provide carefully selected numbers and ask students to discover relationships among their digits. Students then use **place-value reasoning** to explain why their discovered rules work.

(10) **Number Play Mini-Hackathon:** Conduct a team challenge with stations such as **Parity Puzzle, Remainder Mystery, LCM Challenge, Divisibility Detective, Counterexample Hunt, and Algebraic Proof**. Award points not only for correct answers but also for the quality of mathematical justification, encouraging the chapter's central progression from **observation → conjecture → testing → reasoning → generalisation**.

5.17: Practicing Questions/Question Bank :

A. Fill in the Blanks:

1. An even number can be expressed algebraically as ________.
   Answer: $2n$

2. An odd number can be expressed as ________.
   Answer: $2n + 1$

3. An even number that is a multiple of 4 has the form ________.
   Answer: $4n$

4. An even number that is not a multiple of 4 has the form ________.
   Answer: $4n + 2$

5. The sum of two odd numbers is always ________.
   Answer: even

6. The sum of an odd number and an even number is always ________.
   Answer: odd

7. If $a$divides both $M$and N, then $a$also divides M________ N.
   Answer: $+$and $-$

8. A number that leaves remainder 3 when divided by 5 can be written as ________.
   Answer: $5k + 3$

9. If a number is divisible by both $a$and b, it is divisible by ________ of $a$and b.
   Answer: LCM

10. If the digit sum of a number is divisible by 9, the number itself is divisible by ________.
    Answer: 9

11. A number and its digit sum leave the same ________ when divided by 9.
    Answer: remainder

12. The divisibility test for 11 uses the ________ sum of the digits.
    Answer: alternating

13. The four-digit number $dcba$can be expanded as ________.
    Answer: $1000d + 100c + 10b + a$

14. A mathematical example that disproves a general statement is called a ________.
    Answer: counterexample

15. If $A$is divisible by k, every ________ of $A$is also divisible by k.
    Answer: multiple

B. 1-Mark Descriptive Questions:

1. What is meant by parity of a number?
Answer: Parity tells whether an integer is **odd or even**.

2. Write the algebraic form of an even number that is not divisible by 4.
Answer: $4n + 2$.

3. What is the parity of the sum of two odd numbers?
Answer: The sum of two odd numbers is **even**.

4. Write the general form of a number leaving remainder 3 when divided by 5.
Answer: $5k + 3$.

5. State the divisibility rule for 3.
Answer: A number is divisible by 3 if its digit sum is divisible by 3.

6. State the divisibility rule for 9.
Answer: A number is divisible by 9 if its digit sum is divisible by 9.

7. What is used to test divisibility by 11?
Answer: The **alternating sum of the digits** is used.

8. What is a counterexample?
Answer: A counterexample is an example that shows that a proposed general statement is not always true.

9. If a number is divisible by 8 and 12, by which LCM must it be divisible?
Answer: It must be divisible by $\text{LCM}(8, 12) = 24$.

10. What remainder does a number with digit sum 28 leave when divided by 9?
Answer: It leaves remainder **1**.

C. 2-Mark Descriptive Questions:

1. Show that the sum of two odd numbers is even.
Answer:

1. Let the numbers be $2p + 1$and $2q + 1$.
2. Their sum is $2(p + q + 1)$, which is divisible by 2 and hence even.

2. Explain why $4p + 2$represents an even number that is not divisible by 4.
Answer:

1. $4p + 2 = 2(2p + 1)$, so it is even.
2. It leaves remainder 2 when divided by 4, so it is not a multiple of 4.

3. If $a$divides both $M$and N, show that $a$divides $M + N$.
Answer:

1. Write $M = ap$and $N = aq$.
2. Then $M + N = a(p + q)$, so $a$divides $M + N$.

4. A number leaves remainder 3 when divided by 5. What should be added to make it divisible by 5? Explain.
Answer:

1. The number has the form $5k + 3$.
2. Adding 2 gives $5k + 5 = 5(k + 1)$; therefore, **2 must be added**.

5. Why is every number divisible by 9 also divisible by 3?
Answer:

1. Since $9 = 3 \times 3$, 3 is a factor of 9.
2. Therefore, every multiple of 9 is also a multiple of 3.

6. Check whether 738 is divisible by 9.
Answer:

1. Its digit sum is $7 + 3 + 8 = 18$.
2. Since 18 is divisible by 9, **738 is divisible by 9**.

7. Give a counterexample to "Every number divisible by 4 and 6 is divisible by 24."
Answer:

1. Consider **12**, which is divisible by both 4 and 6.
2. But 12 is not divisible by 24; hence it is a counterexample.

8. Explain why the sum of an odd and an even number cannot be a multiple of 6.
Answer:

1. The sum of an odd and an even number is odd.
2. Every multiple of 6 is even; therefore, such a sum cannot be a multiple of 6.

D. 3-Mark Descriptive Questions:

1. Prove that the sum of two even numbers that are not multiples of 4 is a multiple of 4.
Answer:

1. Represent the numbers as $4p + 2$and $4q + 2$.
2. Their sum is $4p + 2 + 4q + 2 = 4p + 4q + 4$.
3. This equals $4(p + q + 1)$, so the sum is divisible by 4.

2. Explain why changing a sign in $a \pm b \pm c \pm d$does not change the parity of the result.
Answer:

1. Changing $+b$to $-b$, for example, changes the result by $2b$.
2. Since $2b$is always even, the original and new results differ by an even number.
3. Hence, both results have the same parity.

3. Prove that if $a$divides $M$and N, then $a$divides their difference.
Answer:

1. Let $M = ap$and $N = aq$.
2. Then $M - N = ap - aq = a(p - q)$.
3. Therefore, $M - N$is divisible by a.

4. Explain why a number and its digit sum leave the same remainder when divided by 9.
Answer:

1. Each power $10, 100, 1000, \ldots$leaves remainder 1 when divided by 9.
2. Therefore, each place-value contribution has the same remainder effect as its digit.
3. Hence, the number and its digit sum have the same remainder modulo 9.

5. A number has digit sum 46. Find its remainder when divided by 9 without dividing the original number.
Answer:

1. A number and its digit sum have the same remainder upon division by 9.
2. $46 = 9 \times 5 + 1$.
3. Therefore, the original number leaves **remainder 1** when divided by 9.

6. Explain how the divisibility test for 11 works.
Answer:

1. Successive powers of 10 behave alternately like $+1$and $-1$when considered with respect to division by 11.
2. Therefore, the digits are combined using an alternating sum.
3. If this alternating sum is 0 or a multiple of 11, the original number is divisible by 11.

E. 5-Mark Descriptive Questions:

1. Explain the important divisibility properties involving factors, multiples, sums, differences and LCM.
Answer:

1. If $a$divides both $M$and N, then $a$divides $M + N$.
2. If $a$divides both $M$and N, then $a$divides $M - N$.
3. If $A$is divisible by k, every multiple of $A$is divisible by k.
4. If $A$is divisible by k, it is divisible by every factor of k.
5. If $A$is divisible by both $k$and m, it is divisible by $\text{LCM}(k, m)$.

2. Explain the divisibility tests for 3 and 9 and the reasoning behind them.
Answer:

1. A number is divisible by 3 when its digit sum is divisible by 3.
2. A number is divisible by 9 when its digit sum is divisible by 9.
3. The powers $10, 100, 1000, \ldots$leave remainder 1 when divided by 9.
4. Therefore, a number and its digit sum have the same remainder when divided by 9.
5. A similar place-value argument explains the digit-sum test for divisibility by 3.

3. Explain the two types of even numbers with respect to divisibility by 4 and analyse their sums.
Answer:

1. Even multiples of 4 can be represented as $4p$.
2. Even numbers that are not multiples of 4 can be represented as $4p + 2$.
3. The sum of two numbers of the form $4p$is divisible by 4.

4. The sum of two numbers of the form $4p + 2$is also divisible by 4.
5. The sum of one number of each type has the form $4k + 2$, so it is even but not divisible by 4.

4. Describe how algebra can be used to study numbers that leave a fixed remainder on division.
Answer:

1. A number leaving remainder $r$when divided by $d$can be represented as $dk + r$.
2. For example, numbers leaving remainder 3 when divided by 5 have the form $5k + 3$.
3. Adding 2 to such a number gives $5k + 5 = 5(k + 1)$.
4. Thus, adding 2 always produces a multiple of 5.
5. Algebraic representation allows the result to be established for all such numbers rather than checking individual examples.

5. Explain how examples, counterexamples and algebraic reasoning help establish mathematical statements.
Answer:

1. Examples help students observe patterns and develop mathematical conjectures.
2. Testing several cases provides evidence about whether the conjecture may be true.
3. A single counterexample is sufficient to show that an “always true” claim is false.
4. Algebra represents numbers generally and can establish why a valid pattern works in all relevant cases.
5. Thus, examples, counterexamples and algebra together develop stronger mathematical reasoning and justification.

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8th Standard Maths 1 Book Chapter 6

Chapter 6: WE DISTRIBUTE, YET THINGS MULTIPLY

6.1 Summary of the Chapter:

Chapter 6: We Distribute, Yet Things Multiply — Summary:

(1) **Distributive Property:** The chapter introduces the distributive property of multiplication over addition, expressed as $\mathbf{a(b + c) = ab + ac}$. It shows how algebra can describe multiplication patterns and justify mathematical relationships.

(2) **Changes in Products:** It investigates how a product changes when one or both factors are increased or decreased. For example, when both factors are increased by 1, $\mathbf{(a + 1)(b + 1) = ab + a + b + 1}$.

(3) **General Expansion of Two Binomials:** The distributive property is generalised to obtain $\mathbf{(a + m)(b + n) = ab + an + bm + mn}$. Thus, changes of any size in the two factors can be analysed algebraically.

(4) **Handling Addition and Subtraction:** The same principle works when terms are subtracted. Important expansions include $\mathbf{(a + u)(b - v) = ab + ub - av - uv}$ and $\mathbf{(a - u)(b - v) = ab - ub - av + uv}$.

(5) **Expansion and Like Terms:** While expanding algebraic expressions, every term in one bracket is multiplied by every relevant term in the other. The resulting expression is simplified by combining **like terms**, i.e., terms having the same letter-numbers.

(6) **Fast Multiplication:** The distributive property provides convenient methods for multiplying numbers by **11, 101, 1001, etc.** For example, a number multiplied by 101 can be viewed as multiplication by $\mathbf{(100 + 1)}$.

(7) **Square of a Sum:** A major special case is the identity $\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$. The chapter explains this both algebraically and geometrically using the areas of squares and rectangles.

(8) **Square of a Difference:** Similarly, distributivity gives the identity $\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$. This identity can be used for expanding expressions and quickly calculating squares of suitable numbers.

(9) **Difference of Two Squares:** Another important identity developed through patterns is $\mathbf{(a + b)(a - b) = a^2 - b^2}$. This can be applied to simplify calculations such as products of numbers equally spaced around a convenient number.

(10) **Discovering and Proving Patterns:** Algebraic identities are used to explain numerical patterns. One example establishes $\mathbf{2(a^2 + b^2) = (a + b)^2 + (a - b)^2}$, showing how apparently different expressions can represent the same quantity.

(11) **Identifying and Correcting Mistakes:** The chapter develops algebraic reasoning by asking students to examine incorrect expansions, identify where the distributive property or simplification has been misapplied, and write the **correct expression**.

(12) **Multiple Ways of Solving Problems:** The chapter demonstrates that a mathematical pattern can often be represented in several different ways. Although the expressions may initially look different, correct simplification can show that they are **equivalent**, encouraging creativity and flexible mathematical thinking.

Key Identities to Remember:

(a + m)(b + n) = ab + an + bm + mn
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$(a + b)(a - b) = a^2 - b^2$
These identities form the central mathematical ideas of **Chapter 6 – "We Distribute, Yet Things Multiply."**

6.2 Important Formulas:

Important Formulas for Solving Quantitative Problems:
The following formulas and algebraic identities are important for solving the numerical and algebraic problems in Chapter 6.

1. **Distributive Property of Multiplication over Addition**

$$\boxed{a(b+c) = ab + ac}$$

Similarly,

$$\boxed{(a+b)c = ac + bc}$$

This is the basic property on which most expansions in the chapter are based.

2. **Product When One Factor is Increased by 1**

$$\boxed{a(b+1) = ab + a}$$

Hence, the **increase in the product** = a.

3. **Product When Both Factors are Increased by 1**

$$\boxed{(a+1)(b+1) = ab + a + b + 1}$$

Therefore,

$$\boxed{\text{Increase} = a + b + 1}$$

4. **One Factor Increased by 1 and the Other Decreased by 1**

$$\boxed{(a+1)(b-1) = ab + b - a - 1}$$

Hence, the change from the original product $ab$is

$$\boxed{b - a - 1}$$

5. **General Product When Both Factors are Increased**

$$\boxed{(a+m)(b+n) = ab + an + bm + mn}$$

Therefore,

$$\boxed{\text{Increase} = an + bm + mn}$$

This is an important general identity in the chapter.

6. **One Factor Increased and the Other Decreased**

$$\boxed{(a+u)(b-v) = ab + ub - av - uv}$$

7. **One Factor Decreased and the Other Increased**

$$\boxed{(a-u)(b+v) = ab - ub + av - uv}$$

8. **Both Factors Decreased**

$$\boxed{(a-u)(b-v) = ab - ub - av + uv}$$

9. **Square of the Sum of Two Terms – Identity 1A**

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

This is useful for expanding algebraic expressions and calculating squares of numbers close to convenient values.

10. **Square of the Difference of Two Terms – Identity 1B**

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

11. **Difference of Two Squares – Identity 1C**

$$\boxed{(a+b)(a-b)=a^2-b^2}$$

Equivalently,

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

12. **Useful Rearrangement for Finding Squares**

From the difference-of-squares identity:

$$\boxed{a^2=(a+b)(a-b)+b^2}$$

This is presented as a quick method for calculating squares. For example,

$$31^2=(31+1)(31-1)+1^2$$

13. **Sum of Square Identities**

Combining the square-of-sum and square-of-difference identities gives:

$$\boxed{(a+b)^2+(a-b)^2=2(a^2+b^2)}$$

or

$$\boxed{2(a^2+b^2)=(a+b)^2+(a-b)^2}$$

14. **Multiplication by 11**

The chapter uses:

$$\boxed{N\times 11=N(10+1)=10N+N}$$

This distributive form helps develop quick multiplication techniques.

15. **Multiplication by 101**

$$\boxed{N\times 101=N(100+1)=100N+N}$$

16. **Multiplication by 1001**

Extending the same pattern:

$$\boxed{N\times 1001=N(1000+1)=1000N+N}$$

This follows the chapter's extension of the method to 1001,10001, ...

Most Important Formula Box

$$\boxed{a(b+c)=ab+ac}$$

$$\boxed{(a+m)(b+n)=ab+an+bm+mn}$$

$$\boxed{(a+b)^2=a^2+2ab+b^2}$$

$$\boxed{(a-b)^2=a^2-2ab+b^2}$$

$$\boxed{(a+b)(a-b)=a^2-b^2}$$

$$\boxed{(a+b)^2+(a-b)^2=2(a^2+b^2)}$$

These are the **core formulas students should remember and practise** for solving quantitative problems from **Chapter 6 – We Distribute, Yet Things Multiply**.

6.3 Fill-Up the Blank LOTS type questions with Answers: (30 Questions)

LOTS – Fill in the Blanks with Answers:

1. The property represented by $a(b+c)=ab+ac$is called the __________ **property** of multiplication over addition.
   Answer: Distributive
2. According to the distributive property, $a(b+c)=ab+$__________.
   Answer: ac
3. The expression $(a+b)c$can be expanded as __________.
   Answer: $ac+bc$
4. If $b$is increased by 1, $a(b+1)=ab+$__________.
   Answer: a

5. When both $a$and $b$are increased by 1, the new product is __________.
   Answer: $(a + 1)(b + 1)$
6. The expansion of $(a + 1)(b + 1)$is __________.
   Answer: $ab + a + b + 1$
7. If both factors $a$and $b$are increased by 1, the product increases by __________.
   Answer: $a + b + 1$
8. If $a$is increased by 1 and $b$is decreased by 1, the new product is __________.
   Answer: $(a + 1)(b - 1)$
9. The expansion of $(a + 1)(b - 1)$is __________.
   Answer: $ab + b - a - 1$
10. Mathematical statements expressing the equality of two algebraic expressions are called __________.
    Answer: Identities
11. The expansion of $(a + m)(b + n)$is __________.
    Answer: $ab + mb + an + mn$
12. In $(a + m)(b + n)$, the increase over the original product $ab$is __________.
    Answer: $an + bm + mn$
13. The expansion of $(a + u)(b - v)$is __________.
    Answer: $ab + ub - av - uv$
14. The expansion of $(a - u)(b + v)$is __________.
    Answer: $ab - ub + av - uv$
15. The expansion of $(a - u)(b - v)$is __________.
    Answer: $ab - ub - av + uv$
16. Terms having the same letter-numbers are called __________ **terms**.
    Answer: Like
17. Since $ba = ab$, the expression $ba + ab$simplifies to __________.
    Answer: $2ab$
18. The expansion of $(a + b)^2$is __________.
    Answer: $a^2 + 2ab + b^2$
19. The identity $(a + b)^2 = a^2 + 2ab + b^2$is called the square of the __________ of two terms.
    Answer: Sum
20. The expansion of $(a - b)^2$is __________.
    Answer: $a^2 - 2ab + b^2$
21. In the identity $(a - b)^2 = a^2 - 2ab + b^2$, the middle term is __________.
    Answer: $-2ab$
22. The identity $(a + b)(a - b) =$__________.
    Answer: $a^2 - b^2$
23. The expression $a^2 - b^2$is known as the __________ **of two squares**.
    Answer: Difference
24. The identity $2(a^2 + b^2) = (a + b)^2 +$__________.
    Answer: $(a - b)^2$
25. To multiply a number $N$by 11 using distributivity, we write $N \times 11 = N \times (10 +$__________$)$.
    Answer: 1
26. $N \times 11$can therefore be written as $10N +$__________.
    Answer: N
27. To multiply a number by 101, we can write $101 = 100 +$__________.
    Answer: 1

28. Using distributivity, $N \times 101 =$__________.
    Answer: $100N + N$
29. The modified difference-of-squares identity used for quickly finding squares is $a^2 = (a+b)(a-b) +$__________.
    Answer: b^2
30. In the chapter's circle pattern, the number of circles at Step $k$is given by __________.
    Answer: $k^2 + 2k$

Answer Key:
1. Distributive; **2.** ac; **3.** $ac + bc$; **4.** a; **5.** $(a+1)(b+1)$; **6.** $ab + a + b + 1$; **7.** $a + b + 1$; **8.** $(a+1)(b-1)$; **9.** $ab + b - a - 1$; **10.** Identities; **11.** $ab + mb + an + mn$; **12.** $an + bm + mn$; **13.** $ab + ub - av - uv$; **14.** $ab - ub + av - uv$; **15.** $ab - ub - av + uv$; **16.** Like; **17.** $2ab$; **18.** $a^2 + 2ab + b^2$; **19.** Sum; **20.** $a^2 - 2ab + b^2$; **21.** $-2ab$; **22.** $a^2 - b^2$; **23.** Difference; **24.** $(a-b)^2$; **25.** 1; **26.** N; **27.** 1; **28.** $100N + N$; **29.** b^2; **30.** $k^2 + 2k$.

6.4 Fill-Up the Blank HOTS type questions with Answers: (30 Questions)

HOTS – Fill in the Blanks with Answers:
The following questions require students to **apply, analyse, connect patterns, and reason using the distributive property and algebraic identities** from the chapter.

1. If $(a+1)(b+1) - ab = 21$and $a = 8$, then $b =$________.
   Answer: 12
   Reason: $a + b + 1 = 21 \Rightarrow 8 + b + 1 = 21 \Rightarrow b = 12$.
2. If both factors of the product $12 \times 15$are increased by 1, the increase in the product is ________.
   Answer: 28
   Reason: Increase $= a + b + 1 = 12 + 15 + 1 = 28$.
3. If $a$is increased by 1 and $b$is decreased by 1, the change in the product $ab$is ________.
   Answer: $b - a - 1$
4. If $(a+1)(b-1) = ab$, then $b - a =$________.
   Answer: 1
   Reason: $ab + b - a - 1 = ab \Rightarrow b - a = 1$.
5. If the numbers 10 and 15 are increased by 2 and 3 respectively, the increase in their product is ________.
   Answer: 66
   Reason: $an + bm + mn = (10)(3) + (15)(2) + (2)(3) = 66$.
6. If $(x+4)(y+3)$is expanded, the term independent of $x$and $y$is ________.
   Answer: 12
7. The coefficient of $x$in the expansion of $(x+3)(x+5)$is ________.
   Answer: 8
   Reason: $(x+3)(x+5) = x^2 + 8x + 15$.
8. If $(x+p)(x+q) = x^2 + 9x + 20$, then $p + q =$________.
   Answer: 9
9. If $(x+4)^2 = x^2 + kx + 16$, then $k =$________.
   Answer: 8
   Reason: $2(x)(4) = 8x$.
10. The missing term in (3x+2)^2=9x^2+\text{________}+4 is ________.
    Answer: $12x$

11. If $(a+b)^2 - a^2 - b^2 = 20$, then $ab =$________.
    Answer: 10
    Reason: The difference is $2ab$, so $2ab = 20$.
12. If $a + b = 10$and $ab = 21$, then $(a+b)^2 = a^2 + b^2 +$________.
    Answer: 42
13. If $(x-5)^2 = x^2 - kx + 25$, then $k =$________.
    Answer: 10
    Reason: $(x-5)^2 = x^2 - 10x + 25$.
14. If $(a-b)^2 = 49$, then the possible values of $a - b$are ________.
    Answer: 7or -7
15. If $a + b = 15$and $a - b = 5$, then $a^2 - b^2 =$________.
    Answer: 75
    Reason: $a^2 - b^2 = (a+b)(a-b) = 15 \times 5 = 75$.
16. Without direct multiplication, 102×98can be written as $100^2 - 2^2 =$________.
    Answer: 9996
17. Using the difference-of-squares identity, $55 \times 45 = 50^2 - 5^2 =$________.
    Answer: 2475
18. If $a^2 - b^2 = 48$and $a - b = 4$, then $a + b =$________.
    Answer: 12
    Reason: $(a-b)(a+b) = 48 \Rightarrow 4(a+b) = 48$.
19. If $a^2 = 961$, using $a^2 = (a+b)(a-b) + b^2$with $a = 31$and $b = 1$, the product $(a+b)(a-b)$is ________.
    Answer: 960
20. If $a = 6$and $b = 2$, then $2(a^2 + b^2)$and $(a+b)^2 + (a-b)^2$are both equal to ________.
    Answer: 80
21. If $(a+b)^2 = 81$and $(a-b)^2 = 25$, then $2(a^2 + b^2) =$________.
    Answer: 106
22. If $a^2 + b^2 = 50$, then $(a+b)^2 + (a-b)^2 =$________.
    Answer: 100
23. If $N \times 101 = 4545$, then $N =$________.
    Answer: 45
    Reason: $45 \times 101 = 45 \times 100 + 45 = 4545$.
24. Using distributivity, $999 \times 37 = (1000 - 1) \times 37 = 37000 -$________.
    Answer: 37
25. If $x^2 + 10x + 25$is written as a perfect square, it becomes ________.
    Answer: $(x+5)^2$
26. The expression $9x^2 - 24xy + 16y^2$can be written as the perfect square ________.
    Answer: $(3x - 4y)^2$
27. The expression $25a^2 - 9b^2$can be factorised as ________.
    Answer: $(5a + 3b)(5a - 3b)$
28. If $x^2 - y^2 = 0$, then $(x+y)(x-y) =$________.
    Answer: 0
29. Four different expressions for the number of circles at Step $k$in the chapter simplify to ________.
    Answer: $k^2 + 2k$
30. According to the circle pattern, the number of circles at Step 15 is $15^2 + 2(15) =$________.
    Answer: 255

Quick Answer Key:
1. 12; **2.** 28; **3.** $b - a - 1$; **4.** 1; **5.** 66; **6.** 12; **7.** 8; **8.** 9; **9.** 8; **10.** $12x$; **11.** 10; **12.** 42; **13.** 10; **14.** 7or -7; **15.** 75; **16.** 9996; **17.** 2475; **18.** 12; **19.** 960; **20.** 80; **21.** 106; **22.** 100; **23.** 45; **24.** 37; **25.** $(x + 5)^2$; **26.** $(3x - 4y)^2$; **27.** $(5a + 3b)(5a - 3b)$; **28.** 0; **29.** $k^2 + 2k$; **30.** 255.

6.5 One-mark questions with Answers of LOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – LOTS Type:

1. **What is the distributive property of multiplication over addition?**
   Answer: The distributive property states that $a(b + c) = ab + ac$.
2. **How can $(a + b)c$ be expanded using the distributive property?**
   Answer: $(a + b)c$ can be expanded as $ac + bc$.
3. **What happens to the product ab when b is increased by 1?**
   Answer: The product increases by a.
4. **By how much does ab increase when both a and b are increased by 1?**
   Answer: The product increases by $a + b + 1$.
5. **Write the expansion of $(a + 1)(b + 1)$.**
   Answer: $(a + 1)(b + 1) = ab + a + b + 1$.
6. **Write the expansion of $(a + 1)(b - 1)$.**
   Answer: $(a + 1)(b - 1) = ab + b - a - 1$.
7. **What is an algebraic identity?**
   Answer: An algebraic identity is a mathematical statement expressing the equality of two algebraic expressions for all permissible values of their letter-numbers.
8. **Write the expansion of $(a + m)(b + n)$.**
   Answer: $(a + m)(b + n) = ab + mb + an + mn$.
9. **By how much does the product ab increase when a and b are increased by m and n, respectively?**
   Answer: The product increases by $an + bm + mn$.
10. **Write the expansion of $(a + u)(b - v)$.**
    Answer: $(a + u)(b - v) = ab + ub - av - uv$.
11. **Write the expansion of $(a - u)(b + v)$.**
    Answer: $(a - u)(b + v) = ab - ub + av - uv$.
12. **Write the expansion of $(a - u)(b - v)$.**
    Answer: $(a - u)(b - v) = ab - ub - av + uv$.
13. **What are like terms in an algebraic expression?**
    Answer: Like terms are terms having exactly the same letter-numbers.
14. **Simplify $ab + ab$.**
    Answer: $ab + ab = 2ab$.
15. **Write the identity for the square of the sum of two terms.**
    Answer: The identity is $(a + b)^2 = a^2 + 2ab + b^2$.
16. **Write the identity for the square of the difference of two terms.**
    Answer: The identity is $(a - b)^2 = a^2 - 2ab + b^2$.
17. **Write the identity for the product of the sum and difference of two terms.**
    Answer: The identity is $(a + b)(a - b) = a^2 - b^2$.
18. **What is the middle term in the expansion of $(a + b)^2$?**
    Answer: The middle term is $2ab$.
19. **What is the middle term in the expansion of $(a - b)^2$?**
    Answer: The middle term is $-2ab$.
20. **How can $a^2 - b^2$ be factorised?**
    Answer: $a^2 - b^2$ can be factorised as $(a + b)(a - b)$.

21. **Write the identity connecting $(a+b)^2$, $(a-b)^2$, a^2, and b^2.**
    Answer: The identity is $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$.
22. **How can 11 be expressed to apply the distributive property for fast multiplication?**
    Answer: 11can be expressed as $10 + 1$.
23. **How can a number Nbe multiplied by 101 using the distributive property?**
    Answer: $N \times 101 = N \times (100+1) = 100N + N$.
24. **What modified identity can be used for quickly calculating the square of a number?**
    Answer: The identity $a^2 = (a+b)(a-b) + b^2$can be used for quick square calculations.
25. **Who gave the first explicit statement of the distributive property according to the chapter?**
    Answer: Brahmagupta gave the first explicit statement of the distributive property in the *Brahmasphuṭasiddhānta*.
26. **What term did Brahmagupta use for multiplication by parts?**
    Answer: Brahmagupta referred to multiplication by parts as **khaṇḍa-guṇanam**.
27. **What term is used in the chapter for Brahmagupta's fast multiplication method?**
    Answer: The fast multiplication method using the distributive property is referred to as **ista-gunana**.
28. **What is the expansion of $(a+b)(a+b)$?**
    Answer: $(a+b)(a+b) = a^2 + 2ab + b^2$.
29. **What expression gives the number of circles at Step $k$in the pattern discussed in the chapter?**
    Answer: The number of circles at Step $k$is $k^2 + 2k$.
30. **Why can different algebraic expressions describe the same pattern?**
    Answer: Different algebraic expressions can describe the same pattern because they may simplify to the same equivalent expression.

6.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – HOTS Type:

1. **If $a = 8$and $b = 6$, by how much does abincrease when both numbers are increased by 1?**
   Answer: The product increases by $a + b + 1 = 8 + 6 + 1 = 15$.
2. **If $(a+1)(b+1) - ab = 18$and $a = 7$, find b.**
   Answer: Since $a + b + 1 = 18$, $b = 18 - 7 - 1 = 10$.
3. **If $a$is increased by 1 and $b$is decreased by 1, when will the product remain unchanged?**
   Answer: The product remains unchanged when $b - a - 1 = 0$, or $b = a + 1$.
4. **Find the increase in $12 \times 15$when the factors are increased by 2 and 3 respectively.**
   Answer: The increase is $12(3) + 15(2) + (2)(3) = 72$.
5. **Without direct multiplication, find 21×19using an algebraic identity.**
   Answer: $21 \times 19 = (20+1)(20-1) = 20^2 - 1^2 = 399$.
6. **Which identity is most suitable for finding 98×102quickly?**
   Answer: The identity $(a+b)(a-b) = a^2 - b^2$is most suitable because $98 \times 102 = (100-2)(100+2)$.
7. **Find 98×102without conventional multiplication.**
   Answer: $98 \times 102 = 100^2 - 2^2 = 9996$.

8. **How can 104^2 be written using the square-of-a-sum identity?**
   Answer: $104^2 = (100+4)^2 = 100^2 + 2(100)(4) + 4^2$.
9. **Find 99^2 using the square-of-a-difference identity.**
   Answer: $99^2 = (100-1)^2 = 10000 - 200 + 1 = 9801$.
10. **If $(x+5)^2 = x^2 + kx + 25$, find k.**
    Answer: Comparing with $(a+b)^2 = a^2 + 2ab + b^2$, we get $k = 10$.
11. **If $(x-7)^2 = x^2 - kx + 49$, find k.**
    Answer: Since $(x-7)^2 = x^2 - 14x + 49$, $k = 14$.
12. **If $a+b = 12$ and $ab = 20$, find $a^2 + b^2$.**
    Answer: Using $(a+b)^2 = a^2 + 2ab + b^2$, $a^2 + b^2 = 144 - 40 = 104$.
13. **If $a-b = 6$ and $ab = 8$, find $a^2 + b^2$.**
    Answer: Using $(a-b)^2 = a^2 - 2ab + b^2$, $a^2 + b^2 = 36 + 16 = 52$.
14. **If $a+b = 15$ and $a-b = 3$, find $a^2 - b^2$.**
    Answer: Using $a^2 - b^2 = (a+b)(a-b)$, we get $15 \times 3 = 45$.
15. **If $a^2 - b^2 = 60$ and $a-b = 5$, find $a+b$.**
    Answer: Since $a^2 - b^2 = (a+b)(a-b)$, $a+b = 60 \div 5 = 12$.
16. **Why are $(a-b)^2$ and $(b-a)^2$ always equal?**
    Answer: They are equal because $b - a = -(a-b)$ and squaring either expression gives the same value.
17. **If $(a+b)^2 = 64$ and $(a-b)^2 = 16$, find $a^2 + b^2$.**
    Answer: Since $2(a^2 + b^2) = 64 + 16 = 80$, $a^2 + b^2 = 40$.
18. **Show in one sentence why $x^2 + 6x + 9$ is a perfect square.**
    Answer: Since $x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$, it equals $(x+3)^2$.
19. **Express $25x^2 - 16y^2$ as a product of two binomials.**
    Answer: $25x^2 - 16y^2 = (5x+4y)(5x-4y)$.
20. **Find 45×55 using the difference-of-squares identity.**
    Answer: $45 \times 55 = (50-5)(50+5) = 50^2 - 5^2 = 2475$.
21. **How would you use distributivity to calculate 999×24 quickly?**
    Answer: Write $999 \times 24 = (1000-1) \times 24 = 24000 - 24 = 23976$.
22. **Why does multiplying a number N by 101 give $100N + N$?**
    Answer: Since $101 = 100 + 1$, distributivity gives $N(100+1) = 100N + N$.
23. **If $x^2 + 10x + 25 = (x+p)^2$, find p.**
    Answer: Comparing with $x^2 + 2px + p^2$, we get $p = 5$.
24. **If $9x^2 - 24xy + 16y^2$ is a perfect square, write it in squared form.**
    Answer: $9x^2 - 24xy + 16y^2 = (3x-4y)^2$.
25. **If $a = 9$ and $b = 4$, find $2(a^2+b^2)$ using $(a+b)^2 + (a-b)^2$.**
    Answer: $2(a^2+b^2) = 13^2 + 5^2 = 169 + 25 = 194$.
26. **Which is greater, $(a-b)^2$ or $(b-a)^2$?**
    Answer: Neither is greater because $(a-b)^2 = (b-a)^2$.
27. **Why can different expressions for the same geometric pattern give the same answer?**
    Answer: Different expressions can represent the same quantity because they simplify to an identical algebraic expression.
28. **Find the number of circles in Step 15 if the pattern is represented by $k^2 + 2k$.**
    Answer: At $k = 15$, the number of circles is $15^2 + 2(15) = 255$.
29. **If $(x+2)(x+5)$ is expanded, what is the coefficient of x?**
    Answer: Since $(x+2)(x+5) = x^2 + 7x + 10$, the coefficient of x is 7.

30. **Why is $a^2 = (a+b)(a-b) + b^2$ useful for mental calculation?**
Answer: It converts a square into a potentially easier product plus a smaller square, making some calculations quicker.

6.7 Two-mark questions with Answers of LOTS Type: (22 Questions)

Two-Mark Descriptive Questions with Answers – LOTS Type:

1. What is the distributive property of multiplication over addition? Give its algebraic form.

Answer:

1. The distributive property states that multiplying a number by a sum is the same as multiplying it separately by each term and then adding the products.
2. Its algebraic form is $a(b+c) = ab + ac$.

2. How can $(a+b)c$ be expanded using the distributive property?

Answer:

1. Using commutativity, $(a+b)c = c(a+b)$.
2. Applying distributivity gives $(a+b)c = ac + bc$.

3. What happens to the product ab when b is increased by 1?

Answer:

1. The new product is $a(b+1) = ab + a$.
2. Therefore, the original product **increases by** a.

4. What happens when both numbers in the product ab are increased by 1?

Answer:

1. The new product is $(a+1)(b+1) = ab + a + b + 1$.
2. Therefore, the product **increases by** $a + b + 1$.

5. Expand $(a+1)(b-1)$.

Answer:

1. Applying the distributive property gives $(a+1)(b-1) = ab + b - a - 1$.
2. Thus, the change from the original product ab is $b - a - 1$.

6. What is an algebraic identity? Give one example.

Answer:

1. An algebraic identity is a mathematical statement expressing the equality of two algebraic expressions for all values of their letter-numbers.
2. One example is $a(b+8) = ab + 8a$.

7. Expand $(a+m)(b+n)$ and state the increase over ab.

Answer:

1. The expansion is $(a+m)(b+n) = ab + mb + an + mn$.
2. Hence, the increase over the original product ab is $an + bm + mn$.

8. Expand $(a+u)(b-v)$ using the distributive property.

Answer:

1. We get $(a+u)(b-v) = ab + ub - av - uv$.
2. Each term of $a + u$ is multiplied by each term of $b - v$, with the appropriate signs.

9. Write the expansions of $(a-u)(b+v)$ and $(a-u)(b-v)$.

Answer:

1. $(a-u)(b+v) = ab - ub + av - uv$.
2. $(a-u)(b-v) = ab - ub - av + uv$.

10. What are like terms? Give an example from the chapter.

Answer:

1. Terms having exactly the same letter-numbers are called **like terms**.
2. For example, ba and ab are like terms because $ba = ab$, so $ba + ab = 2ab$.

11. State and explain the identity for the square of a sum.
Answer:
1. The identity for the square of a sum is $(a+b)^2 = a^2 + 2ab + b^2$.
2. It is obtained by expanding $(a+b)(a+b)$using the distributive property.

12. Expand $(6x+5)^2$.
Answer:
1. Using $(a+b)^2 = a^2 + 2ab + b^2$, $(6x+5)^2 = (6x)^2 + 2(6x)(5) + 5^2$.
2. Therefore, $(6x+5)^2 = 36x^2 + 60x + 25$.

13. State the identity for the square of a difference and explain it.
Answer:
1. The identity is $(a-b)^2 = a^2 - 2ab + b^2$.
2. It is obtained by expanding $(a-b)(a-b)$using the distributive property.

14. Use an identity to explain how 55^2can be calculated from $60$and 5.
Answer:
1. Using $(a-b)^2 = a^2 - 2ab + b^2$, $55^2 = (60-5)^2 = 60^2 - 2(60)(5) + 5^2$.
2. Hence, $55^2 = 3600 - 600 + 25 = 3025$.

15. State the difference-of-two-squares identity and give its reverse form.
Answer:
1. The identity is $(a+b)(a-b) = a^2 - b^2$.
2. In reverse form, it can be written as $a^2 - b^2 = (a+b)(a-b)$.

16. State the identity relating the sum of two squares to the square of the sum and difference.
Answer:
1. The identity is $2(a^2+b^2) = (a+b)^2 + (a-b)^2$.
2. It follows because the $+2ab$and $-2ab$terms cancel when the two squared expressions are added.

17. How can the distributive property be used to multiply a number by 11 quickly?
Answer:
1. Since $11 = 10 + 1$, we can write $N \times 11 = N(10+1)$.
2. Applying distributivity gives $N \times 11 = 10N + N$.

18. How can the distributive property be used for multiplication by 101?
Answer:
1. Since $101 = 100 + 1$, we write $N \times 101 = N(100+1)$.
2. Therefore, $N \times 101 = 100N + N$.

19. Explain the identity used by Sridharacharya for quickly finding squares.
Answer:
1. The identity is $a^2 = (a+b)(a-b) + b^2$, obtained from the difference-of-squares identity.
2. For example, $31^2 = (31+1)(31-1) + 1^2 = 32 \times 30 + 1 = 961$.

20. What does the chapter teach about different expressions representing the same pattern?
Answer:
1. A single pattern can be interpreted in several ways, producing apparently different algebraic expressions.
2. When simplified correctly, these expressions can give the **same equivalent expression**.

21. What is the algebraic expression for the number of circles at Step $k$in the pattern discussed in the chapter?
Answer:

1. Different methods used to count the circles simplify to $k^2 + 2k$.
2. Therefore, the number of circles at Step $k$is $k^2 + 2k$.

22. What is the purpose of the "Mind the Mistake, Mend the Mistake" activity?
Answer:
1. Students are asked to check algebraic simplifications and identify any mistakes in them.
2. They must explain what went wrong and then write the **correct expression**.

6.8. Two-mark questions with Answers of HOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – HOTS Type:
1. The factors of a product are 12 and 18. If both are increased by 1, find the increase in the product.
Answer:
1. The increase is $a + b + 1 = 12 + 18 + 1 = 31$.
2. Hence, the product increases by **31**.

2. If one factor $a$is increased by 1 and the other factor $b$is decreased by 1, under what condition will the product remain unchanged?
Answer:
1. Since $(a + 1)(b - 1) = ab + b - a - 1$, the change is $b - a - 1$.
2. The product remains unchanged when $b = a + 1$.

3. Two numbers are 10 and 15; the first is increased by 2 and the second by 3. Find the increase in their product.
Answer:
1. Using $an + bm + mn$, the increase is $10(3) + 15(2) + (2)(3)$.
2. Therefore, the increase is $30 + 30 + 6 = 66$.

4. Expand and simplify $(x + 4)(x + 7)$.
Answer:
1. By distributivity, $(x + 4)(x + 7) = x^2 + 7x + 4x + 28$.
2. Combining like terms gives $x^2 + 11x + 28$.

5. Expand and simplify $(3 - x)(x - 6)$.
Answer:
1. Applying distributivity gives $3x - 18 - x^2 + 6x$.
2. Combining like terms gives $-x^2 + 9x - 18$.

6. Find 104^2using a suitable algebraic identity.
Answer:
1. Using $(a + b)^2 = a^2 + 2ab + b^2$, $104^2 = (100 + 4)^2 = 10000 + 800 + 16$.
2. Therefore, $104^2 = 10816$.

7. Find 99^2without using ordinary multiplication.
Answer:
1. Using $(a - b)^2$, $99^2 = (100 - 1)^2 = 10000 - 200 + 1$.
2. Therefore, $99^2 = 9801$.

8. Find 98×102using an appropriate identity.
Answer:
1. $98 \times 102 = (100 - 2)(100 + 2) = 100^2 - 2^2$.
2. Hence, $98 \times 102 = 10000 - 4 = 9996$.

9. Find 45×55using the difference-of-squares identity.
Answer:
1. $45 \times 55 = (50 - 5)(50 + 5) = 50^2 - 5^2$.
2. Therefore, $45 \times 55 = 2500 - 25 = 2475$.

10. If $a + b = 14$and $a - b = 6$, find $a^2 - b^2$.
Answer:
1. Using $a^2 - b^2 = (a + b)(a - b)$, we get 14×6.
2. Therefore, $a^2 - b^2 = 84$.

11. If $a + b = 10$and $ab = 21$, find $a^2 + b^2$.
Answer:
1. From $(a + b)^2 = a^2 + 2ab + b^2$, $a^2 + b^2 = 10^2 - 2(21)$.
2. Therefore, $a^2 + b^2 = 100 - 42 = 58$.

12. If $a - b = 8$and $ab = 9$, find $a^2 + b^2$.
Answer:
1. Using $(a - b)^2 = a^2 - 2ab + b^2$, $a^2 + b^2 = 8^2 + 2(9)$.
2. Hence, $a^2 + b^2 = 64 + 18 = 82$.

13. If $(a + b)^2 = 100$and $(a - b)^2 = 36$, find $a^2 + b^2$.
Answer:
1. Using $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$, we get $2(a^2 + b^2) = 136$.
2. Therefore, $a^2 + b^2 = 68$.

14. Show that $(a - b)^2$and $(b - a)^2$are equal.
Answer:
1. Since $b - a = -(a - b)$, we have $(b - a)^2 = [-(a - b)]^2$.
2. Squaring removes the negative sign, so $(a - b)^2 = (b - a)^2$.

15. Recognise $x^2 + 10x + 25$as a perfect-square expression.
Answer:
1. The expression has the form $a^2 + 2ab + b^2$, since $10x = 2(x)(5)$.
2. Therefore, $x^2 + 10x + 25 = (x + 5)^2$.

16. Express $9x^2 - 24xy + 16y^2$as a perfect square.
Answer:
1. Here, $9x^2 = (3x)^2$, $16y^2 = (4y)^2$, and $24xy = 2(3x)(4y)$.
2. Therefore, $9x^2 - 24xy + 16y^2 = (3x - 4y)^2$.

17. Factorise $25a^2 - 9b^2$using an identity.
Answer:
1. Recognising it as a difference of squares, $25a^2 - 9b^2 = (5a)^2 - (3b)^2$.
2. Hence, $25a^2 - 9b^2 = (5a + 3b)(5a - 3b)$.

18. Explain how 197^2can be calculated quickly using the method discussed in the chapter.
Answer:
1. Using $a^2 = (a + b)(a - b) + b^2$, take $a = 197$and $b = 3$, giving $197^2 = 200 \times 194 + 9$.
2. Therefore, $197^2 = 38809$.

19. Use distributivity to calculate 3874×11.
Answer:
1. Since $11 = 10 + 1$, $3874 \times 11 = 3874 \times 10 + 3874$.
2. Therefore, $3874 \times 11 = 42614$.

20. Use distributivity to calculate 89×101.
Answer:
1. Since $101 = 100 + 1$, $89 \times 101 = 8900 + 89$.
2. Therefore, $89 \times 101 = 8989$.

21. Four different methods give expressions $(k + 1)^2 - 1$, $k^2 + 2k$, $k(k + 1) + k$, and $k(k + 2)$; what conclusion can you draw?
Answer:

1. Simplifying each expression gives $k^2 + 2k$.
2. Hence, apparently different algebraic expressions can represent the **same mathematical pattern**.

22. Find the number of circles at Step 15 if Step $k$contains $k^2 + 2k$circles.

Answer:

1. Substituting $k = 15$, the number is $15^2 + 2(15) = 225 + 30$.
2. Therefore, **Step 15 contains 255 circles**.

23. A student writes $(a + b)^2 = a^2 + b^2$; explain the mistake.

Answer:

1. The student has omitted the two cross-products $ab$and ba, which together give $2ab$.
2. The correct expansion is $(a + b)^2 = a^2 + 2ab + b^2$.

24. A student simplifies $5w^2 + 6w$as $11w^2$; why is this incorrect?

Answer:

1. $5w^2$and $6w$are **unlike terms** because their powers of $w$are different.
2. Therefore, they cannot be combined, and the expression remains $5w^2 + 6w$.

25. Two methods give the shaded area as $(m + n)^2 - 4mn$and $(n - m)^2$; show why they are equivalent.

Answer:

1. Expanding the first expression gives $m^2 + 2mn + n^2 - 4mn = m^2 - 2mn + n^2$.
2. Since this equals $(n - m)^2$, both methods give the same area.

6.9. Three-mark questions with Answers of LOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – LOTS Type:

1. Explain the distributive property of multiplication over addition with an example.

Answer:

1. The distributive property states that $a(b + c) = ab + ac$.
2. It means that $a$is multiplied separately by both $b$and c, and the products are then added.
3. For example, $5(3 + 2) = 5 \times 3 + 5 \times 2 = 15 + 10 = 25$.

2. Explain how the product abchanges when both $a$and $b$are increased by 1.

Answer:

1. When both numbers are increased by 1, the new product becomes $(a + 1)(b + 1)$.
2. Expanding gives $(a + 1)(b + 1) = ab + a + b + 1$.
3. Therefore, the product increases by $a + b + 1$.

3. Explain what happens when one factor is increased by 1 and the other is decreased by 1.

Answer:

1. If $a$is increased by 1 and $b$is decreased by 1, the new product is $(a + 1)(b - 1)$.
2. Expanding gives $(a + 1)(b - 1) = ab + b - a - 1$.
3. Therefore, the change in the original product is $b - a - 1$.

4. What is an algebraic identity? Explain with examples from the chapter.

Answer:

1. An identity is a mathematical statement expressing the equality of two algebraic expressions for all values of their letter-numbers.
2. One example is $a(b + 8) = ab + 8a$.
3. Another example is $(a + 1)(b - 1) = ab + b - a - 1$.

5. Derive the expansion of $(a + m)(b + n)$.

Answer:

1. Applying distributivity, $(a + m)(b + n) = (a + m)b + (a + m)n$.
2. Expanding further gives $ab + mb + an + mn$.

3. Hence, $(a+m)(b+n) = ab + mb + an + mn$.

6. Explain the expansion of $(a+u)(b-v)$.

Answer:

1. Using distributivity, $(a+u)(b-v) = (a+u)b - (a+u)v$.
2. Expanding the two products gives $ab + ub - av - uv$.
3. Therefore, $(a+u)(b-v) = ab + ub - av - uv$.

7. What are like terms? Explain how they are combined.

Answer:

1. Terms having exactly the same letter-numbers are called **like terms**.
2. For example, $ba$and $ab$are like terms because $ba = ab$.
3. Hence, $ba + ab = ab + ab = 2ab$.

8. Derive the identity for the square of the sum of two terms.

Answer:

1. Write $(a+b)^2$as $(a+b)(a+b)$.
2. Using distributivity, it becomes $a^2 + ab + ba + b^2$.
3. Since $ab = ba$, $(a+b)^2 = a^2 + 2ab + b^2$.

9. Expand $(6x+5)^2$using the square-of-a-sum identity.

Answer:

1. Using $(a+b)^2 = a^2 + 2ab + b^2$, $(6x+5)^2 = (6x)^2 + 2(6x)(5) + 5^2$.
2. Simplifying gives $36x^2 + 60x + 25$.
3. Therefore, $(6x+5)^2 = 36x^2 + 60x + 25$.

10. Derive the identity for the square of the difference of two terms.

Answer:

1. Write $(a-b)^2$as $(a-b)(a-b)$.
2. Applying distributivity gives $a^2 - ab - ba + b^2$.
3. Since $ab = ba$, $(a-b)^2 = a^2 - 2ab + b^2$.

11. Explain how 55^2can be calculated using the identity for the square of a difference.

Answer:

1. Write $55^2 = (60-5)^2$and use $(a-b)^2 = a^2 - 2ab + b^2$.
2. Thus, $55^2 = 60^2 - 2(60)(5) + 5^2 = 3600 - 600 + 25$.
3. Therefore, $55^2 = 3025$.

12. Derive the difference-of-two-squares identity.

Answer:

1. Expand $(a+b)(a-b)$using distributivity to get $a^2 - ab + ab - b^2$.
2. The terms $-ab$and $+ab$cancel each other.
3. Therefore, $(a+b)(a-b) = a^2 - b^2$.

13. Derive the identity $2(a^2+b^2) = (a+b)^2 + (a-b)^2$.

Answer:

1. We know $(a+b)^2 = a^2 + 2ab + b^2$and $(a-b)^2 = a^2 - 2ab + b^2$.
2. Adding them cancels $+2ab$and $-2ab$, leaving $2a^2 + 2b^2$.
3. Hence, $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$.

14. Explain how the distributive property helps in multiplying a number by 11.

Answer:

1. Write 11as $10+1$, so $N \times 11 = N(10+1)$.
2. By distributivity, $N(10+1) = 10N + N$.
3. For example, $3874 \times 11 = 38740 + 3874 = 42614$.

15. Explain how the distributive property can be used for multiplication by 101.

Answer:

1. Write $101 = 100 + 1$, so $N \times 101 = N(100+1)$.

2. Applying distributivity gives $N \times 101 = 100N + N$.
3. Thus, multiplying by 101 can be performed by adding the original number to **100 times the number**.

16. Explain Sridharacharya's method for calculating squares using an identity.
Answer:
1. The method uses the identity $a^2 = (a+b)(a-b) + b^2$.
2. For 31^2, take $a = 31$and $b = 1$, giving $31^2 = 32 \times 30 + 1$.
3. Therefore, $31^2 = 961$.

17. Explain why $(a - b)^2 = (b - a)^2$.
Answer:
1. We know that $b - a = -(a - b)$.
2. Therefore, $(b - a)^2 = [-(a - b)]^2$.
3. Since the square of a negative quantity is positive, $(a - b)^2 = (b - a)^2$.

18. Explain how different expressions can represent the same circle pattern in the chapter.
Answer:
1. The circle pattern can be counted in different ways, producing expressions such as $(k+1)^2 - 1$, $k(k+1) + k$, and $k(k+2)$.
2. On expansion and simplification, all these expressions become $k^2 + 2k$.
3. Hence, the number of circles at Step $k$is $k^2 + 2k$irrespective of the counting method.

19. Explain why $(a + b)^2$is not equal to $a^2 + b^2$.
Answer:
1. Expanding $(a+b)^2$gives $a^2 + ab + ba + b^2$.
2. Since $ab + ba = 2ab$, the complete expansion is $a^2 + 2ab + b^2$.
3. Therefore, $(a+b)^2 \neq a^2 + b^2$in general.

20. Explain the purpose of checking mistakes while simplifying algebraic expressions.
Answer:
1. Checking each step helps identify errors in applying distributivity, multiplication, signs, or combining terms.
2. After locating an error, the incorrect step can be explained and corrected.
3. This process helps students obtain the **correct simplified algebraic expression**.

6.10. Three-mark questions with Answers of HOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – HOTS Type:

1. The product of two numbers is ab. Both numbers are increased by 2. Find the increase in the product.
Answer:
1. The new product is $(a+2)(b+2) = ab + 2a + 2b + 4$.
2. Subtracting the original product ab, the increase is $2a + 2b + 4$.
3. Therefore, the product increases by $2a + 2b + 4$.

2. One factor $a$is increased by 1 and the other factor $b$is decreased by 1. Find the condition under which the product remains unchanged.
Answer:
1. The new product is $(a+1)(b-1) = ab + b - a - 1$.
2. For the product to remain unchanged, $b - a - 1 = 0$.
3. Therefore, the required condition is $b = a + 1$.

3. Without ordinary multiplication, calculate 103×97using a suitable identity.
Answer:
1. Write $103 \times 97 = (100 + 3)(100 - 3)$.
2. Using $(a+b)(a-b) = a^2 - b^2$, this becomes $100^2 - 3^2$.

3. Therefore, $103 \times 97 = 10000 - 9 = 9991$.

4. Find 98^2 using an algebraic identity instead of direct multiplication.

Answer:

1. Write $98^2 = (100 - 2)^2$.
2. Using $(a - b)^2 = a^2 - 2ab + b^2$, we get $10000 - 400 + 4$.
3. Therefore, $98^2 = 9604$.

5. A student claims that $(x + 6)^2 = x^2 + 36$. Identify the error and give the correct answer.

Answer:

1. The student has omitted the middle term $2(x)(6) = 12x$.
2. Using $(a + b)^2 = a^2 + 2ab + b^2$, we get $(x + 6)^2 = x^2 + 12x + 36$.
3. Hence, the correct expansion is $x^2 + 12x + 36$.

6. If $a + b = 18$ and $a - b = 4$, find $a^2 - b^2$ without finding a and b separately.

Answer:

1. Use the identity $a^2 - b^2 = (a + b)(a - b)$.
2. Substituting the given values gives $a^2 - b^2 = 18 \times 4$.
3. Therefore, $a^2 - b^2 = 72$.

7. If $a + b = 12$ and $ab = 25$, find $a^2 + b^2$.

Answer:

1. Using $(a + b)^2 = a^2 + 2ab + b^2$, we get $144 = a^2 + b^2 + 50$.
2. Therefore, $a^2 + b^2 = 144 - 50$.
3. Hence, $a^2 + b^2 = 94$.

8. If $(a + b)^2 = 121$ and $(a - b)^2 = 49$, find $a^2 + b^2$.

Answer:

1. Use $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.
2. Thus, $121 + 49 = 2(a^2 + b^2)$, giving $170 = 2(a^2 + b^2)$.
3. Therefore, $a^2 + b^2 = 85$.

9. Show that $x^2 + 14x + 49$ is a perfect-square expression.

Answer:

1. Here, $x^2 = x^2$, $49 = 7^2$, and $14x = 2(x)(7)$.
2. Thus, the expression matches the identity $a^2 + 2ab + b^2 = (a + b)^2$.
3. Therefore, $x^2 + 14x + 49 = (x + 7)^2$.

10. Factorise $49x^2 - 25y^2$ using an appropriate identity.

Answer:

1. Rewrite the expression as $(7x)^2 - (5y)^2$.
2. Applying $a^2 - b^2 = (a + b)(a - b)$, we get $(7x + 5y)(7x - 5y)$.
3. Hence, $49x^2 - 25y^2 = (7x + 5y)(7x - 5y)$.

11. Find 198^2 quickly using the identity $a^2 = (a + b)(a - b) + b^2$.

Answer:

1. Take $a = 198$ and $b = 2$, so $198^2 = (200)(196) + 2^2$.
2. This gives $198^2 = 39200 + 4$.
3. Therefore, $198^2 = 39204$.

12. Use distributivity to calculate 999×47 efficiently.

Answer:

1. Write $999 = 1000 - 1$, so $999 \times 47 = (1000 - 1) \times 47$.
2. Applying distributivity gives $47000 - 47$.
3. Therefore, $999 \times 47 = 46953$.

13. If $(x + p)(x + q) = x^2 + 11x + 24$, what can you determine about p and q?

Answer:

1. Expanding gives $(x+p)(x+q) = x^2 + (p+q)x + pq$.
2. Comparing coefficients gives $p + q = 11$and $pq = 24$.
3. Hence, $p = 3, q = 8$or $p = 8, q = 3$.

14. A student simplifies $7x^2 + 5x$as $12x^2$. Analyse the mistake.

Answer:

1. The terms $7x^2$and $5x$are unlike terms because the powers of $x$are different.
2. Unlike terms cannot be added by simply adding their numerical coefficients.
3. Therefore, the correct expression remains $7x^2 + 5x$.

15. Show that $(a-b)^2$and $(b-a)^2$always have the same value.

Answer:

1. Since $b - a = -(a-b)$, we can write $(b-a)^2 = [-(a-b)]^2$.
2. Squaring a negative expression gives the same result as squaring the corresponding positive expression.
3. Hence, $(a-b)^2 = (b-a)^2$.

16. A circle pattern has $k^2 + 2k$circles at Step k. Find the number of circles at Step 20 and explain your calculation.

Answer:

1. Substitute $k = 20$in the expression $k^2 + 2k$.
2. This gives $20^2 + 2(20) = 400 + 40$.
3. Therefore, **Step 20 contains 440 circles**.

17. Show that $k(k+2)$and $(k+1)^2 - 1$represent the same expression.

Answer:

1. Expanding $k(k+2)$gives $k^2 + 2k$.
2. Expanding $(k+1)^2 - 1$gives $k^2 + 2k + 1 - 1 = k^2 + 2k$.
3. Therefore, **both expressions are equivalent and can represent the same pattern**.

18. The shaded area of a figure is written as $(m+n)^2 - 4mn$. Show that it can also be written as $(n-m)^2$.

Answer:

1. Expanding gives $(m+n)^2 - 4mn = m^2 + 2mn + n^2 - 4mn$.
2. Simplifying gives $m^2 - 2mn + n^2$.
3. Therefore, the shaded area equals $(n-m)^2$.

19. If $a^2 - b^2 = 96$and $a - b = 8$, find $a + b$without finding $a$and b.

Answer:

1. Using $a^2 - b^2 = (a+b)(a-b)$, we get $96 = (a+b) \times 8$.
2. Dividing both sides by 8 gives $a + b = 96 \div 8$.
3. Therefore, $a + b = 12$.

20. Compare $51^2$and 49^2without calculating the two squares separately.

Answer:

1. Their difference is $51^2 - 49^2 = (51+49)(51-49)$.
2. Using the difference-of-squares identity gives $100 \times 2 = 200$.
3. Therefore, 51^2**is 200 greater than** 49^2.

6.11 Five-mark questions with Answers of LOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – LOTS Type:

1. Explain the distributive property of multiplication over addition with an example.

Answer:

1. The **distributive property** connects multiplication with addition and helps us expand algebraic expressions.

2. It is written algebraically as $a(b + c) = ab + ac$.
3. Similarly, when the sum occurs in the first factor, $(a + b)c = ac + bc$.
4. For example, $5(7 + 3) = 5 \times 7 + 5 \times 3 = 35 + 15$.
5. Thus, both sides give **50**, showing how multiplication can be distributed over addition.

2. Explain how the product abchanges when both factors are increased by 1.

Answer:

1. The original product of the two factors is ab.
2. When both factors are increased by 1, the new product becomes $(a + 1)(b + 1)$.
3. Using distributivity, $(a + 1)(b + 1) = ab + a + b + 1$.
4. The difference between the new and original products is $(ab + a + b + 1) - ab = a + b + 1$.
5. Therefore, when both factors are increased by 1, the product **increases by** $a + b + 1$.

3. Explain the general expansion of $(a + m)(b + n)$and the resulting increase in the product.

Answer:

1. Consider the original product $ab$and increase its factors by $m$and n, respectively.
2. The new product is $(a + m)(b + n)$.
3. Applying the distributive property gives $(a + m)(b + n) = ab + mb + an + mn$.
4. Comparing this with the original product ab, the additional terms are $mb + an + mn$.
5. Hence, the increase in the product is $an + bm + mn$.

4. Explain the different expansions obtained when quantities are added to or subtracted from two factors.

Answer:

1. When quantities are added to both factors, $(a + u)(b + v) = ab + ub + av + uv$.
2. When $u$is added and $v$is subtracted, $(a + u)(b - v) = ab + ub - av - uv$.
3. When $u$is subtracted and $v$is added, $(a - u)(b + v) = ab - ub + av - uv$.
4. When quantities are subtracted from both factors, $(a - u)(b - v) = ab - ub - av + uv$.
5. These expansions follow from repeatedly applying the **distributive property** and the rules of signs.

5. Derive the identity for the square of the sum of two terms.

Answer:

1. The square of a sum can be written as $(a + b)^2 = (a + b)(a + b)$.
2. Applying distributivity gives $a(a + b) + b(a + b)$.
3. Expanding further gives $a^2 + ab + ba + b^2$.
4. Since $ab = ba$, the two like terms combine to give $2ab$.
5. Therefore, $(a + b)^2 = a^2 + 2ab + b^2$.

6. Derive the identity for the square of the difference of two terms.

Answer:

1. The square of a difference is written as $(a - b)^2 = (a - b)(a - b)$.
2. Applying distributivity gives $a(a - b) - b(a - b)$.
3. Expanding gives $a^2 - ab - ab + b^2$.
4. Combining the two like terms $-ab$and $-ab$gives $-2ab$.
5. Hence, $(a - b)^2 = a^2 - 2ab + b^2$.

7. Derive and explain the difference-of-two-squares identity.

Answer:

1. Consider the product $(a + b)(a - b)$.
2. Applying distributivity gives $a(a - b) + b(a - b)$.
3. Expanding further gives $a^2 - ab + ab - b^2$.

4. The terms $-ab$ and $+ab$ cancel each other, leaving $a^2 - b^2$.
5. Therefore, $(a+b)(a-b) = a^2 - b^2$ or $a^2 - b^2 = (a+b)(a-b)$.

8. Derive the identity $2(a^2 + b^2) = (a+b)^2 + (a-b)^2$.

Answer:

1. We know that $(a+b)^2 = a^2 + 2ab + b^2$.
2. We also know that $(a-b)^2 = a^2 - 2ab + b^2$.
3. Adding these expressions gives $a^2 + 2ab + b^2 + a^2 - 2ab + b^2$.
4. The terms $+2ab$ and $-2ab$ cancel, leaving $2a^2 + 2b^2$.
5. Therefore, $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$.

9. Explain how the distributive property can be used for fast multiplication by 11 and 101.

Answer:

1. To multiply a number N by 11, write 11 as $10 + 1$.
2. Then $N \times 11 = N(10+1) = 10N + N$.
3. Similarly, write 101 as $100 + 1$.
4. Thus, $N \times 101 = N(100+1) = 100N + N$.
5. Therefore, the distributive property provides convenient methods for performing such multiplications quickly.

10. Explain the identity used for finding squares quickly and illustrate it with 31^2.

Answer:

1. The difference-of-squares identity can be rearranged as $a^2 = (a+b)(a-b) + b^2$.
2. This form can make square calculations easier by choosing a convenient value of b.
3. For 31^2, choose $a = 31$ and $b = 1$, giving $31^2 = (31+1)(31-1) + 1^2$.
4. Thus, $31^2 = 32 \times 30 + 1 = 960 + 1$.
5. Therefore, $31^2 = 961$.

11. Explain how different algebraic expressions can represent the same circle pattern.

Answer:

1. The chapter shows that the same arrangement of circles can be viewed and counted in different ways.
2. Different counting methods produce apparently different algebraic expressions involving the step number k.
3. When these expressions are expanded and like terms are combined, they simplify to the same expression.
4. The common simplified expression for the number of circles is $k^2 + 2k$.
5. Thus, different algebraic representations can describe the **same mathematical pattern**.

12. Explain the importance of identifying and correcting mistakes in algebraic simplification.

Answer:

1. Algebraic expressions must be simplified carefully by following the distributive property and rules of operations.
2. Students should check whether each multiplication and sign has been handled correctly.
3. They should also ensure that only **like terms** are combined during simplification.
4. If a mistake is found, the incorrect step should be identified, explained, and corrected.
5. The chapter's **“Mind the Mistake, Mend the Mistake”** activity develops this habit of checking and correcting algebraic work.

6.12 Five-mark questions with Answers of HOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – HOTS Type:

1. Two numbers are a and b. If both are increased by 3, determine the increase in their product and verify it for $a = 10$ and $b = 12$.

Answer:

1. The original product of the two numbers is ab, while the new product is $(a+3)(b+3)$.
2. Using distributivity, $(a+3)(b+3) = ab + 3a + 3b + 9$.
3. Hence, the increase in the product is $3a + 3b + 9$.
4. For $a = 10$and $b = 12$, the increase is $3(10) + 3(12) + 9 = 75$.
5. Indeed, $13 \times 15 - 10 \times 12 = 195 - 120 = 75$, verifying the result.

2. A product is ab. If $a$is increased by 1 and $b$is decreased by 1, determine when the new product is greater than, equal to, or less than the original product.

Answer:

1. The new product is $(a+1)(b-1) = ab + b - a - 1$.
2. Therefore, the change in the product is $b - a - 1$.
3. The new product is greater than $ab$when $b > a + 1$.
4. The new product is equal to $ab$when $b = a + 1$.
5. The new product is less than $ab$when $b < a + 1$.

3. Calculate 103×97using an algebraic identity and explain why the method is efficient.

Answer:

1. The numbers 103 and 97 are equally distant from 100, so write them as $100 + 3$and $100 - 3$.
2. Thus, $103 \times 97 = (100+3)(100-3)$.
3. Using $(a+b)(a-b) = a^2 - b^2$, the product becomes $100^2 - 3^2$.
4. Therefore, $103 \times 97 = 10000 - 9 = 9991$.
5. This method is efficient because it converts a lengthy multiplication into the simple subtraction of two squares.

4. A student says that $(2x+5)^2 = 4x^2 + 25$. Analyse the mistake and obtain the correct expression.

Answer:

1. The student has incorrectly squared only the two individual terms and omitted their cross-product.
2. The correct identity is $(a+b)^2 = a^2 + 2ab + b^2$.
3. Therefore, $(2x+5)^2 = (2x)^2 + 2(2x)(5) + 5^2$.
4. Simplifying gives $4x^2 + 20x + 25$.
5. Hence, the correct result is $(2x+5)^2 = 4x^2 + 20x + 25$.

5. If $a + b = 20$and $ab = 64$, find $a^2 + b^2$and explain the identity used.

Answer:

1. We use the identity $(a+b)^2 = a^2 + 2ab + b^2$.
2. Substituting $a + b = 20$and $ab = 64$gives $20^2 = a^2 + b^2 + 2(64)$.
3. Thus, $400 = a^2 + b^2 + 128$.
4. Therefore, $a^2 + b^2 = 400 - 128 = 272$.
5. This shows that the sum of the squares can be determined without finding the individual values of $a$and b.

6. If $a + b = 18$and $a - b = 6$, find $a^2 + b^2$without first finding $a$and b.

Answer:

1. Use the identity $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$.
2. Substituting the given values gives $18^2 + 6^2 = 2(a^2 + b^2)$.
3. Thus, $324 + 36 = 2(a^2 + b^2)$.
4. Hence, $360 = 2(a^2 + b^2)$, so $a^2 + b^2 = 180$.
5. Therefore, $a^2 + b^2 = 180$without separately calculating $a$and b.

7. Show that $x^2+16x+64$is a perfect square and use this observation to evaluate it when $x=12$.
Answer:
1. Compare $x^2+16x+64$with the identity $a^2+2ab+b^2=(a+b)^2$.
2. Since $16x=2(x)(8)$and $64=8^2$, the expression is a perfect square.
3. Therefore, $x^2+16x+64=(x+8)^2$.
4. For $x=12$, its value is $(12+8)^2=20^2$.
5. Hence, the value of the expression is **400**.

8. Factorise $81x^2-49y^2$and verify the factorisation by expansion.
Answer:
1. Recognise $81x^2-49y^2$as $(9x)^2-(7y)^2$.
2. Using $a^2-b^2=(a+b)(a-b)$, it becomes $(9x+7y)(9x-7y)$.
3. Expanding the factors gives $81x^2-63xy+63xy-49y^2$.
4. The terms $-63xy$and $+63xy$cancel each other.
5. Hence, the expansion gives $81x^2-49y^2$, verifying the factorisation.

9. Find 198^2using the rearranged difference-of-squares identity and explain the advantage of the method.
Answer:
1. Use the identity $a^2=(a+b)(a-b)+b^2$.
2. Taking $a=198$and $b=2$, we get $198^2=(198+2)(198-2)+2^2$.
3. Thus, $198^2=200\times196+4$.
4. Therefore, $198^2=39200+4=39204$.
5. The method is convenient because it replaces squaring 198 directly with multiplication involving the round number 200.

10. Different students obtain $k(k+2)$and $(k+1)^2-1$while counting the same circle pattern. Prove that both expressions are equivalent.
Answer:
1. Expanding the first expression gives $k(k+2)=k^2+2k$.
2. Expanding the second expression gives $(k+1)^2-1=k^2+2k+1-1$.
3. Therefore, $(k+1)^2-1=k^2+2k$.
4. Hence, both $k(k+2)$and $(k+1)^2-1$simplify to k^2+2k.
5. This demonstrates that different ways of viewing the same pattern can produce different-looking but equivalent algebraic expressions.

11. A shaded region has area $(m+n)^2-4mn$. Show that its area can also be expressed as $(n-m)^2$.
Answer:
1. Start with the given expression $(m+n)^2-4mn$.
2. Expanding the square gives $m^2+2mn+n^2-4mn$.
3. Combining like terms gives $m^2-2mn+n^2$.
4. Using the identity $a^2-2ab+b^2=(a-b)^2$, this becomes $(m-n)^2$.
5. Since $(m-n)^2=(n-m)^2$, the shaded area is $(n-m)^2$.

12. Compare $52^2$and 48^2without calculating the two squares separately, and determine their difference.
Answer:
1. The required difference is 52^2-48^2.
2. Using $a^2-b^2=(a+b)(a-b)$, it becomes $(52+48)(52-48)$.
3. This simplifies to 100×4.
4. Therefore, $52^2-48^2=400$.
5. Hence, 52^2**is 400 greater than** 48^2.

6.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – Applied Type:

1. A rectangular school garden is $x$metres long and $y$metres wide. The school decides to increase its length by 3 m and width by 2 m. Find the increase in the area of the garden.

Answer:

1. The original area of the garden is xy**square metres**.
2. After extension, the new dimensions are $(x + 3)$m and $(y + 2)$m.
3. The new area is $(x + 3)(y + 2) = xy + 2x + 3y + 6$.
4. The increase in area is $(xy + 2x + 3y + 6) - xy = 2x + 3y + 6$.
5. Therefore, the garden's area increases by $(2x + 3y + 6)$**square metres**.

2. A square playground has a side of 48 m. Use a suitable algebraic identity to calculate its area without directly multiplying 48 × 48.

Answer:

1. The area of the square is 48^2square metres.
2. Write $48 = 50 - 2$, so $48^2 = (50 - 2)^2$.
3. Using $(a - b)^2 = a^2 - 2ab + b^2$, we get $50^2 - 2(50)(2) + 2^2$.
4. Thus, $48^2 = 2500 - 200 + 4 = 2304$.
5. Therefore, the area of the playground is **2304 m²**.

3. A shopkeeper has 98 boxes, each containing 102 pencils. Find the total number of pencils using an algebraic identity.

Answer:

1. The total number of pencils is 98×102.
2. Write the product as $(100 - 2)(100 + 2)$.
3. Using $(a - b)(a + b) = a^2 - b^2$, we get $100^2 - 2^2$.
4. Thus, $10000 - 4 = 9996$.
5. Therefore, the shopkeeper has **9,996 pencils** in all.

4. A square floor has a side of $(x + 5)$metres. Find an expression for its area and explain the contribution of each part.

Answer:

1. The area of the square is $(x + 5)^2$square metres.
2. Using $(a + b)^2 = a^2 + 2ab + b^2$, we get $(x + 5)^2 = x^2 + 10x + 25$.
3. The term x^2represents the area of a square of side x.
4. The term $10x$represents the combined area of the two rectangular portions, while 25represents the 5×5square.
5. Therefore, the total area of the floor is $x^2 + 10x + 25$**m²**.

5. A school needs 101 notebooks for each of its 75 students. Use the distributive property to calculate the total number of notebooks required.

Answer:

1. The total number of notebooks required is 75×101.
2. Write 101as $100 + 1$, giving $75(100 + 1)$.
3. Using distributivity, $75(100 + 1) = 75 \times 100 + 75 \times 1$.
4. Thus, the total is $7500 + 75 = 7575$.
5. Therefore, the school requires **7,575 notebooks**.

6. A square park has a side of 97 m. Use the quick-square method discussed in the chapter to calculate its area.

Answer:

1. The area of the park is 97^2square metres.
2. Using $a^2 = (a + b)(a - b) + b^2$, choose $a = 97$and $b = 3$.

3. Then $97^2 = (97 + 3)(97 - 3) + 3^2$.
4. Thus, $97^2 = 100 \times 94 + 9 = 9409$.
5. Therefore, the area of the park is **9,409 m²**.

7. A decorative arrangement follows the circle pattern described in the chapter, where Step $k$contains $k^2 + 2k$circles. How many circles are required for Step 25?

Answer:

1. The number of circles at Step $k$is represented by $k^2 + 2k$.
2. For Step 25, substitute $k = 25$to get $25^2 + 2(25)$.
3. This gives $625 + 50$.
4. Therefore, the arrangement requires 675circles.
5. Hence, **675 circles** are needed to construct Step 25 of the pattern.

8. A square courtyard has side $(m + n)$metres, and four rectangular flower beds of total area $4mn$m² are removed. Find the remaining area in its simplest form.

Answer:

1. The total area of the square courtyard is $(m + n)^2$square metres.
2. After removing the flower beds, the remaining area is $(m + n)^2 - 4mn$.
3. Expanding gives $m^2 + 2mn + n^2 - 4mn = m^2 - 2mn + n^2$.
4. Using the identity $a^2 - 2ab + b^2 = (a - b)^2$, this becomes $(m - n)^2$.
5. Therefore, the remaining area is $(m - n)^2$**square metres**.

9. A farmer compares two square plots with sides 52 m and 48 m. Find how much greater the area of the larger plot is without calculating the two areas separately.

Answer:

1. The difference between the areas is $52^2 - 48^2$.
2. Using $a^2 - b^2 = (a + b)(a - b)$, this becomes $(52 + 48)(52 - 48)$.
3. Therefore, the difference is 100×4.
4. This gives an area difference of 400square metres.
5. Hence, the larger plot has **400 m² more area** than the smaller plot.

10. A student wants to calculate 999×64mentally rather than using long multiplication. Show how the distributive property can help.

Answer:

1. Write 999as $1000 - 1$, so $999 \times 64 = (1000 - 1) \times 64$.
2. Applying distributivity gives $1000 \times 64 - 1 \times 64$.
3. This simplifies to $64000 - 64$.
4. Therefore, the product is $63{,}936$.
5. Hence, using distributivity, $999 \times 64 = 63{,}936$without long multiplication.

6.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams: (20 MCQs of LOTS & 20 MCQs of HOTS Type)

Olympiad-Type Multiple Choice Questions with Answers:

The following questions emphasize **application, logical reasoning, algebraic identities, pattern recognition, simplification, and mental computation** based on the chapter.

1. If both factors of the product $ab$are increased by 1, by how much does the product increase?

A) $a + b$

B) $a + b + 1$

C) $ab + 1$

D) $2a + 2b$

Answer: B) $a + b + 1$

2. If $(a+1)(b-1)=ab$, which relation must be true?
A) $a=b$
B) $a=b+1$
C) $b=a+1$
D) $a+b=1$
Answer: C) $b=a+1$
3. If the factors 12 and 15 are increased by 2 and 3 respectively, what is the increase in their product?
A) 60
B) 62
C) 66
D) 72
Answer: C) 66, since the increase is $12(3)+15(2)+(2)(3)=66$.
4. Which is the correct expansion of $(a+3)(b+4)$?
A) $ab+3a+4b+12$
B) $ab+4a+3b+12$
C) $ab+3a+4b+7$
D) $ab+4a+3b+7$
Answer: B) $ab+4a+3b+12$
5. The coefficient of $x$in $(x+4)(x+7)$is:
A) 4
B) 7
C) 11
D) 28
Answer: C) 11
6. Which of the following is equivalent to $(a+b)^2$?
A) a^2+b^2
B) a^2+ab+b^2
C) $a^2+2ab+b^2$
D) $2a^2+2b^2$
Answer: C) $a^2+2ab+b^2$
7. If $(x+5)^2=x^2+kx+25$, what is k?
A) 5
B) 10
C) 15
D) 25
Answer: B) 10
8. Which expression is a perfect square?
A) $x^2+8x+16$
B) x^2+8x+8
C) $x^2+4x+16$
D) x^2+16
Answer: A) $x^2+8x+16=(x+4)^2$
9. The expression $9x^2-24xy+16y^2$is equal to:
A) $(3x+4y)^2$
B) $(3x-4y)^2$
C) $(9x-16y)^2$
D) $(3x-8y)^2$
Answer: B) $(3x-4y)^2$

10. Without direct multiplication, the value of $103 \times 97$is:
A) 9991
B) 9997
C) 10009
D) 10091
Answer: A) 9991, because $(100 + 3)(100 - 3) = 100^2 - 3^2$.
11. The value of $51^2 - 49^2$is:
A) 100
B) 196
C) 200
D) 400
Answer: C) 200, since $(51 + 49)(51 - 49) = 100 \times 2$.
12. Which is the correct factorisation of $49x^2 - 25y^2$?
A) $(7x - 5y)^2$
B) $(7x + 5y)^2$
C) $(7x + 5y)(7x - 5y)$
D) $(49x + 25y)(x - y)$
Answer: C) $(7x + 5y)(7x - 5y)$
13. If $a + b = 15$and $a - b = 5$, then $a^2 - b^2$equals:
A) 20
B) 50
C) 75
D) 100
Answer: C) 75
14. If $a^2 - b^2 = 84$and $a - b = 7$, then $a + b$equals:
A) 6
B) 12
C) 14
D) 21
Answer: B) 12
15. If $a + b = 10$and $ab = 21$, then $a^2 + b^2$equals:
A) 42
B) 58
C) 79
D) 100
Answer: B) 58, since $a^2 + b^2 = (a + b)^2 - 2ab = 100 - 42$.
16. If $(a + b)^2 = 100$and $(a - b)^2 = 36$, then $a^2 + b^2$is:
A) 32
B) 64
C) 68
D) 136
Answer: C) 68, because $2(a^2 + b^2) = 100 + 36$.
17. Which statement is always true?
A) $(a - b)^2 = -(b - a)^2$
B) $(a - b)^2 = (b - a)^2$
C) $(a - b)^2 = a^2 - b^2$
D) $(a - b)^2 = a^2 + b^2$
Answer: B) $(a - b)^2 = (b - a)^2$

18. Using the identity $a^2 = (a+b)(a-b) + b^2$, the quickest useful representation of $198^2$is:
A) $199 \times 197 + 1$
B) $200 \times 196 + 4$
C) $208 \times 188 + 100$
D) Both A and B
Answer: D) Both A and B, as both expressions equal 198^2.
19. The value of 999×37can be obtained most conveniently as:
A) $1000 \times 37 + 37$
B) $1000 \times 37 - 37$
C) $100 \times 37 - 37$
D) $999 \times (40 - 3)$only
Answer: B) $1000 \times 37 - 37$
20. If $N \times 101 = 4545$, then $N$is:
A) 35
B) 45
C) 55
D) 405
Answer: B) 45, since $45 \times 101 = 4545$.
21. Which pair of terms can be directly combined as like terms?
A) $3x$and $5x^2$
B) $4ab$and $7ba$
C) $6a$and $6b$
D) $x^2y$and xy^2
Answer: B) $4ab$**and** $7ba$, because $ab = ba$.
22. If $(x+p)(x+q) = x^2 + 9x + 20$, which could be the values of $p$and q?
A) 2 and 10
B) 4 and 5
C) 3 and 6
D) 1 and 20
Answer: B) 4 and 5
23. If $x^2 - 14x + k$is a perfect square, the value of $k$is:
A) 7
B) 14
C) 28
D) 49
Answer: D) 49, because $x^2 - 14x + 49 = (x-7)^2$.
24. Which expression is equivalent to $k(k+2)$?
A) $(k+1)^2 + 1$
B) $(k+1)^2 - 1$
C) $(k-1)^2 - 1$
D) $k^2 + k + 2$
Answer: B) $(k+1)^2 - 1$, since both simplify to $k^2 + 2k$.
25. If the number of circles at Step $k$is $k^2 + 2k$, how many circles are present at Step 12?
A) 144
B) 156
C) 168
D) 172
Answer: C) 168, since $12^2 + 2(12) = 144 + 24 = 168$.

26. Which of the following is equal to $(m+n)^2 - 4mn$?
A) $(m+n)^2$
B) $m^2 + n^2$
C) $(m-n)^2$
D) $m^2 - n^2$
Answer: C) $(m-n)^2$
27. A student writes $5x^2 + 7x = 12x^2$. Why is the simplification incorrect?
A) The coefficients cannot be added.
B) $5x^2$and $7x$are unlike terms.
C) $x^2 = x$.
D) The answer should be $35x^3$.
Answer: B) $5x^2$and $7x$are unlike terms.
28. If $x + y = 13$and $x - y = 7$, what is xy?
A) 21
B) 24
C) 30
D) 42
Answer: C) 30, since $(x+y)^2 - (x-y)^2 = 4xy$, so $169 - 49 = 4xy$.
29. If $a^2 + b^2 = 50$and $ab = 24$, what is $(a-b)^2$?
A) 2
B) 4
C) 26
D) 98
Answer: A) 2, since $(a-b)^2 = a^2 + b^2 - 2ab = 50 - 48$.
30. A square has side $(x+3)$units. By how much is its area greater than that of a square of side $x$units?
A) $3x + 9$
B) $6x + 9$
C) $6x + 6$
D) $9x + 9$
Answer: B) $6x + 9$, since $(x+3)^2 - x^2 = 6x + 9$.

Answer Key:
1-B, 2-C, 3-C, 4-B, 5-C, 6-C, 7-B, 8-A, 9-B, 10-A, 11-C, 12-C, 13-C, 14-B, 15-B, 16-C, 17-B, 18-D, 19-B, 20-B, 21-B, 22-B, 23-D, 24-B, 25-C, 26-C, 27-B, 28-C, 29-A, 30-B.

6.15 Best Practices Opportunity List from the Chapter: (10 Ideas)

"Best Practices" Opportunities While Teaching Chapter 6 – We Distribute, Yet Things Multiply:

(1) **Concrete-to-Algebra Approach:** Begin distributivity with familiar numerical examples and rectangular/area models before moving to expressions such as $a(b+c) = ab + ac$. This helps students understand the meaning of expansion rather than merely memorising a rule.
(2) **Visual Area Models for Identities:** Use square grids, coloured paper, or algebra tiles to demonstrate $(a+b)^2 = a^2 + 2ab + b^2$. Students can physically identify the a^2, b^2, and two abregions.
(3) **Pattern–Observe–Generalise Method:** Let students calculate how a product changes when one or both factors are increased or decreased, record their observations in a table,

and then derive the general algebraic rule. This develops the ability to move from numerical patterns to algebraic generalisation.

(4) **Identity Discovery Instead of Rote Learning:** Encourage students to derive $(a+b)^2$, $(a-b)^2$, and $(a+b)(a-b)$through distributivity before asking them to use the identities in problems. This strengthens conceptual understanding and reduces dependence on memorisation.

(5) **Mental-Maths Challenge:** Regularly provide calculations such as multiplication by 11 or 101 and products of numbers close to convenient round numbers. Ask students to use distributivity and identities to develop faster and more flexible calculation strategies.

(6) **"Find and Fix the Error" Practice:** Give deliberately incorrect expansions or simplifications and ask students to locate, explain, and correct the mistake. This follows the chapter's **"Mind the Mistake, Mend the Mistake"** approach and develops mathematical reasoning and self-checking habits.

(7) **Emphasise Like-Term Recognition:** Before simplifying expanded expressions, train students to identify and group like terms, such as recognising that $ba = ab$and hence $ba + ab = 2ab$. This prevents common errors during algebraic simplification.

(8) **Multiple-Solution Strategy:** Present a pattern and ask different groups to derive its algebraic expression in different ways, then simplify and compare their answers. The chapter's circle pattern shows how several representations can simplify to the same $k^2 + 2k$, reinforcing that mathematics can have multiple valid approaches.

(9) **Peer Explanation and Think–Pair–Share:** After solving an identity or expansion problem individually, students should explain their method to a partner and compare approaches. This is especially suitable for the chapter's emphasis on viewing and solving the same mathematical situation in different ways.

(10) **Application-Based Exit Tickets:** End the lesson with one short problem requiring students to choose the most appropriate identity—for example, calculating $51^2 - 49^2$or expanding $(x+5)^2$. This provides a quick assessment of whether students can **select and apply**, rather than merely recall, the appropriate algebraic idea.

6.16 Innovations Opportunity List from the Chapter: (12 Ideas)

"Innovations" Opportunities While Teaching-Learning Chapter 6 – We Distribute, Yet Things Multiply:

(1) **Algebra-Tile Identity Laboratory:** Let students create movable paper or digital algebra tiles representing a^2, ab, and b^2, then rearrange them to visually discover identities such as $(a+b)^2 = a^2 + 2ab + b^2$. This converts an abstract identity into a visual construction.

(2) **"Identity Detective" Challenge:** Display expressions such as $x^2 + 10x + 25$, $9x^2 - 24xy + 16y^2$, and $a^2 - b^2$, and ask teams to identify the hidden algebraic identity. Students can earn points for identifying, explaining, and verifying each identity through expansion.

(3) **Human Distributive Property Activity:** Assign students the roles of a, b, c, multiplication, and addition and have them physically rearrange themselves to demonstrate $a(b+c) = ab + ac$. The activity can then progress from numerical examples to letter-number expressions used in the chapter.

(4) **Mental-Maths Speed Lab:** Organise timed challenges involving multiplication by 11, 101, 1001, or numbers close to 10, 100, and 1000. Students must explain the distributive strategy used rather than merely state the answer, encouraging mathematical flexibility.

(5) **"Mistake Museum" Classroom Wall:** Students can anonymously contribute incorrect algebraic solutions, after which the class identifies, explains, and repairs each error. This creatively extends the chapter's **"Mind the Mistake, Mend the Mistake"** activity into collaborative error analysis.

(6) **Digital Pattern-to-Algebra Exploration:** Use a spreadsheet or simple dynamic geometry tool to generate successive stages of a visual pattern and let students record the number of objects at each stage. They can then predict Step $k$and compare different expressions that simplify to the chapter's $k^2 + 2k$.

(7) **Multiple-Methods Mathematics Gallery:** Give different groups the same visual pattern or algebraic problem but require each group to solve it using a different representation—area model, distributivity, identity, numerical pattern, or algebraic expansion. Display the solutions together to reinforce the chapter's idea that mathematics often permits multiple ways of viewing and solving the same problem.

(8) **Design-Your-Own Algebra Puzzle:** Students create puzzles in which classmates must determine a missing term, coefficient, or factor—for example, $x^2 + \square x + 25 = (x + 5)^2$. Creating the puzzle requires students to understand the identity from both the expanded and factorised forms.

(9) **Real-Life Area Design Project:** Ask students to design a square garden, classroom floor, courtyard, or decorative border with dimensions such as $(a + b)$and represent its area in different algebraic forms. This naturally connects geometric area with identities such as $(a + b)^2$and $(a - b)^2$, reflecting the chapter's geometric treatment of identities.

(10) **Student-Created "Algebra Strategy Cards":** Each student prepares a card containing one identity, its visual meaning, one worked example, one common mistake, and one mental-maths application. Combining the cards creates a student-generated revision resource for the chapter.

(11) **QR-Based Algebra Challenge Stations:** Set up classroom stations containing QR codes that reveal progressively harder problems on expansion, like terms, identities, fast multiplication, and pattern generalisation. Students rotate in teams, solve each problem, and unlock the next challenge only after explaining their reasoning.

(12) **Ancient Mathematics Connection Corner:** Students can prepare a small timeline or presentation showing how distributive ideas appeared historically and how Brahmagupta described multiplication by parts as **khaṇḍa-guṇanam**. This integrates mathematics with the historical perspective presented in the chapter and can encourage interdisciplinary learning.

6.17: Practicing Questions/Question Bank

A. Fill in the Blanks:

1. $a(b + c) = ab + ac$.
2. $(a + 1)(b + 1) - ab = a + b + 1$.
3. $(a + m)(b + n) = ab + mb + an + mn$.
4. Terms containing exactly the same letter-numbers are called **like terms**.
5. $(a + b)^2 = a^2 + 2ab + b^2$.
6. $(a - b)^2 = a^2 - 2ab + b^2$.
7. $(a + b)(a - b) = a^2 - b^2$.

8. $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$.

9. $N \times 101 = 100N + N$.

10. If the circle pattern contains $k^2 + 2k$circles at Step k, Step 10 contains **120** circles.

B. One-Mark Descriptive Questions:

1. **What is the distributive property?**
Answer: The distributive property is $a(b + c) = ab + ac$.

2. **What are like terms?**
Answer: Terms having exactly the same letter-numbers are called like terms.

3. **Expand** $(x + 3)^2$**.**
Answer: $(x + 3)^2 = x^2 + 6x + 9$.

4. **Factorise** $x^2 - 25$**.**
Answer: $x^2 - 25 = (x + 5)(x - 5)$.

5. **Find** 99×101**using an identity.**
Answer: $99 \times 101 = (100 - 1)(100 + 1) = 9999$.

6. **Simplify** $5ab + 3ba$**.**
Answer: Since $ab = ba$, $5ab + 3ba = 8ab$.

7. **Write** $x^2 - 12x + 36$**as a perfect square.**
Answer: $x^2 - 12x + 36 = (x - 6)^2$.

8. **Find the number of circles at Step 8 if the rule is** $k^2 + 2k$**.**
Answer: Step 8 contains $8^2 + 2(8) = 80$circles.

C. Two-Mark Descriptive Questions:

1. **Expand and simplify** $(x + 4)(x + 6)$**.**
Answer: $(x + 4)(x + 6) = x^2 + 6x + 4x + 24$.
Hence, the simplified expression is $x^2 + 10x + 24$.

2. **Find** 102×98**using an algebraic identity.**
Answer: $102 \times 98 = (100 + 2)(100 - 2)$.
Therefore, $102 \times 98 = 100^2 - 2^2 = 9996$.

3. **If** $a + b = 12$**and** $a - b = 4$**, find** $a^2 - b^2$**.**
Answer: $a^2 - b^2 = (a + b)(a - b)$.
Hence, $a^2 - b^2 = 12 \times 4 = 48$.

4. **Correct the statement** $(x + 5)^2 = x^2 + 25$**.**
Answer: The middle term $2(x)(5) = 10x$has been omitted.
Therefore, $(x + 5)^2 = x^2 + 10x + 25$.

5. **Show that** $k(k + 2)$**and** $k^2 + 2k$**are equivalent.**
Answer: Applying distributivity gives $k(k + 2) = k^2 + 2k$.
Hence, both expressions represent the same quantity.

6. **Calculate** 56×11**using distributivity.**
Answer: $56 \times 11 = 56(10 + 1)$.
Therefore, $56 \times 11 = 560 + 56 = 616$.

D. Three-Mark Descriptive Questions:

1. **Derive the identity** $(a+b)^2 = a^2 + 2ab + b^2$**.**
   Answer: $(a+b)^2 = (a+b)(a+b)$.
   Applying distributivity gives $a^2 + ab + ab + b^2$.
   Combining like terms gives $a^2 + 2ab + b^2$.

2. **Calculate** 97^2**using the square-of-a-difference identity.**
   Answer: Write $97^2 = (100-3)^2$.
   Thus, $97^2 = 10000 - 600 + 9$.
   Therefore, $97^2 = 9409$.

3. **If** $a + b = 16$**and** $ab = 48$**, find** $a^2 + b^2$**.**
   Answer: Using $(a+b)^2 = a^2 + 2ab + b^2$, $256 = a^2 + b^2 + 96$.
   Therefore, $a^2 + b^2 = 256 - 96$.
   Hence, $a^2 + b^2 = 160$.

4. **Factorise** $64x^2 - 25y^2$**and explain the identity used.**
   Answer: $64x^2 - 25y^2 = (8x)^2 - (5y)^2$.
   Using $a^2 - b^2 = (a+b)(a-b)$, it factorises into two binomials.
   Therefore, $64x^2 - 25y^2 = (8x+5y)(8x-5y)$.

5. **Explain why** $(a-b)^2 = (b-a)^2$**.**
   Answer: Since $b - a = -(a-b)$, the two expressions differ only in sign before squaring.
   Squaring either expression removes this difference in sign.
   Therefore, $(a-b)^2 = (b-a)^2$.

E. Five-Mark Descriptive Questions:

1. **A square garden has side 96 m. Find its area using an algebraic identity instead of direct multiplication.**
   Answer:
   1. The area of the garden is 96^2.
   2. Write $96 = 100 - 4$.
   3. Using $(a-b)^2 = a^2 - 2ab + b^2$, $96^2 = 100^2 - 2(100)(4) + 4^2$.
   4. Thus, $96^2 = 10000 - 800 + 16 = 9216$.
   5. Therefore, the area is **9,216 m²**.

2. **A rectangular field measures** x**m by** y**m. Its length is increased by 4 m and width by 3 m. Find the increase in area.**
   Answer:
   1. The original area is xym².
   2. The new dimensions are $(x+4)$m and $(y+3)$m.
   3. The new area is $(x+4)(y+3) = xy + 3x + 4y + 12$.
   4. The increase is $(xy + 3x + 4y + 12) - xy$.
   5. Therefore, the area increases by $3x + 4y + 12$**m²**.

3. **Prove that $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$.**
   Answer:
   1. Expand $(a+b)^2$as $a^2 + 2ab + b^2$.
   2. Expand $(a-b)^2$as $a^2 - 2ab + b^2$.
   3. Adding them gives $2a^2 + 2b^2$, as $+2ab$and $-2ab$cancel.
   4. Factorising 2 gives $2(a^2+b^2)$.
   5. Hence, $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$.
4. **A student writes $(3x-4)^2 = 9x^2 - 16$. Analyse and correct the solution.**
   Answer:
   1. The student has omitted the middle term while squaring the binomial.
   2. The correct identity is $(a-b)^2 = a^2 - 2ab + b^2$.
   3. Therefore, $(3x-4)^2 = (3x)^2 - 2(3x)(4) + 4^2$.
   4. Simplifying gives $9x^2 - 24x + 16$.
   5. Hence, the correct answer is $9x^2 - 24x + 16$.
5. **Different students describe the same circle pattern by $k(k+2)$and $(k+1)^2 - 1$. Prove that both methods are correct.**
   Answer:
   1. Expanding the first expression gives $k(k+2) = k^2 + 2k$.
   2. Expanding the second gives $(k+1)^2 - 1 = k^2 + 2k + 1 - 1$.
   3. Therefore, the second expression also simplifies to $k^2 + 2k$.
   4. Hence, both expressions give the same number for every value of k.
   5. This demonstrates that **different ways of viewing a pattern can produce equivalent algebraic expressions**.

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8th Standard Maths 1 Book Chapter 7

Chapter 7: PROPORTIONAL REASONING-1

7.1 Summary of the Chapter:

Chapter 7 – Proportional Reasoning–1: Summary:

(1) **Proportional reasoning** helps us understand how two related quantities change while maintaining the same relationship. For example, an image remains similar when its width and height are changed by the **same multiplication factor**.

(2) A **ratio** compares two quantities and is written in the form **a : b**. Here, *a* and *b* are called the **terms of the ratio**, meaning that for every *a* units of the first quantity, there are *b* units of the second quantity.

(3) A ratio can be reduced to its **simplest form** by dividing both its terms by their **HCF (Highest Common Factor)**. For example, 60 : 40 simplifies to **3 : 2**.

(4) Two ratios are said to be **in proportion** when their simplest forms are the same. Proportion is represented using the symbol **::**, as in **a : b :: c : d**.

(5) In proportional relationships, both terms of a ratio must change by the **same factor**. Merely adding or subtracting the same number from both terms does not necessarily produce a proportional ratio.

(6) **Proportional reasoning** can be used to solve many practical problems involving quantities such as ingredients, distance, price, measurements and mixtures by identifying the common factor of change.

(7) The **Rule of Three (Trairāsika)** is used when three quantities in a proportional relationship are known and the fourth has to be determined. It was used in ancient Indian mathematics for solving problems involving proportionality.

(8) For a proportion **a : b :: c : d**, cross multiplication gives **ad = bc**. Therefore, an unknown fourth term can be calculated using **d = (b × c)/a**.

(9) Before forming a proportion, the quantities being compared should be expressed in **compatible units**. For example, hours may need to be converted into minutes before comparing time ratios.

(10) Ratios can also be used to **divide a whole quantity in a given ratio**. If a quantity **x** is divided in the ratio **m : n**, the two shares are **mx/(m+n)** and **nx/(m+n)**.

(11) Proportional reasoning has many **real-life applications**, such as sharing profits according to investment, preparing mixtures, scaling recipes, comparing prices, determining quantities of materials and making similar drawings.

(12) **Unit conversion** is important in proportional problems. The chapter introduces conversions involving **length, area, volume and temperature**, including Celsius–Fahrenheit conversions, to help solve practical mathematical situations.

7.2 Important Formulas:

Important Formulas for Solving Quantitative Problems:

1. **Ratio of two quantities**

$$\boxed{a : b = \frac{a}{b}}$$

Here, **a** and **b** are the two terms of the ratio.

2. **Simplifying a ratio**

$$\boxed{a : b = \frac{a}{HCF(a,b)} : \frac{b}{HCF(a,b)}}$$

Divide both terms by their **HCF** to obtain the simplest form.

3. **Proportion**

$$\boxed{a : b :: c : d}$$

Two ratios are proportional when their corresponding terms change by the **same factor** or their simplest forms are identical.

4. **Cross-multiplication rule**

$$\boxed{a : b :: c : d \Rightarrow ad = bc}$$

This is one of the most important formulas for solving proportional reasoning problems.

5. **Finding the fourth proportional**
   If

$$a : b :: c : x$$

then

$$\boxed{x = \frac{b \times c}{a}}$$

This is the **Rule of Three (Trairāsika)**.

6. **Factor of change**

$$\boxed{\text{Factor of change} = \frac{\text{New quantity}}{\text{Original quantity}}}$$

For proportional quantities, the corresponding quantity must change by the **same factor**.

7. **Dividing a quantity in a given ratio**

If a total quantity **x** is divided in the ratio **m : n**:

$$\boxed{\text{First part} = \frac{m}{m+n} \times x}$$

$$\boxed{\text{Second part} = \frac{n}{m+n} \times x}$$

8. **Value of one ratio-part**

$$\boxed{\text{Value of one part} = \frac{\text{Total quantity}}{m+n}}$$

Therefore,

$$\boxed{\text{First share} = m \times \text{value of one part}}$$

$$\boxed{\text{Second share} = n \times \text{value of one part}}$$

This method is useful for **profit sharing, mixtures, money and distribution problems**.

9. **Celsius to Fahrenheit**

$$\boxed{F = \frac{9}{5}C + 32}$$

10. **Fahrenheit to Celsius**

$$\boxed{C = \frac{5}{9}(F - 32)}$$

Important Unit Conversions

Quantity	**Formula / Conversion**
Length	$1 \text{ metre} = 3.281 \text{ feet}$
Area	$1 \text{ m}^2 = 10.764 \text{ ft}^2$
Area	$1 \text{ acre} = 43{,}560 \text{ ft}^2$
Area	$1 \text{ hectare} = 10{,}000 \text{ m}^2$
Area	$1 \text{ hectare} = 2.471 \text{ acres}$

Quantity Formula / Conversion

Volume 1 mL = 1 cm^3 = 1 cc

Volume 1 litre = 1,000 mL = 1,000 cc

These conversions are specifically provided in the chapter for solving proportional-reasoning problems involving different units.

Most important formulas to remember:

$$\boxed{a:b::c:d \Rightarrow ad = bc}$$

$$\boxed{\text{Fourth proportional} = \frac{\text{Second term} \times \text{Third term}}{\text{First term}}}$$

$$\boxed{\text{Parts when } x \text{ is divided in } m:n = \frac{mx}{m+n}, \frac{nx}{m+n}}$$

7.3 Fill-Up the Blank LOTS type questions with Answers: (35 Questions)

Fill in the Blanks – LOTS Type:

1. A comparison of two quantities using division is called a __________.
   Answer: Ratio
2. The numbers being compared in a ratio are called the __________ of the ratio.
   Answer: Terms
3. The ratio of width to height of Image A, having width 60 mm and height 40 mm, is __________.
   Answer: 60 : 40
4. The simplest form of the ratio 60 : 40 is __________.
   Answer: 3 : 2
5. To reduce a ratio to its simplest form, we divide both terms by their __________.
   Answer: HCF
6. Two ratios having the same simplest form are said to be in __________.
   Answer: Proportion
7. The symbol used to indicate that two ratios are proportional is __________.
   Answer: ::
8. In proportional ratios, corresponding terms change by the __________ factor.
   Answer: Same
9. The ratios 3 : 4 and 72 : 96 are __________.
   Answer: Proportional
10. The simplest form of the ratio 72 : 96 is __________.
    Answer: 3 : 4
11. If $a:b::c:d$, then by cross multiplication, $ad =$__________.
    Answer: bc
12. The method of finding an unknown fourth quantity when three proportional quantities are known is called the __________.
    Answer: Rule of Three (Trairāsika)
13. If 6 glasses of lemonade require 10 spoons of sugar, 18 glasses of lemonade require __________ spoons to maintain the same sweetness.
    Answer: 30 spoons
14. The simplest form of the ratio 14 : 21 is __________.
    Answer: 2 : 3
15. The proportional ratio 14: 21: : 28:__________ is completed by 42.
    Answer: 42

16. If 120 students require 15 kg of rice, 80 students require __________ kg of rice.
    Answer: 10 kg
17. A car travelling 90 km in 150 minutes will travel __________ km in 240 minutes at the same speed.
    Answer: 144 km
18. Before comparing quantities using ratios, quantities of the same kind should be expressed in the __________ units.
    Answer: Same
19. When 12 counters are divided in the ratio 3 : 1, the two shares are __________ and __________.
    Answer: 9 and 3
20. When 42 counters are divided in the ratio 4 : 3, the two shares are __________ and __________.
    Answer: 24 and 18
21. If a quantity $x$is divided in the ratio m: n, the total number of ratio parts is __________.
    Answer: $m + n$
22. If a quantity $x$is divided in the ratio m: n, the first part is __________.
    Answer: $\frac{m}{m+n} \times x$
23. If a quantity $x$is divided in the ratio m: n, the second part is __________.
    Answer: $\frac{n}{m+n} \times x$
24. Prashanti and Bhuvan invested ₹75,000 and ₹25,000 respectively. Their investment ratio in simplest form is __________.
    Answer: 3 : 1
25. If a profit of ₹4,000 is divided in the ratio 3 : 1, Prashanti receives **₹**__________.
    Answer: ₹3,000
26. In a 40 kg mixture of sand and cement in the ratio 3 : 1, the quantity of sand is __________ kg.
    Answer: 30 kg
27. In the same 40 kg mixture with sand and cement in the ratio 3 : 1, the quantity of cement is __________ kg.
    Answer: 10 kg
28. One millilitre is equal to one cubic centimetre or one __________.
    Answer: cc
29. One litre is equal to __________ millilitres.
    Answer: 1,000
30. One hectare is equal to __________ square metres.
    Answer: 10,000
31. One acre is equal to __________ square feet.
    Answer: 43,560
32. One metre is approximately equal to __________ feet.
    Answer: 3.281
33. The formula for converting Celsius to Fahrenheit is $F =$__________.
    Answer: $\frac{9}{5}C + 32$
34. The formula for converting Fahrenheit to Celsius is $C =$__________.
    Answer: $\frac{5}{9}(F - 32)$
35. A temperature of $25°C$is equal to __________**°F**.
    Answer: 77°F

These questions cover the chapter's basic recall and understanding areas—**ratios, simplest form, proportion, Rule of Three, cross multiplication, sharing in a given ratio, proportional applications, and unit conversions**.

7.4 Fill-Up the Blank HOTS type questions with Answers: (30 Questions)

Fill in the Blanks with Answers – HOTS Type:

1. A photograph has width and height in the ratio **3 : 2**. If its new width is 120 cm, its height should be __________ **cm** to retain the same shape.
   Answer: 80 cm
2. A rectangle measuring 60 cm × 40 cm is enlarged proportionally to a width of 150 cm. Its new height will be __________ **cm**.
   Answer: 100 cm
3. If $8:12::x:30$, then the value of $x$is __________.
   Answer: 20
4. If $15:25::21:x$, then $x =$__________.
   Answer: 35
5. The ratio 18 : 24 is proportional to $27:x$. Therefore, $x =$__________.
   Answer: 36
6. If both terms of the ratio 7 : 9 are multiplied by 5, the resulting proportional ratio is __________.
   Answer: 35 : 45
7. A student claims that 12 : 18 and 20 : 30 are proportional. Since both ratios simplify to __________, the student's claim is correct.
   Answer: 2 : 3
8. If $a:b::c:d$, then the equality obtained by cross multiplication is __________.
   Answer: $ad = bc$
9. If 9 notebooks cost ₹135 and the price is proportional to the number of notebooks, 15 notebooks will cost **₹**__________.
   Answer: ₹225
10. A recipe requires flour and sugar in the ratio 4 : 1. If 600 g of flour is used, __________ **g** of sugar is required to maintain the same ratio.
    Answer: 150 g
11. A fruit drink contains orange juice and apple juice in the ratio 2 : 3. If 1,200 mL of apple juice is used, the quantity of orange juice required is __________ **mL**.
    Answer: 800 mL
12. If 120 students require 15 kg of rice, then 200 students will require __________ **kg** of rice, assuming the quantity of rice is directly proportional to the number of students.
    Answer: 25 kg
    This applies the same proportional reasoning used in the chapter's mid-day meal example.
13. A car travels 90 km in 150 minutes at a constant speed. In 5 hours, it will travel __________ **km**.
    Answer: 180 km
14. If a car covers 144 km in 240 minutes at constant speed, it will cover __________ **km** in 100 minutes.
    Answer: 60 km
    The chapter demonstrates this time-distance proportional relationship using the 90 km in 150 minutes example.

15. ₹5,400 is divided between two students in the ratio 4 : 5. The larger share is ₹__________.
    Answer: ₹3,000
16. If ₹7,200 is divided in the ratio 5 : 3, the difference between the two shares is ₹__________.
    Answer: ₹1,800
17. A total quantity of 84 kg is divided in the ratio 3 : 4. The smaller part is __________ **kg**.
    Answer: 36 kg
18. If 72 chocolates are shared between A and B in the ratio 5 : 3, A receives __________ **chocolates**.
    Answer: 45 chocolates
19. Two partners invest money in the ratio 3 : 2. If they earn a profit of ₹25,000 and divide it according to their investments, the second partner receives ₹__________.
    Answer: ₹10,000
    The chapter applies this principle to sharing business profit in the ratio of investment.
20. A 48 kg mixture contains two substances in the ratio 5 : 3. The quantities of the two substances are __________ **kg and** __________ **kg**.
    Answer: 30 kg and 18 kg
21. Sand and cement are mixed in the ratio 3 : 1. If there are 45 kg of sand, the amount of cement required is __________ **kg**.
    Answer: 15 kg
22. A 40 kg mixture contains sand and cement in the ratio 3 : 1. If the sand remains unchanged and the new sand-to-cement ratio is 5 : 2, the amount of cement that must be added is __________ **kg**.
    Answer: 2 kg
23. A student says that adding 5 to both terms of the ratio 3 : 5 gives a proportional ratio 8 : 10. Since $3:5 \neq 8:10$, the student's statement is __________.
    Answer: Incorrect
    Adding the same quantity to both terms does not necessarily preserve a ratio.
24. If Neelima and her mother's ages are initially 3 and 30 years, their ratio is 1 : 10. Nine years later, their ages are 12 and 39, giving the simplified ratio __________.
    Answer: 4 : 13
25. A tea packet weighing 200 g costs ₹200. At the same rate, 1 kg of the tea would cost ₹__________.
    Answer: ₹1,000
26. If another 1 kg packet of tea costs ₹800, then compared with the ₹200-per-200-g tea, the cheaper tea is the one costing ₹__________ **per kg**.
    Answer: ₹800
27. If 1 metre = 3.281 feet, then 5 metres is approximately __________ **feet**.
    Answer: 16.405 feet
28. Since 1 hectare = 10,000 m², an area of 2.5 hectares is __________ **m²**.
    Answer: 25,000 m²
29. A 10-litre bucket has a capacity of __________ **mL**.
    Answer: 10,000 mL
30. A tap fills 500 mL in 15 seconds. At the same rate, it will take __________ **seconds** to fill a 10-litre bucket.
    Answer: 300 seconds
    This quantitative situation is included among the chapter exercises.

31. Using $F = \frac{9}{5}C + 32$, a temperature of $40^\circ C$ is equal to __________ **°F**.
 Answer: 104°F
32. Using $C = \frac{5}{9}(F - 32)$, a temperature of $95^\circ F$ is equal to __________ **°C**.
 Answer: 35°C
33. If the ratio of coffee decoction to milk is 15 : 35, its simplest form is __________.
 Answer: 3 : 7
34. Coffee prepared in the ratio 20 : 30 has a greater proportion of decoction than coffee prepared in the ratio 15 : 35. Therefore, the 20 : 30 mixture will taste __________.
 Answer: Stronger
35. A person travels a fixed distance at a higher speed. The time required will __________, so speed and time for a fixed distance should not be treated as the direct proportion used in the chapter's Rule of Three examples.
 Answer: Decrease

7.5 One-mark questions with Answers of LOTS Type: (35 Questions)

One-Mark Descriptive Questions with Answers – LOTS Type:

1. **What is a ratio?**
 Answer: A ratio is a mathematical way of comparing two quantities and is written in the form **a : b**.
2. **What are the numbers in a ratio called?**
 Answer: The numbers being compared in a ratio are called the **terms of the ratio**.
3. **What does the ratio a : b represent?**
 Answer: The ratio **a : b** means that for every **a** units of the first quantity, there are **b** units of the second quantity.
4. **What is meant by proportional change?**
 Answer: A change is proportional when the related quantities change by the **same multiplication factor**.
5. **How can a ratio be reduced to its simplest form?**
 Answer: A ratio is reduced to its simplest form by dividing both terms by their **HCF**.
6. **What is the simplest form of 60 : 40?**
 Answer: The simplest form of **60 : 40 is 3 : 2**.
7. **When are two ratios said to be in proportion?**
 Answer: Two ratios are in proportion when their **simplest forms are the same**.
8. **Which symbol is used to represent proportion?**
 Answer: The symbol **::** is used to indicate that two ratios are proportional.
9. **Write the general form of a proportion.**
 Answer: The general form of a proportion is **a : b :: c : d**.
10. **What is the cross-multiplication rule for a : b :: c : d?**
 Answer: If **a : b :: c : d**, then **ad = bc**.
11. **What is meant by the Rule of Three?**
 Answer: The **Rule of Three** is a method used to find a fourth unknown quantity when three quantities in a proportional relationship are known.
12. **What was the Rule of Three called in ancient Indian mathematics?**
 Answer: The Rule of Three was called **Trairāsika** in ancient Indian mathematics.
13. **What is the simplest form of the ratio 14 : 21?**
 Answer: The simplest form of **14 : 21 is 2 : 3**.
14. **How many spoons of sugar are required for 18 glasses of lemonade if 6 glasses require 10 spoons?**
 Answer: 30 spoons of sugar are required for 18 glasses of lemonade.

15. **What happens to a ratio when the same number is added to both its terms?**
    Answer: Adding the same number to both terms **changes the ratio and does not necessarily produce a proportional ratio**.
16. **How much rice is required for 80 students if 15 kg is required for 120 students?**
    Answer: 10 kg of rice is required for 80 students.
17. **What distance does a car cover in 4 hours if it covers 90 km in 150 minutes at the same speed?**
    Answer: The car covers **144 km in 4 hours**.
18. **Why should units be made the same before forming a proportion?**
    Answer: Units should be made the same so that the corresponding quantities can be **compared correctly**.
19. **What is the ratio of equal sharing between two people?**
    Answer: The ratio of equal sharing between two people is **1 : 1**.
20. **How is 12 divided in the ratio 3 : 1?**
    Answer: When 12 is divided in the ratio **3 : 1**, the two parts are **9 and 3**.
21. **How is 42 divided in the ratio 4 : 3?**
    Answer: When 42 is divided in the ratio **4 : 3**, the two parts are **24 and 18**.
22. **What is the formula for the first part when x is divided in the ratio m : n?**
    Answer: The first part is $\frac{m}{m+n} \times x$.
23. **What is the formula for the second part when x is divided in the ratio m : n?**
    Answer: The second part is $\frac{n}{m+n} \times x$.
24. **In what ratio did Prashanti and Bhuvan invest in their food cart business?**
    Answer: Prashanti and Bhuvan invested in the simplified ratio **3 : 1**.
25. **How was a profit of ₹4,000 shared between Prashanti and Bhuvan?**
    Answer: Prashanti received **₹3,000** and Bhuvan received **₹1,000**.
26. **What is the ratio of sand to cement in the 40 kg mixture discussed in the chapter?**
    Answer: The ratio of sand to cement in the mixture is **3 : 1**.
27. **How much sand is present in a 40 kg mixture with sand and cement in the ratio 3 : 1?**
    Answer: The mixture contains **30 kg of sand**.
28. **How many millilitres are there in one litre?**
    Answer: There are **1,000 millilitres in one litre**.
29. **How many square metres are there in one hectare?**
    Answer: There are **10,000 square metres in one hectare**.
30. **How many square feet are there in one acre?**
    Answer: There are **43,560 square feet in one acre**.
31. **What is the relation between a millilitre and a cubic centimetre?**
    Answer: 1 millilitre is equal to 1 cubic centimetre (1 cc).
32. **Write the formula for converting Celsius into Fahrenheit.**
    Answer: The formula is $F = \frac{9}{5}C + 32$.
33. **Write the formula for converting Fahrenheit into Celsius.**
    Answer: The formula is $C = \frac{5}{9}(F - 32)$.
34. **What is 25°C in Fahrenheit?**
    Answer: 25°C is equal to 77°F.
35. **What is the simplest ratio of 600 mL of orange juice to 900 mL of apple juice?**
    Answer: The simplest ratio of orange juice to apple juice is **2 : 3**.

7.6 One-mark questions with Answers of HOTS Type: (30 Questions)

One-Mark Descriptive Questions with Answers – HOTS Type:

1. **A photograph of size 60 mm × 40 mm is resized to 90 mm × 60 mm. Will its shape remain similar? Why?**
   Answer: Yes, because both width and height are multiplied by the same factor of **1.5**.
2. **Why does a 40 mm × 20 mm image not remain similar to a 60 mm × 40 mm image?**
   Answer: It is not similar because its width and height have **not changed by the same factor**.
3. **Are the ratios 18 : 24 and 27 : 36 proportional? Give a reason.**
   Answer: Yes, because both ratios simplify to **3 : 4**.
4. **If 5 : 8 :: 15 : x, what is the value of x?**
   Answer: The value of **x is 24**, because both terms must increase by the same factor.
5. **A student says 6 : 9 and 10 : 15 are not proportional because their terms are different; is the student correct?**
   Answer: No, because both ratios simplify to **2 : 3** and are therefore proportional.
6. **Can 4 : 7 and 12 : 21 form a proportion? Justify your answer.**
   Answer: Yes, because multiplying both terms of **4 : 7 by 3** gives **12 : 21**.
7. **If both terms of 8 : 12 are doubled, does the ratio change? Why?**
   Answer: No, because multiplying both terms by the same non-zero factor produces an **equivalent ratio**.
8. **Why does adding 5 to both terms of 5 : 10 not preserve the original ratio?**
   Answer: Adding the same number gives **10 : 15 = 2 : 3**, which is different from the original **1 : 2**.
9. **If 6 glasses of lemonade need 10 spoons of sugar, how many spoons are required for 12 glasses with the same sweetness?**
   Answer: 20 spoons are required because the number of glasses has doubled.
10. **Why must the sugar quantity change proportionally when more lemonade is prepared?**
    Answer: The sugar must change proportionally to maintain the **same sweetness or glasses-to-sugar ratio**.
11. **If 120 students need 15 kg of rice, how much rice is required for 40 students?**
    Answer: 5 kg of rice is required because 40 is one-third of 120.
12. **Why can cross multiplication be used to test whether a : b and c : d are proportional?**
    Answer: Cross multiplication works because proportional ratios satisfy the condition **ad = bc**.
13. **If 12 : 18 :: x : 30, find x.**
    Answer: x = 20, since $12 \times 30 = 18 \times 20$.
14. **A car travels 90 km in 150 minutes; how far will it travel in 300 minutes at the same speed?**
    Answer: The car will travel **180 km**, because the travel time has doubled.
15. **Why should 4 hours be converted into 240 minutes before comparing it with 150 minutes?**
    Answer: The conversion is necessary because corresponding quantities must be expressed in the **same units** before forming the proportion.
16. **If 200 g of tea costs ₹200, what should 500 g cost at the same proportional rate?**
    Answer: 500 g should cost ₹500 at the same proportional rate.

17. **Tea costs ₹1,000 per kg in one place and ₹800 per kg in another; which is more expensive?**
Answer: The tea costing **₹1,000 per kg** is more expensive because the prices are compared for the same weight.
18. **Why is comparing ₹200 for 200 g directly with ₹800 for 1 kg misleading?**
Answer: It is misleading because the prices correspond to **different weights**, so the weights must first be made equal.
19. **If 12 objects are shared in the ratio 3 : 1, why does the larger share equal 9?**
Answer: The larger share is 9 because $12 \times \frac{3}{3+1} = 9$.
20. **If 42 counters are divided in the ratio 4 : 3, how many counters are in one ratio-part?**
Answer: One ratio-part contains **6 counters**, because $42 \div (4 + 3) = 6$.
21. **If ₹6,000 is shared in the ratio 2 : 1, what is the larger share?**
Answer: The larger share is **₹4,000**, because it represents two of the three total ratio-parts.
22. **Two partners invest in the ratio 3 : 1; what fraction of the total profit should the first partner receive?**
Answer: The first partner should receive $\frac{3}{4}$of the total profit.
23. **A 40 kg sand-cement mixture is in the ratio 3 : 1; how much sand does it contain?**
Answer: It contains **30 kg of sand**, because sand represents three-fourths of the mixture.
24. **Why does adding 2 kg of cement to the chapter's 40 kg mixture change the sand-cement ratio from 3 : 1 to 5 : 2?**
Answer: The quantities become **30 kg sand and 12 kg cement**, whose ratio simplifies to **5 : 2**.
25. **If orange juice and apple juice are mixed in the ratio 2 : 3, how much apple juice is needed with 400 mL of orange juice?**
Answer: 600 mL of apple juice is needed to maintain the ratio 2 : 3.
26. **Why is coffee mixed in the ratio 20 : 30 stronger than coffee mixed in the ratio 15 : 35?**
Answer: The **20 : 30 mixture contains a greater proportion of coffee decoction relative to milk**.
27. **If a 10-litre bucket is filled at the same rate as a 500 mL mug, how many mugfuls are equivalent to one bucket?**
Answer: The bucket is equivalent to **20 mugfuls**, since 10 litres equals 10,000 mL.
28. **If a tap fills 500 mL in 15 seconds, how long will it take to fill 1 litre at the same rate?**
Answer: It will take **30 seconds**, because 1 litre is twice 500 mL.
29. **A vehicle's speed increases from 50 km/h to 75 km/h for the same journey; should the travel time increase proportionally?**
Answer: No, the travel time should **decrease as the speed increases** for the same distance.
30. **Why can the speed-time situation** $50:2::75:x$**not be treated as the direct proportion used in the Rule of Three examples?**
Answer: It cannot be treated as direct proportion because for a fixed distance, **increasing speed decreases travel time**.

7.7 Two-mark questions with Answers of LOTS Type: (25 Questions)

Two-Mark Descriptive Questions with Answers – LOTS Type:

1. What is a ratio? Explain with an example.

Answer:

1. A **ratio** compares two quantities and is written in the form **a : b**.
2. For example, the ratio of width to height of an image measuring 60 mm × 40 mm is **60 : 40**.

2. What are the terms of a ratio? Give an example.

Answer:

1. The two numbers used in a ratio are called the **terms of the ratio**.
2. In the ratio **60 : 40**, 60 and 40 are its two terms.

3. How do you reduce a ratio to its simplest form?

Answer:

1. Find the **HCF** of the two terms of the ratio.
2. Divide both terms by their HCF; for example, $60:40 = 3:2$.

4. When are two ratios said to be proportional?

Answer:

1. Two ratios are proportional when they have the **same simplest form**.
2. For example, **60 : 40** and **30 : 20** are proportional because both simplify to **3 : 2**.

5. What does the notation $a:b::c:d$mean?

Answer:

1. The notation $a:b::c:d$means that the ratios **a : b** and **c : d** are proportional.
2. In such a proportion, the corresponding terms change by the **same factor**.

6. Check whether 3 : 4 and 72 : 96 are proportional.

Answer:

1. Dividing 72 and 96 by their HCF 24 gives **3 : 4**.
2. Therefore, **3 : 4 and 72 : 96 are proportional**.

7. Explain how the same sweetness is maintained while increasing the quantity of lemonade.

Answer:

1. The ratio of the number of glasses of lemonade to the number of spoons of sugar must remain **proportional**.
2. Since 6 glasses require 10 spoons, 18 glasses require **30 spoons of sugar**.

8. What happens when the same number is added to both terms of a ratio?

Answer:

1. Adding the same number to both terms generally **changes the ratio**.
2. Therefore, the new ratio is not necessarily proportional to the original ratio.

9. Find the missing term in $14:21::28:x$.

Answer:

1. Since 14 is multiplied by 2 to obtain 28, 21 must also be multiplied by 2.
2. Therefore, $x = 21 \times 2 = 42$.

10. What is the Rule of Three (Trairāsika)?

Answer:

1. The **Rule of Three** is used when three quantities in a proportional relationship are known and the fourth is unknown.
2. The unknown quantity is found by maintaining the same proportional relationship between the quantities.

11. State the cross-multiplication rule for two proportional ratios.

Answer:

1. If $a:b::c:d$, then by cross multiplication $ad = bc$.

2. Hence, the unknown fourth term can be calculated as $d = \frac{bc}{a}$.

12. How much rice should be cooked for 80 students if 15 kg is required for 120 students?

Answer:

1. The number of students decreases by a factor of $\frac{80}{120} = \frac{2}{3}$.
2. Therefore, the rice required is $15 \times \frac{2}{3} = 10\ kg$.

13. Why should units be the same before forming a proportion? Give an example.

Answer:

1. Corresponding quantities must be expressed in the **same units** for a valid comparison.
2. For example, 4 hours must be converted to **240 minutes** before comparing it with 150 minutes.

14. How do you divide a quantity $x$in the ratio $m\!:n$?

Answer:

1. The first part is $\frac{m}{m+n} \times x$.
2. The second part is $\frac{n}{m+n} \times x$.

15. Divide 12 counters in the ratio 3 : 1.

Answer:

1. The total number of ratio-parts is $3 + 1 = 4$, so each part contains $12 \div 4 = 3$counters.
2. Therefore, the two shares are **9 counters and 3 counters**.

16. Divide 42 counters in the ratio 4 : 3.

Answer:

1. There are $4 + 3 = 7$parts, so each part contains $42 \div 7 = 6$counters.
2. Therefore, the two shares are **24 counters and 18 counters**.

17. How did Prashanti and Bhuvan share their ₹4,000 profit?

Answer:

1. Their investments of ₹75,000 and ₹25,000 are in the simplified ratio **3 : 1**.
2. Hence, Prashanti received **₹3,000** and Bhuvan received **₹1,000**.

18. Find the quantities of sand and cement in a 40 kg mixture in the ratio 3 : 1.

Answer:

1. Sand $= \frac{3}{4} \times 40 = 30\ kg$.
2. Cement $= \frac{1}{4} \times 40 = 10\ kg$.

19. State any two important conversions related to area given in the chapter.

Answer:

1. **1 acre = 43,560 square feet.**
2. **1 hectare = 10,000 square metres.**

20. State the relationship between litre, millilitre and cubic centimetre.

Answer:

1. **1 mL = 1 cubic centimetre (cc).**
2. **1 litre = 1,000 mL = 1,000 cc.**

21. Write the formulas for converting Celsius and Fahrenheit temperatures.

Answer:

1. Celsius to Fahrenheit: $F = \frac{9}{5}C + 32$.
2. Fahrenheit to Celsius: $C = \frac{5}{9}(F - 32)$.

22. What is the difference between regular and stronger filter coffee described in the chapter?
Answer:
1. Regular coffee uses decoction and milk in the ratio **15 : 35**.
2. Stronger coffee uses the ratio **20 : 30**, giving a greater proportion of coffee decoction.

23. Why are the walls built by Nitin and Hari considered equally strong in the example?
Answer:
1. Nitin's length-to-cement ratio $60:3$simplifies to **20 : 1**, while Hari's $40:2$also simplifies to **20 : 1**.
2. Since the ratios are proportional, the chapter concludes that the two walls are **equally strong**.

24. How can proportionality of two ratios be checked using their simplest forms?
Answer:
1. Reduce each ratio to its **simplest form** by dividing its terms by their HCF.
2. If the resulting simplest ratios are identical, the original ratios are **proportional**.

25. Why can the speed-time problem $50:2::75:x$**not be treated as a direct proportion?**
Answer:
1. For the same distance, increasing the speed causes the travel time to **decrease**.
2. Therefore, speed and time in this situation do not follow the direct proportional relationship used in the Rule of Three examples.

7.8. Two-mark questions with Answers of HOTS Type: (15 Questions)

Two-Mark Descriptive Questions with Answers – HOTS Type:

1. A photograph measuring 60 mm × 40 mm is resized to 120 mm × 80 mm. Will the new photograph look similar to the original? Justify.
Answer:
1. The width and height are both multiplied by the same factor **2**.
2. Therefore, the dimensions remain proportional and the new photograph will look **similar to the original**.

2. A student says that 40 : 20 and 60 : 40 are proportional because the difference between their terms is 20. Is the student correct?
Answer:
1. No, because $40:20 = 2:1$, whereas $60:40 = 3:2$.
2. Equal differences do not establish proportion; the corresponding terms must change by the **same multiplication factor**.

3. Without using decimals, determine whether 18 : 24 and 27 : 36 are proportional.
Answer:
1. Simplifying gives $18:24 = 3:4$and $27:36 = 3:4$.
2. Since their simplest forms are identical, the two ratios are **proportional**.

4. Find the missing term in $15:25::21:x$**and justify your answer.**
Answer:
1. By cross multiplication, $15x = 25 \times 21 = 525$.
2. Therefore, $x = 525 \div 15 = 35$.

5. A student changes the ratio 4 : 7 to 12 : 20 and claims that it is proportional. Evaluate the claim.
Answer:
1. The first term is multiplied by 3, but the second should then be $7 \times 3 = 21$, not 20.
2. Therefore, **12 : 20 is not proportional** to 4 : 7.

6. Six glasses of lemonade require 10 spoons of sugar. How much sugar is required for 24 glasses to retain the same sweetness?

Answer:

1. The number of glasses increases by a factor of $24 \div 6 = 4$.
2. Hence, the sugar must also increase by the same factor, giving $10 \times 4 = 40$spoons.

7. Why does adding the same number to both terms of a ratio not necessarily preserve proportionality? Illustrate using 3 : 6.

Answer:

1. Adding 3 to both terms gives $6:9 = 2:3$, whereas $3:6 = 1:2$.
2. Thus, equal addition changes the multiplicative relationship and does **not necessarily preserve the ratio**.

8. For 120 students, 15 kg of rice is prepared. How much rice should be prepared for 200 students?

Answer:

1. Using proportion, $120:15::200:x$, so $x = \frac{15 \times 200}{120}$.
2. Therefore, **25 kg of rice** should be prepared.

9. A car covers 90 km in 150 minutes. How far will it travel in 5 hours at the same speed?

Answer:

1. Five hours equals 300 minutes, so $150:90::300:x$.
2. Since the time doubles, the distance also doubles; hence the car travels **180 km**.

10. Why is it incorrect to directly write $150:90::4:x$when a car covers 90 km in 150 minutes and travels for 4 hours?

Answer:

1. The time quantities are expressed in different units—**150 minutes and 4 hours**.
2. We must first convert 4 hours to **240 minutes** before forming the proportion.

11. A 200 g tea packet costs ₹200, while a 1 kg packet of another tea costs ₹800. Which tea is more expensive for the same weight?

Answer:

1. At the first rate, 1 kg costs $5 \times ₹200 = ₹1{,}000$, while the second tea costs ₹800 per kg.
2. Therefore, the **first tea is more expensive by ₹200 per kg**.

12. Divide ₹4,500 in the ratio 2 : 3 and explain how you obtained the shares.

Answer:

1. There are $2 + 3 = 5$equal ratio-parts, so each part is $₹4{,}500 \div 5 = ₹900$.
2. Therefore, the two shares are **₹1,800 and ₹2,700**.

13. A 240 mL solution contains acid and water in the ratio 1 : 5. Find the quantity of each.

Answer:

1. There are $1 + 5 = 6$parts, so each part equals $240 \div 6 = 40$mL.
2. Therefore, the solution contains **40 mL acid and 200 mL water**.

14. Blue and yellow paints are mixed in the ratio 3 : 5 to make 40 mL of green paint. Find the quantity of each colour.

Answer:

1. The total number of ratio-parts is 8, so each part represents $40 \div 8 = 5$mL.
2. Hence, **15 mL of blue paint and 25 mL of yellow paint** are required.

15. Rice and urad dal are mixed in the ratio 2 : 1. How much of each is required to prepare 6 cups of mixture?

Answer:

1. The three total ratio-parts represent 6 cups, so each part equals **2 cups**.
2. Therefore, **4 cups of rice and 2 cups of urad dal** are required.

16. Prashanti invests ₹75,000 and Bhuvan ₹25,000 in a business. Why should a ₹12,000 profit be shared as ₹9,000 and ₹3,000?

Answer:

1. Their investments simplify to the ratio $75{,}000 : 25{,}000 = 3 : 1$.
2. Dividing ₹12,000 in the ratio 3 : 1 gives **₹9,000 to Prashanti and ₹3,000 to Bhuvan**.

17. A 40 kg mixture contains sand and cement in the ratio 3 : 1. If 2 kg of cement is added, show that the new ratio is 5 : 2.

Answer:

1. Initially, the mixture contains **30 kg sand and 10 kg cement**, so after adding 2 kg cement the quantities are 30 kg and 12 kg.
2. The new ratio $30 : 12$simplifies to **5 : 2**.

18. A drink contains 600 mL orange juice and 900 mL apple juice. If 300 mL more orange juice is added, what is the new ratio?

Answer:

1. The new quantity of orange juice is $600 + 300 = 900$mL, while apple juice remains 900 mL.
2. Therefore, the new ratio is $900 : 900 = 1 : 1$.

19. A tap fills a 500 mL mug in 15 seconds. How long will it take to fill a 10-litre bucket at the same rate?

Answer:

1. Since 10 litres = 10,000 mL, the bucket holds $10{,}000 \div 500 = 20$mugfuls.
2. Therefore, the tap takes $20 \times 15 = 300$seconds, or **5 minutes**.

20. A ₹10 coin has a mass of 7.74 g and contains copper and nickel in the ratio 3 : 1. Find the mass of each metal.

Answer:

1. Four ratio-parts represent 7.74 g, so one part is $7.74 \div 4 = 1.935$g.
2. Hence, the coin contains **5.805 g of copper and 1.935 g of nickel**.

21. A student's age and her brother's age are 1 year and 5 years respectively. After how many years will their ages be in the ratio 1 : 2?

Answer:

1. After 3 years, their ages will be **4 years and 8 years** respectively.
2. Since $4 : 8 = 1 : 2$, the required time is **3 years**.

22. A farmer needs 10 tonnes of manure for one acre. Why must the area of a rectangular field measured in feet first be converted into acres before calculating manure?

Answer:

1. The manure requirement is stated **per acre**, while the field dimensions give its area in square feet.
2. Therefore, the area must be converted using **1 acre = 43,560 ft²** before applying proportional reasoning.

23. A motorcycle takes 2 hours at 50 km/h for a fixed journey. Why is $50 : 2 :: 75 : x$**not an appropriate direct proportion?**

Answer:

1. For a fixed distance, an increase in speed causes the required travel time to **decrease rather than increase**.
2. Hence, speed and time in this situation do not follow the **direct proportional relationship** represented by that proportion.

24. Two rectangles have dimensions 12 cm × 8 cm and 18 cm × 12 cm. Are they similar? Give a reason.
Answer:
1. Their width-to-height ratios are $12: 8 = 3: 2$and $18: 12 = 3: 2$.
2. Since the ratios are proportional, the two rectangles are **similar in shape**.

25. Regular filter coffee has decoction and milk in the ratio 15 : 35, while another cup has the ratio 20 : 30. Which cup is stronger and why?
Answer:
1. The ratios simplify to **3 : 7** and **2 : 3**, respectively, so the second mixture has more decoction relative to milk.
2. Therefore, the coffee prepared in the ratio **20 : 30 is stronger**.

7.9. Three-mark questions with Answers of LOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – LOTS Type:
1. What is a ratio? Explain how a ratio can be written in its simplest form.
Answer:
1. A **ratio** compares two quantities and is written in the form **a : b**, where a and b are its terms.
2. To obtain the simplest form, divide both terms of the ratio by their **HCF**.
3. For example, $60: 40$becomes $3: 2$after dividing both terms by 20.

2. Explain when two ratios are said to be proportional.
Answer:
1. Two ratios are said to be **proportional** when their simplest forms are the same.
2. Proportional ratios are represented using the symbol **::**, as in $a: b: : c: d$.
3. For example, $60: 40: : 30: 20$, because both ratios simplify to **3 : 2**.

3. Explain why Images A, C and D discussed in the chapter appear similar.
Answer:
1. Image A has dimensions 60 mm × 40 mm, Image C has 30 mm × 20 mm, and Image D has 90 mm × 60 mm.
2. Their width-to-height ratios all simplify to **3 : 2**.
3. Hence, their widths and heights have changed by the same factor, making the images **proportional and similar**.

4. Show that the ratios 3 : 4 and 72 : 96 are proportional.
Answer:
1. The ratio **3 : 4** is already in its simplest form.
2. The HCF of 72 and 96 is 24, so $72: 96 = 3: 4$.
3. Since both ratios have the same simplest form, **3 : 4 and 72 : 96 are proportional**.

5. Explain the lemonade example used to illustrate proportional reasoning.
Answer:
1. Kesang uses **10 spoons of sugar for 6 glasses of lemonade**.
2. For 18 glasses, the number of glasses increases by a factor of $18 \div 6 = 3$, so the sugar must also increase by the same factor.
3. Therefore, she needs $10 \times 3 = 30$spoons of sugar to maintain the same sweetness.

6. Explain why Nitin and Hari's walls are considered equally strong in the chapter.
Answer:
1. Nitin uses 3 bags of cement for a 60-ft wall, giving the ratio $60: 3 = 20: 1$.
2. Hari uses 2 bags of cement for a 40-ft wall, giving the ratio $40: 2 = 20: 1$.
3. Since both ratios are the same, the chapter concludes that the walls are **equally strong**.

7. Explain why adding the same number to both terms of a ratio does not necessarily maintain proportionality.

Answer:

1. Neelima's age and her mother's age are initially 3 and 30 years, giving the ratio **1 : 10**.
2. Nine years later, their ages become 12 and 39 years, giving the simplified ratio **4 : 13**.
3. Thus, adding the same number to both terms changes the ratio and does not necessarily maintain **proportionality**.

8. Find the missing terms in the ratios proportional to 14 : 21: ___ : 42 and 2 : ___.

Answer:

1. Since 21 is multiplied by 2 to obtain 42, 14 must also be multiplied by 2, giving **28 : 42**.
2. The simplest form of $14: 21$is obtained by dividing both terms by 7, giving **2 : 3**.
3. Therefore, the missing terms are **28 and 3**, respectively.

9. Explain the Rule of Three (Trairāsika) using the school mid-day meal example.

Answer:

1. For 120 students, 15 kg of rice is required, while the quantity required for 80 students is unknown.
2. We form the proportion $120: 15:: 80: x$, and the factor of change is $80/120 = 2/3$.
3. Therefore, $x = 15 \times \frac{2}{3} = 10\ kg$of rice.

10. Explain the cross-multiplication method for solving proportions.

Answer:

1. If $a: b:: c: d$, the corresponding terms change by the same proportional factor.
2. Cross multiplication gives the relation $ad = bc$.
3. Therefore, if d is unknown, it can be calculated using $d = \frac{bc}{a}$.

11. Explain how Āryabhaṭa's Rule of Three is expressed mathematically.

Answer:

1. In the ancient formulation, the three known quantities were called **pramāṇa, phala and ichchhā**.
2. The unknown quantity was called **ichchhāphala**, represented by $pramāṇa: phala:: ichchhā: ichchhāphala$.
3. Thus, $ichchhāphala = \frac{phala \times ichchhā}{pramāṇa}$.

12. A car travels 90 km in 150 minutes. Find the distance it covers in 4 hours at the same speed.

Answer:

1. First convert 4 hours into minutes: $4 \times 60 = 240$minutes.
2. Form the proportion $150: 90:: 240: x$, giving $150x = 240 \times 90$.
3. Therefore, $x = \frac{240 \times 90}{150} = 144\ km$.

13. Explain how 42 counters can be divided in the ratio 4 : 3.

Answer:

1. The total number of ratio-parts is $4 + 3 = 7$.
2. Each part contains $42 \div 7 = 6$counters.
3. Therefore, the two shares are $4 \times 6 = 24$and $3 \times 6 = 18$counters.

14. State and explain the general method of dividing a quantity x in the ratio m : n.

Answer:

1. The total number of ratio-parts is $m + n$.
2. The first part is $\frac{m}{m+n} \times x$.
3. The second part is $\frac{n}{m+n} \times x$.

15. Explain how Prashanti and Bhuvan share a profit of ₹4,000 based on their investments.
Answer:
1. Their investments are ₹75,000 and ₹25,000, giving the simplified ratio **3 : 1**.
2. The value of one ratio-part is ₹4,000 ÷ 4 = ₹1,000.
3. Therefore, Prashanti receives **₹3,000** and Bhuvan receives **₹1,000**.

16. Find the quantities of sand and cement in a 40 kg mixture having the ratio 3 : 1.
Answer:
1. The total number of ratio-parts is $3 + 1 = 4$.
2. Sand $= \frac{3}{4} \times 40 = 30\ kg$.
3. Cement $= \frac{1}{4} \times 40 = 10\ kg$.

17. State three important area conversions given in the chapter.
Answer:
1. **1 square metre = 10.764 square feet.**
2. **1 acre = 43,560 square feet.**
3. **1 hectare = 10,000 square metres = 2.471 acres.**

18. State the important volume and temperature conversions given in the chapter.
Answer:
1. **1 mL = 1 cubic centimetre (cc)** and **1 litre = 1,000 mL or 1,000 cc**.
2. Celsius to Fahrenheit is calculated using $F = \frac{9}{5}C + 32$.
3. Fahrenheit to Celsius is calculated using $C = \frac{5}{9}(F - 32)$.

19. Explain the three main ideas about ratios given in the chapter summary.
Answer:
1. A ratio **a : b** means that for every a units of the first quantity, there are b units of the second quantity.
2. Two ratios $a\!:\!b$and $c\!:\!d$are proportional when $ad = bc$.
3. A quantity x divided in the ratio m : n gives the parts $\frac{mx}{m+n}$and $\frac{nx}{m+n}$.

20. Explain how regular, stronger and lighter filter coffee are represented using ratios.
Answer:
1. Regular filter coffee uses coffee decoction and milk in the ratio **15 : 35**.
2. Stronger coffee uses the ratio **20 : 30**, which has a greater proportion of decoction.
3. Lighter coffee uses the ratio **10 : 40**, which has a smaller proportion of decoction.

7.10. Three-mark questions with Answers of HOTS Type: (20 Questions)

Three-Mark Descriptive Questions with Answers – HOTS Type:

1. An image measuring 60 mm × 40 mm is resized to 75 mm × 50 mm. Determine whether the resized image will remain similar to the original.
Answer:
1. The original ratio of width to height is $60\!:\!40 = 3\!:\!2$.
2. The resized image has the ratio $75\!:\!50 = 3\!:\!2$.
3. Since both ratios are equal, the resized image **remains similar to the original**.

2. A student claims that 24 : 36 and 40 : 60 are not proportional because all four numbers are different. Analyse the claim.
Answer:
1. Simplifying $24\!:\!36$gives **2 : 3**.
2. Simplifying $40\!:\!60$also gives **2 : 3**.

3. Therefore, the student's claim is incorrect because ratios with different terms can still be **proportional** when their simplest forms are equal.

3. A recipe uses 10 spoons of sugar for 6 glasses of lemonade. Riya uses 25 spoons for 15 glasses. Will her lemonade have the same sweetness? Justify.

Answer:

1. The original ratio of sugar to glasses is $10:6 = 5:3$.
2. Riya's ratio is $25:15 = 5:3$.
3. Since the ratios are proportional, Riya's lemonade will have the **same sweetness**.

4. A student changes the ratio 3 : 5 to 6 : 8 by adding 3 to both terms. Explain why the two ratios are not proportional.

Answer:

1. The original ratio is **3 : 5**, while $6:8$simplifies to **3 : 4**.
2. Adding the same number to both terms does not preserve their multiplicative relationship.
3. Therefore, **3 : 5 and 6 : 8 are not proportional**.

5. Find the missing term in $\mathbf{18:27::30:}x$using proportional reasoning.

Answer:

1. From $18:27::30:x$, cross multiplication gives $18x = 27 \times 30$.
2. Therefore, $x = \frac{27\times30}{18}$.
3. Hence, the missing term is **45**.

6. A school needs 15 kg of rice for 120 students. If 32 students are absent, calculate the quantity of rice required for the students present.

Answer:

1. The number of students present is $120 - 32 = 88$.
2. Using proportion, the rice required is $15 \times \frac{88}{120}$.
3. Therefore, the school requires **11 kg of rice**.

7. A car covers 90 km in 150 minutes. How far will it travel in 6 hours at the same speed?

Answer:

1. Convert 6 hours into minutes: $6 \times 60 = 360$minutes.
2. Using proportion, $150:90::360:x$, so $x = \frac{360\times90}{150}$.
3. Therefore, the car will travel **216 km**.

8. A student directly compares ₹200 for 200 g of tea with ₹800 for 1 kg and concludes that the ₹800 tea is more expensive. Evaluate the conclusion.

Answer:

1. At ₹200 for 200 g, 1 kg of the first tea would cost $5 \times$ ₹$200 =$ ₹$1{,}000$.
2. The second tea costs **₹800 for 1 kg**.
3. Therefore, the student's conclusion is incorrect; the **₹1,000-per-kg tea is more expensive**.

9. A sum of ₹7,200 is to be divided between two people in the ratio 5 : 3. Find their shares.

Answer:

1. The total number of ratio-parts is $5 + 3 = 8$, so each part is ₹$7{,}200 \div 8 =$ ₹900.
2. The first person's share is $5 \times$ ₹$900 =$ ₹$4{,}500$.
3. The second person's share is $3 \times$ ₹$900 =$ ₹$2{,}700$.

10. Prashanti and Bhuvan invest in the ratio 3 : 1. If they earn a profit of ₹20,000, determine how the profit should be shared.

Answer:

1. The total number of investment parts is $3 + 1 = 4$.
2. Prashanti receives $\frac{3}{4} \times$ ₹$20{,}000 =$ ₹$15{,}000$.

3. Bhuvan receives $\frac{1}{4} \times ₹20{,}000 = ₹5{,}000$.

11. A 40 kg mixture contains sand and cement in the ratio 3 : 1. How much cement must be added to change the ratio to 5 : 2?

Answer:

1. Initially, the mixture contains **30 kg sand and 10 kg cement**.
2. For the new ratio $5:2$, 30 kg of sand corresponds to $30 \times \frac{2}{5} = 12$kg of cement.
3. Therefore, $12 - 10 = 2$kg of cement must be added.

12. A mixture contains 600 mL of orange juice and 900 mL of apple juice. How much orange juice must be added to make their ratio 1 : 1?

Answer:

1. The present ratio $600:900$simplifies to **2 : 3**.
2. For a ratio of 1 : 1, the orange juice must also become **900 mL**.
3. Therefore, $900 - 600 = 300$mL of orange juice must be added.

13. Regular coffee has decoction and milk in the ratio 15 : 35, while strong coffee has the ratio 20 : 30. Explain mathematically why the second coffee is stronger.

Answer:

1. The regular coffee ratio simplifies from $15:35$to **3 : 7**.
2. The strong coffee ratio simplifies from $20:30$to **2 : 3**, giving relatively more decoction for the amount of milk.
3. Therefore, the **20 : 30 mixture is stronger** because it contains a greater proportion of decoction.

14. A tap fills 500 mL of water in 15 seconds. Find how long it will take to fill a 10-litre bucket at the same rate.

Answer:

1. Convert 10 litres to millilitres: $10 \times 1000 = 10{,}000$mL.
2. The bucket contains $10{,}000 \div 500 = 20$quantities of 500 mL.
3. Therefore, the required time is $20 \times 15 = 300$seconds or **5 minutes**.

15. A student says that if a motorcycle takes 2 hours at 50 km/h, it should take 3 hours at 75 km/h because 50 : 2 :: 75 : 3. Explain the error.

Answer:

1. For a fixed journey, increasing the speed should **decrease**, not increase, the travel time.
2. Therefore, speed and time in this situation do not follow the direct proportion $50:2::75:3$.
3. The student's reasoning is incorrect because the chapter specifically warns that this speed-time situation **cannot be modelled as direct proportion**.

16. A paint mixture uses blue and yellow paint in the ratio 3 : 5. Find the quantities required to prepare 64 mL of the mixture.

Answer:

1. The total number of ratio-parts is $3 + 5 = 8$, so each part represents $64 \div 8 = 8$mL.
2. Blue paint required is $3 \times 8 = 24$mL.
3. Yellow paint required is $5 \times 8 = 40$mL.

17. A ₹10 coin has a mass of 7.74 g and contains copper and nickel in the ratio 3 : 1. Determine the amount of each metal.

Answer:

1. The total number of ratio-parts is 4, so one part weighs $7.74 \div 4 = 1.935$g.
2. Copper weighs $3 \times 1.935 = 5.805$g.
3. Nickel weighs **1.935 g**, and together the two quantities total 7.74 g.

18. Neelima is 3 years old and her mother is 30 years old. Nine years later, will their age ratio remain 1 : 10? Explain.

Answer:

1. Their initial age ratio is $3:30 = 1:10$.
2. Nine years later, their ages are 12 and 39, giving $12:39 = 4:13$.
3. Therefore, the age ratio does **not remain 1 : 10**, showing that adding the same number to both terms does not preserve a ratio.

19. A farmer requires 10 tonnes of manure per acre. Explain the steps needed to find the manure required for a rectangular field measuring 200 ft × 500 ft.

Answer:

1. First calculate the field area as $200 \times 500 = 100{,}000$square feet.
2. Convert this area into acres using **1 acre = 43,560 square feet**.
3. Then multiply the area in acres by **10 tonnes per acre** to determine the manure required.

20. Two rectangles measure 15 cm × 10 cm and 24 cm × 16 cm. Determine whether they are similar using proportional reasoning.

Answer:

1. The first rectangle has the ratio $15:10 = 3:2$.
2. The second rectangle has the ratio $24:16 = 3:2$.
3. Since their corresponding dimensions have the same ratio, the two rectangles are **similar**.

7.11 Five-mark questions with Answers of LOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – LOTS Type:

1. What is a ratio? Explain how ratios are simplified and used to identify proportion.

Answer:

1. A **ratio** compares two quantities and is written in the form **a : b**, where a and b are called its terms.
2. A ratio is reduced to its simplest form by dividing both terms by their **HCF**.
3. For example, $60:40$simplifies to $3:2$by dividing both terms by 20.
4. Two ratios are said to be **proportional** when they have the same simplest form.
5. Thus, $60:40::30:20$, because both ratios simplify to **3 : 2**.

2. Explain proportional change using the similar images discussed in the chapter.

Answer:

1. Image A has dimensions **60 mm** × **40 mm**, while Image C has dimensions **30 mm** × **20 mm**.
2. The width and height of Image C are each **half** the corresponding dimensions of Image A.
3. Image D has dimensions **90 mm** × **60 mm**, where both dimensions are 1.5 times those of Image A.
4. Therefore, Images A, C and D have the same width-to-height ratio of **3 : 2**.
5. Since both dimensions change by the same factor, the images retain their shape and demonstrate **proportional change**.

3. Explain the lemonade example to show how proportional reasoning is used.

Answer:

1. Kesang prepares **6 glasses of lemonade using 10 spoons of sugar**.
2. She wants to prepare 18 glasses while keeping the sweetness unchanged.
3. Since $18 \div 6 = 3$, the number of glasses has increased by a factor of **3**.
4. Therefore, the amount of sugar must also be multiplied by 3, giving $10 \times 3 = 30$spoons.
5. Hence, **30 spoons of sugar** are required for 18 glasses to maintain the same proportion and sweetness.

4. Explain the Rule of Three (Trairāsika) with the school meal example.
Answer:
1. The **Rule of Three** is used to find an unknown fourth quantity when three quantities in a proportional relationship are known.
2. In the chapter, **120 students require 15 kg of rice**, and the rice required for 80 students has to be found.
3. Since $\frac{80}{120} = \frac{2}{3}$, the number of students becomes $\frac{2}{3}$of the original number.
4. Therefore, the rice required is $15 \times \frac{2}{3} = 10$kg.
5. Hence, **80 students require 10 kg of rice**, illustrating the Rule of Three.

5. Explain the algebraic method of solving a proportion using cross multiplication.
Answer:
1. A general proportion can be written as $a: b:: c: d$.
2. If the corresponding quantities change by the same factor, then $\frac{c}{a} = \frac{d}{b}$.
3. Cross multiplication of these fractions gives the relationship $ad = bc$.
4. If d is unknown, it can be calculated using $d = \frac{bc}{a}$.
5. Thus, cross multiplication provides a convenient method for finding a **missing term in a proportion**.

6. A car travels 90 km in 150 minutes. Find the distance it travels in 4 hours at the same speed.
Answer:
1. The car covers **90 km in 150 minutes**.
2. Convert 4 hours into minutes: $4 \times 60 = 240$minutes.
3. Form the proportion $150: 90:: 240: x$.
4. By cross multiplication, $x = \frac{90 \times 240}{150} = 144$.
5. Therefore, the car travels **144 km in 4 hours** at the same speed.

7. Explain how a quantity can be divided in a given ratio using the example of 42 counters divided in the ratio 4 : 3.
Answer:
1. To divide 42 counters in the ratio **4 : 3**, first add the ratio terms: $4 + 3 = 7$.
2. Thus, the 42 counters are considered as **7 equal groups**.
3. Each group contains $42 \div 7 = 6$counters.
4. The first share is $4 \times 6 = 24$, while the second share is $3 \times 6 = 18$.
5. Therefore, 42 counters divided in the ratio 4 : 3 give the shares **24 and 18**.

8. Derive the general formula for dividing a quantity x in the ratio m : n.
Answer:
1. Suppose a quantity **x** has to be divided into two parts in the ratio **m : n**.
2. The total number of ratio-parts is $m + n$.
3. Therefore, the value of one ratio-part is $\frac{x}{m+n}$.
4. The first share is $\frac{mx}{m+n}$.
5. The second share is $\frac{nx}{m+n}$.

9. Explain how Prashanti and Bhuvan share their profit according to their investments.
Answer:
1. Prashanti invests **₹75,000**, while Bhuvan invests **₹25,000**.
2. Their investment ratio is $75{,}000: 25{,}000 = 3: 1$.
3. If their total profit is ₹4,000, it must be divided into $3 + 1 = 4$equal ratio-parts.

4. Prashanti receives $\frac{3}{4} \times ₹4{,}000 = ₹3{,}000$, while Bhuvan receives $\frac{1}{4} \times ₹4{,}000 = ₹1{,}000$.
5. Thus, the profit is shared as **₹3,000 and ₹1,000** according to their investment ratio.

10. Explain how the ratio of sand to cement in a 40 kg mixture is changed from 3 : 1 to 5 : 2.

Answer:

1. A 40 kg mixture in the ratio **3 : 1** contains 30kg of sand and 10kg of cement.
2. The amount of sand remains unchanged at **30 kg**.
3. For the new ratio $5:2$, 30 kg of sand represents 5 parts, so one part is $30 \div 5 = 6$kg.
4. Cement must therefore be $2 \times 6 = 12$kg.
5. Hence, $12 - 10 = 2$kg of cement must be added to obtain the ratio **5 : 2**.

11. Explain the important unit conversions given in the chapter.

Answer:

1. **1 metre = 3.281 feet** and **1 square metre = 10.764 square feet**.
2. **1 acre = 43,560 square feet**.
3. **1 hectare = 10,000 square metres = 2.471 acres**.
4. **1 mL = 1 cubic centimetre (cc)** and **1 litre = 1,000 mL = 1,000 cc**.
5. These conversions help express quantities in suitable common units before applying proportional reasoning.

12. Explain the conversion between Celsius and Fahrenheit temperatures.

Answer:

1. Celsius and Fahrenheit are two different scales used for measuring **temperature**.
2. Celsius can be converted to Fahrenheit using $F = \frac{9}{5}C + 32$.
3. Fahrenheit can be converted to Celsius using $C = \frac{5}{9}(F - 32)$.
4. For example, $25^\circ C$ gives $\frac{9}{5}(25) + 32 = 77^\circ F$.
5. Therefore, **25°C is equivalent to 77°F**.

7.12 Five-mark questions with Answers of HOTS Type: (12 Questions)

Five-Mark Descriptive Questions with Answers – HOTS Type:

1. A rectangular image of size 60 mm × 40 mm is resized to 105 mm × 70 mm. Determine whether the new image is similar to the original and justify your answer.

Answer:

1. The width-to-height ratio of the original image is $60:40 = 3:2$.
2. The ratio of the resized image is $105:70 = 3:2$.
3. The width has changed by a factor of $105/60 = 7/4$.
4. The height has also changed by the same factor, $70/40 = 7/4$.
5. Therefore, both dimensions change proportionally, so the **resized image remains similar to the original**.

2. A student says that 18 : 30 and 27 : 45 are not proportional because their terms are different. Analyse the statement and justify your conclusion.

Answer:

1. Simplifying the first ratio gives $18:30 = 3:5$.
2. Simplifying the second ratio gives $27:45 = 3:5$.
3. Thus, both ratios have exactly the same simplest form.
4. Ratios do not need to have identical terms to be proportional; their corresponding terms must have the same multiplicative relationship.
5. Therefore, the student's statement is **incorrect**, and $18:30::27:45$.

3. Kesang uses 10 spoons of sugar for 6 glasses of lemonade. For a gathering, she wants to prepare 42 glasses with the same sweetness. Calculate the sugar required and explain your reasoning.

Answer:

1. The original relationship is **6 glasses : 10 spoons of sugar**.
2. The number of glasses increases from 6 to 42 by a factor of $42 \div 6 = 7$.
3. To maintain the same sweetness, the sugar must also increase by the same factor.
4. Therefore, the required sugar is $10 \times 7 = 70$spoons.
5. Hence, Kesang needs **70 spoons of sugar for 42 glasses** to maintain the original proportion.

4. Neelima is 3 years old and her mother is 30 years old. Nine years later, determine their new age ratio and explain why adding the same number to both terms does not preserve a ratio.

Answer:

1. Initially, their age ratio is $3: 30 = 1: 10$.
2. Nine years later, their ages become $3 + 9 = 12$years and $30 + 9 = 39$years.
3. Their new age ratio is $12: 39 = 4: 13$.
4. The new ratio 4 : 13 is not equal to the original ratio 1 : 10.
5. Therefore, **adding the same number to both terms of a ratio does not necessarily preserve proportionality**.

5. A school requires 15 kg of rice for 120 students. On a particular day, only 96 students attend. Calculate the rice required using proportional reasoning.

Answer:

1. The original relationship is **120 students : 15 kg of rice**.
2. For 96 students, write the proportion $120: 15: : 96: x$.
3. By cross multiplication, $120x = 15 \times 96$.
4. Therefore, $x = \frac{15\times96}{120} = 12$kg.
5. Hence, the school should prepare **12 kg of rice for 96 students**.

6. A car covers 90 km in 150 minutes at a constant speed. How far will it travel in 7 hours? Explain all the steps.

Answer:

1. First convert 7 hours into minutes: $7 \times 60 = 420$minutes.
2. Form the proportion $150: 90: : 420: x$.
3. By cross multiplication, $150x = 90 \times 420$.
4. Therefore, $x = \frac{90\times420}{150} = 252$km.
5. Hence, the car will travel **252 km in 7 hours** at the same speed.

7. Tea from Himachal Pradesh costs ₹200 for 200 g, while tea from Meghalaya costs ₹800 for 1 kg. Compare their prices fairly and determine which is more expensive.

Answer:

1. To compare the prices fairly, both teas must be considered for the **same weight**.
2. Since 1 kg = 1,000 g, five packets of 200 g are required to make 1 kg of Himachal tea.
3. Therefore, 1 kg of Himachal tea costs $5 \times ₹200 = ₹1{,}000$.
4. One kilogram of Meghalaya tea costs **₹800**, which is ₹200 less.
5. Hence, **Himachal tea is more expensive**, costing ₹1,000 per kg compared with ₹800 per kg.

8. Prashanti invests ₹75,000 and Bhuvan invests ₹25,000 in a business. If they earn a profit of ₹28,000, determine how the profit should be shared fairly.

Answer:

1. Their investment ratio is $75{,}000: 25{,}000 = 3: 1$.

2. The total number of ratio-parts is $3 + 1 = 4$.
3. The value of one part of the profit is ₹28,000 ÷ 4 = ₹7,000.
4. Prashanti receives 3 × ₹7,000 = ₹21,000, while Bhuvan receives **₹7,000**.
5. Therefore, the ₹28,000 profit should be divided as **₹21,000 and ₹7,000** according to their investment ratio.

9. A 40 kg mixture contains sand and cement in the ratio 3 : 1. How much cement must be added to change the ratio to 5 : 2? Explain.

Answer:

1. The original 40 kg mixture contains $\frac{3}{4} \times 40 = 30$kg of sand and $\frac{1}{4} \times 40 = 10$kg of cement.
2. The quantity of sand remains unchanged at 30 kg.
3. In the required ratio 5 : 2, if 5 parts equal 30 kg, one part equals $30 \div 5 = 6$kg.
4. Therefore, the required cement is $2 \times 6 = 12$kg.
5. Since there are already 10 kg of cement, **2 kg of cement must be added**.

10. A tap fills 500 mL of water in 15 seconds. Determine how long it will take to fill a 10-litre bucket at the same rate.

Answer:

1. Convert 10 litres into millilitres: $10 \times 1{,}000 = 10{,}000$mL.
2. The bucket contains $10{,}000 \div 500 = 20$quantities of 500 mL.
3. Each 500 mL quantity takes 15 seconds to fill.
4. Therefore, the total time required is $20 \times 15 = 300$seconds.
5. Hence, the tap will take **300 seconds or 5 minutes** to fill the bucket.

11. A ₹10 coin has a mass of 7.74 g and contains copper and nickel in the ratio 3 : 1. Calculate the mass of each metal and verify your answer.

Answer:

1. The total number of ratio-parts is $3 + 1 = 4$.
2. The mass represented by one part is $7.74 \div 4 = 1.935$g.
3. The mass of copper is $3 \times 1.935 = 5.805$g.
4. The mass of nickel is $1 \times 1.935 = 1.935$g.
5. Since $5.805 + 1.935 = 7.74$g, the required masses are **5.805 g of copper and 1.935 g of nickel**.

12. A student argues that if a journey takes 2 hours at 50 km/h, it should take 3 hours at 75 km/h because $50 : 2 :: 75 : 3$. Evaluate the reasoning.

Answer:

1. The student's calculation treats speed and time as though they increase in **direct proportion**.
2. For the same journey, however, increasing the speed should cause the travel time to **decrease**.
3. Therefore, travelling at 75 km/h cannot require more time than travelling at 50 km/h for the same distance.
4. The proportion $50 : 2 :: 75 : 3$does not correctly represent the relationship between speed and time in this situation.
5. Hence, the student's reasoning is **incorrect because this speed-time situation is not a direct proportion**.

7.13 Five-mark questions with Answers of Applied Type: (10 Questions)

Five-Mark Descriptive Questions with Answers – Applied Type:

1. A school kitchen uses 18 kg of rice to prepare lunch for 144 students. On a particular day, 200 students are expected. How much rice should the kitchen prepare if the quantity per student remains the same?

Answer:

1. The number of students and the quantity of rice are in **direct proportion**.
2. Form the proportion $144:18::200:x$.
3. By cross multiplication, $144x = 18 \times 200$.
4. Therefore, $x = \frac{18\times200}{144} = 25$kg.
5. Hence, the school kitchen should prepare **25 kg of rice for 200 students**.

2. A fruit drink is prepared by mixing orange juice and apple juice in the ratio 2 : 3. If a school wants to prepare 15 litres of the drink for an event, calculate the quantity of each juice required.

Answer:

1. The total number of ratio-parts is $2 + 3 = 5$.
2. Each part represents $15 \div 5 = 3$litres.
3. Orange juice required is $2 \times 3 = 6$litres.
4. Apple juice required is $3 \times 3 = 9$litres.
5. Therefore, **6 litres of orange juice and 9 litres of apple juice** should be mixed.

3. Two students invest ₹60,000 and ₹40,000 in a small business project. At the end of the year, they earn a profit of ₹25,000. How should the profit be shared according to their investments?

Answer:

1. Their investment ratio is $60{,}000: 40{,}000 = 3: 2$.
2. The total number of ratio-parts is $3 + 2 = 5$.
3. The value of each part of the profit is $₹25{,}000 \div 5 = ₹5{,}000$.
4. The first student receives $3 \times ₹5{,}000 = ₹15{,}000$, while the second receives $2 \times ₹5{,}000 = ₹10{,}000$.
5. Hence, the profit should be shared as **₹15,000 and ₹10,000** according to their investment ratio.

4. A builder prepares 80 kg of a sand-cement mixture in the ratio 3 : 1. Find the quantities of sand and cement needed.

Answer:

1. The total number of ratio-parts is $3 + 1 = 4$.
2. The value of one part is $80 \div 4 = 20$kg.
3. Sand required is $3 \times 20 = 60$kg.
4. Cement required is $1 \times 20 = 20$kg.
5. Therefore, the builder needs **60 kg of sand and 20 kg of cement**.

5. A tap fills a 500 mL bottle in 15 seconds. At the same rate, how long will it take to fill a 20-litre water container?

Answer:

1. Convert 20 litres into millilitres: $20 \times 1000 = 20{,}000$mL.
2. The container holds $20{,}000 \div 500 = 40$quantities of 500 mL.
3. Each 500 mL quantity takes 15 seconds to fill.
4. Hence, the total time is $40 \times 15 = 600$seconds.
5. Therefore, it will take **600 seconds or 10 minutes** to fill the 20-litre container.

6. A car travels 90 km in 150 minutes at a constant speed. A family plans to travel for 8 hours at the same speed. Find the distance they can cover.

Answer:

1. Convert 8 hours into minutes: $8 \times 60 = 480$minutes.

2. Form the proportion $150:90::480:x$.
3. By cross multiplication, $150x = 90 \times 480$.
4. Therefore, $x = \frac{90\times480}{150} = 288\text{km}$.
5. Hence, the family can travel **288 km in 8 hours** at the same speed.

7. A school needs to resize a rectangular poster measuring 60 cm × 40 cm. If its width is increased to 150 cm, what should its height be so that the poster does not get distorted?
Answer:
1. The original width-to-height ratio is $60:40 = 3:2$.
2. Let the required new height be x, so $60:40::150:x$.
3. By cross multiplication, $60x = 40 \times 150$.
4. Therefore, $x = \frac{40\times150}{60} = 100\text{cm}$.
5. Hence, the poster should measure **150 cm** × **100 cm** to retain its original proportions.

8. A café prepares regular filter coffee by mixing decoction and milk in the ratio 15 : 35. How much decoction and milk are needed to prepare 2 litres of coffee in the same proportion?
Answer:
1. The ratio $15:35$simplifies to **3 : 7**, giving a total of $3 + 7 = 10$parts.
2. Convert 2 litres to **2,000 mL**, so each part represents $2{,}000 \div 10 = 200\text{mL}$.
3. Decoction required is $3 \times 200 = 600\text{mL}$.
4. Milk required is $7 \times 200 = 1{,}400\text{mL}$.
5. Therefore, the café needs **600 mL of decoction and 1,400 mL of milk**.

9. A farmer has a rectangular field measuring 200 ft × 500 ft and needs 10 tonnes of manure per acre. Estimate the manure required using 1 acre = 43,560 ft².
Answer:
1. The area of the field is $200 \times 500 = 100{,}000$square feet.
2. The area in acres is $\frac{100{,}000}{43{,}560} \approx 2.30$acres.
3. The manure requirement is **10 tonnes for each acre**.
4. Therefore, manure required is approximately $2.30 \times 10 = 23$tonnes.
5. Hence, the farmer needs approximately **23 tonnes of manure** for the field.

10. A ₹10 coin has a mass of 7.74 g and contains copper and nickel in the ratio 3 : 1. A collection contains 100 such coins. Find the total mass of copper and nickel separately.
Answer:
1. In one coin, the total number of ratio-parts is $3 + 1 = 4$, so one part weighs $7.74 \div 4 = 1.935\text{g}$.
2. Copper in one coin is $3 \times 1.935 = 5.805\text{g}$, while nickel is **1.935 g**.
3. Copper in 100 coins is $5.805 \times 100 = 580.5\text{g}$.
4. Nickel in 100 coins is $1.935 \times 100 = 193.5\text{g}$.
5. Therefore, 100 coins contain **580.5 g of copper and 193.5 g of nickel**.

7.14 Multiple Choice Questions with answers for Olympiad type Competitive Exams: (20 MCQs of LOTS & 20 MCQs of HOTS Type)

Olympiad-Type Multiple Choice Questions:
Below are **30 MCQs** covering ratios, proportions, proportional change, Rule of Three, sharing in ratios, mixtures, unit conversions, and application-based reasoning from the chapter.

1. Which of the following ratios is equivalent to $18:24$?
A) 2 : 3
B) 3 : 4

C) 4 : 5
D) 5 : 6
Answer: B) 3 : 4
2. If $5:8::x:40$**, the value of** x**is:**
A) 20
B) 24
C) 25
D) 32
Answer: C) 25
3. Which pair of ratios is NOT proportional?
A) 2 : 3 and 8 : 12
B) 4 : 5 and 20 : 25
C) 3 : 7 and 15 : 35
D) 5 : 6 and 25 : 36
Answer: D) 5 : 6 and 25 : 36
4. A rectangle measures 60 cm × 40 cm. Which dimensions will produce a similar rectangle?
A) 90 cm × 50 cm
B) 90 cm × 60 cm
C) 100 cm × 50 cm
D) 120 cm × 60 cm
Answer: B) 90 cm × 60 cm
5. If $a:b::c:d$**, which relation must be true?**
A) $a+b=c+d$
B) $ac=bd$
C) $ad=bc$
D) $ab=cd$
Answer: C) $ad=bc$
6. If $14:21::x:63$**, then** x**is:**
A) 28
B) 36
C) 42
D) 49
Answer: C) 42
7. Six glasses of lemonade require 10 spoons of sugar. How many spoons are required for 30 glasses with the same sweetness?
A) 40
B) 45
C) 50
D) 60
Answer: C) 50
8. The ratio $72:96$**in simplest form is:**
A) 2 : 3
B) 3 : 4
C) 4 : 5
D) 6 : 8
Answer: B) 3 : 4
9. If 120 students require 15 kg of rice, how much rice is required for 200 students?
A) 20 kg
B) 22.5 kg

C) 25 kg
D) 30 kg
Answer: C) 25 kg
10. Which operation will always produce a ratio equivalent to $7:11$?
A) Add 3 to both terms
B) Subtract 2 from both terms
C) Multiply both terms by 4
D) Square both terms
Answer: C) Multiply both terms by 4
11. The ratio of two ages is $1:10$. If the same number of years is added to both ages, which statement is generally correct?
A) The ratio always remains 1 : 10
B) The ratio always becomes 2 : 20
C) The ratio need not remain the same
D) The ratio becomes 10 : 1
Answer: C) The ratio need not remain the same
12. A car travels 90 km in 150 minutes. At the same speed, how far will it travel in 5 hours?
A) 150 km
B) 180 km
C) 200 km
D) 225 km
Answer: B) 180 km
13. If ₹8,400 is divided in the ratio $3:4$, the larger share is:
A) ₹3,200
B) ₹3,600
C) ₹4,200
D) ₹4,800
Answer: D) ₹4,800
14. If 42 objects are divided in the ratio $4:3$, the difference between the two shares is:
A) 4
B) 6
C) 8
D) 12
Answer: B) 6
The shares are 24 and 18, so the difference is 6.
15. A quantity $x$is divided in the ratio $m:n$. The first share is:
A) $\frac{x}{m+n}$
B) $\frac{mx}{m+n}$
C) $\frac{nx}{m+n}$
D) $\frac{(m+n)x}{m}$
Answer: B) $\frac{mx}{m+n}$
16. Two business partners invest ₹75,000 and ₹25,000. If the profit is ₹16,000, the first partner's share is:
A) ₹4,000
B) ₹8,000
C) ₹10,000
D) ₹12,000

Answer: D) ₹12,000
The investment ratio is 3 : 1.
17. A 40 kg mixture contains sand and cement in the ratio 3: 1. How much cement is present?
A) 5 kg
B) 8 kg
C) 10 kg
D) 12 kg
Answer: C) 10 kg
18. A 40 kg sand-cement mixture is in the ratio 3: 1. How much cement should be added to make the ratio 5: 2?
A) 1 kg
B) 2 kg
C) 4 kg
D) 5 kg
Answer: B) 2 kg
19. A drink contains orange and apple juice in the ratio 2: 3. If the total quantity is 2.5 litres, how much apple juice is present?
A) 1 L
B) 1.25 L
C) 1.5 L
D) 2 L
Answer: C) 1.5 L
20. Regular coffee uses decoction and milk in the ratio 15: 35. Its simplest form is:
A) 2 : 5
B) 3 : 5
C) 3 : 7
D) 5 : 7
Answer: C) 3 : 7
21. Which coffee mixture has the greatest proportion of decoction?
A) 15 : 35
B) 20 : 30
C) 10 : 40
D) 5 : 25
Answer: B) 20 : 30
22. A tap fills 500 mL in 15 seconds. How long will it take to fill 5 litres at the same rate?
A) 75 seconds
B) 100 seconds
C) 150 seconds
D) 300 seconds
Answer: C) 150 seconds
23. One hectare is equal to:
A) 1,000 m^2
B) 5,000 m^2
C) 10,000 m^2
D) 43,560 m^2
Answer: C) 10,000 m^2
24. One acre is equal to approximately:
A) 10,000 ft^2
B) 21,780 ft^2

C) 43,560 ft²
D) 100,000 ft²
Answer: C) 43,560 ft²
25. If $C = 25°$, the corresponding Fahrenheit temperature is:
A) 57°F
B) 67°F
C) 77°F
D) 87°F
Answer: C) 77°F
26. A ₹10 coin weighs 7.74 g and contains copper and nickel in the ratio $3:1$. How much nickel does it contain?
A) 1.290 g
B) 1.935 g
C) 2.580 g
D) 5.805 g
Answer: B) 1.935 g
27. A mixture has blue and yellow paint in the ratio $3:5$. If 64 mL of paint is prepared, how much blue paint is required?
A) 16 mL
B) 20 mL
C) 24 mL
D) 40 mL
Answer: C) 24 mL
28. Tea A costs ₹200 for 200 g, while Tea B costs ₹800 for 1 kg. Which statement is correct?
A) Tea A is cheaper by ₹200/kg
B) Tea B is cheaper by ₹200/kg
C) Both cost the same per kg
D) Tea B is twice as expensive
Answer: B) Tea B is cheaper by ₹200/kg
Tea A costs ₹1,000/kg, whereas Tea B costs ₹800/kg.
29. A motorcycle takes 2 hours for a fixed journey at 50 km/h. Which statement best describes what happens if its speed increases to 75 km/h?
A) Time must increase in direct proportion
B) Time remains 2 hours
C) Time should decrease
D) Time becomes 3 hours
Answer: C) Time should decrease
For a fixed distance, the speed-time situation is not a direct proportion.
30. Three numbers $x, 24, 18,$ and 36 satisfy $x : 24 :: 18 : 36$. What is x?
A) 8
B) 10
C) 12
D) 16
Answer: C) 12
Using $x \times 36 = 24 \times 18$, *we get* $x = 12$.

7.15 Best Practices Opportunity List from the Chapter: (12 Ideas)

"Best Practices" Opportunities for Teaching:

The following practices align well with the chapter's focus on **ratios, proportional change, Rule of Three, sharing in a ratio, mixtures, and unit conversions**.

(1) **Begin with Visual Comparison of Similar Shapes:** Use photographs or rectangular cards of different dimensions and ask students to compare their width-to-height ratios to discover why some images remain similar while others become distorted. This directly reinforces the chapter's introduction to proportional change.

(2) **Use Ratio Manipulatives:** Give students counters, beads, buttons, or coloured blocks and ask them to physically create ratios such as **2 : 3, 3 : 4, and 4 : 3**. They can then scale the groups up or down to understand equivalent ratios and proportions.

(3) **Teach Simplification Before Proportion:** Encourage students to reduce every ratio to its **simplest form using the HCF** before deciding whether two ratios are proportional. This develops a systematic habit and reduces calculation errors.

(4) **Use Recipe-Based Learning:** Conduct a lemonade or mock coffee-mixing activity where students scale ingredients while maintaining the same ratio. The chapter's lemonade example provides an effective context for understanding how proportional quantities change together.

(5) **Connect Mathematics with School-Life Situations:** Frame problems around school meals, classroom materials, sharing prizes, recipes, travel, and other familiar contexts. For example, the chapter uses the requirement of **15 kg of rice for 120 students** to introduce proportional reasoning through the Rule of Three.

(6) **Practise Multiple Methods:** Ask students to solve the same proportional problem using the **factor-of-change method, equivalent ratios, and cross multiplication**. Comparing methods helps students understand the reasoning behind the rule $ad = bc$, rather than merely memorising it.

(7) **Use Hands-On Sharing Activities:** Give groups 12, 24, 36, or 42 counters and ask them to divide the objects in specified ratios. Students can physically verify, for example, that dividing 42 in the ratio **4 : 3** produces **24 and 18**.

(8) **Include Real-Life Financial Applications:** Use imaginary investments and profits to teach division in a given ratio. The chapter's Prashanti–Bhuvan example can help students understand how proportional sharing is applied to business situations.

(9) **Emphasise Unit Consistency:** Train students to check and convert units **before setting up a proportion**. Problems involving hours and minutes, litres and millilitres, acres and square feet, or Celsius and Fahrenheit provide useful practice.

(10) **Teach Students to Identify Non-Examples:** Present situations that look proportional but are not direct proportions, such as **speed and travel time for a fixed distance**. Asking students to explain why the Rule of Three cannot be applied directly develops conceptual understanding rather than mechanical calculation.

(11) **Encourage Estimation and Verification:** After solving a proportion, students should estimate whether the answer is reasonable and substitute it back into the ratios. This builds the habit of **self-checking mathematical solutions**.

(12) **Use Peer Explanation and Error Analysis:** Give students intentionally incorrect solutions—for example, adding the same number to both terms and assuming the ratio remains unchanged—and ask them to identify and correct the mistake. The chapter's age example is particularly useful for showing that equal addition does not preserve a ratio.

7.16 Innovations Opportunity List from the Chapter: (12 Ideas)

"Innovations" Opportunities in Teaching–Learning for Grade 8:

The chapter provides strong opportunities for innovative learning through **visual similarity, ratios and proportions, Rule of Three, proportional sharing, mixtures, and unit conversions**.

(1) Digital Image-Resizing Lab: Let students resize the same digital image to different widths and heights using a computer or tablet and record when distortion occurs. They can calculate the width-to-height ratios and discover experimentally that an image retains its shape when both dimensions change by the **same factor**.

(2) "Ratio Café" Experiential Activity: Create a classroom café where groups design imaginary lemonade or coffee recipes using different ingredient ratios. Students can scale their recipes for different numbers of servings and compare which coffee is stronger or lighter, extending the chapter's lemonade and coffee examples.

(3) Ratio Sharing Challenge: Provide counters or tokens and challenge teams to divide quantities such as 24, 36, or 42 in ratios displayed on task cards. Students can first solve physically and then derive the mathematical rule $\frac{mx}{m+n}$and $\frac{nx}{m+n}$.

(4) Interactive "Find the Missing Term" Game: Display proportions such as $3:5::12:x$on a smartboard and allow teams to race to find the missing term using scaling or cross multiplication. Students should also explain *why* their answer maintains proportionality, reinforcing the chapter's relation $ad = bc$.

(5) School Meal Planning Simulation: Turn the chapter's rice example into a mini-project where students act as school meal planners and calculate food requirements for changing student attendance. This provides a meaningful application of the **Rule of Three (Trairāsika)**.

(6) Virtual Business Profit-Sharing Project: Give student groups imaginary investment amounts and profits and ask them to design a fair profit-sharing plan. They calculate investment ratios, divide profits accordingly, and present their reasoning, extending the Prashanti–Bhuvan example.

(7) Mixture Design Laboratory: Provide simulated quantities of sand and cement, coloured water, or virtual ingredients and ask students to modify one ingredient to achieve a target ratio. The chapter's change from a **3 : 1 sand-cement mixture to 5 : 2** is an excellent challenge model.

(8) QR-Code Ratio Treasure Hunt: Place QR codes or task cards around the classroom containing ratio puzzles, missing-term problems, sharing situations, and unit-conversion challenges. Teams move from station to station, solve each problem, and use the answer as a clue to reach the next station.

(9) "Spot the Wrong Reasoning" Challenge: Present deliberately incorrect statements such as "adding 5 to both terms keeps a ratio unchanged" and ask students to disprove them with examples. This builds mathematical argumentation and connects directly with the chapter's demonstration that adding the same number to two ages does not preserve their ratio.

(10) Direct Proportion or Not? Sorting Game: Give students situation cards and ask them to classify each as a direct proportional relationship or a non-example. The chapter's warning about **speed and travel time for a fixed journey** can be used as an important misconception-buster.

(11) Unit-Conversion Escape Room: Design a sequence of puzzles involving **metres–feet, acres–square feet, hectares–square metres, litres–millilitres, and Celsius–Fahrenheit**. Each correct conversion reveals a code or clue needed to solve the next challenge.

(12) Student-Created Olympiad Challenge: After learning the chapter, each group creates one challenging MCQ on proportional reasoning with **four options, the correct answer, and an explanation**. The best questions can be combined into a class-generated **"Proportional Reasoning Olympiad Quiz"**, encouraging higher-order thinking, peer learning, and assessment design.

7.17: Practicing Questions/Question Bank:

A. Fill in the Blanks:

1. A comparison of two quantities using division is called a __________.
   Answer: Ratio

2. The simplest form of $60: 40$is __________.
   Answer: 3 : 2

3. Two ratios having the same simplest form are said to be in __________.
   Answer: Proportion

4. If $a: b: : c: d$, then $ad =$__________.
   Answer: bc

5. If $3: 5: : 12: x$, then $x =$__________.
   Answer: 20

6. The ancient Indian Rule of Three is known as __________.
   Answer: Trairāsika

7. If 6 glasses of lemonade require 10 spoons of sugar, 18 glasses require __________ spoons.
   Answer: 30

8. When 42 is divided in the ratio 4 : 3, the larger share is __________.
   Answer: 24

9. If a quantity $x$is divided in the ratio $m: n$, the first share is __________.
   Answer: $\frac{mx}{m+n}$

10. One hectare is equal to __________ square metres.
    Answer: 10,000

11. One litre is equal to __________ millilitres.
    Answer: 1,000

12. $25°C$is equal to __________ **°F**.
    Answer: 77°F

B. One-Mark Descriptive Questions:

1. What is a ratio?
Answer: A ratio compares two quantities and is written in the form **a : b**.

2. What are the terms of a ratio?
Answer: The quantities represented by a and b in **a : b** are called the terms of the ratio.

3. When are two ratios proportional?
Answer: Two ratios are proportional when they have the **same simplest form**.

4. Write the cross-multiplication condition for $a: b: : c: d$.
Answer: The condition is $ad = bc$.

5. Find the simplest form of $14:21$.
Answer: The simplest form is **2 : 3**.

6. What is the Rule of Three used for?
Answer: It is used to find an unknown fourth quantity when three quantities in a proportional relationship are known.

7. How many kilograms of rice are required for 80 students if 120 students require 15 kg?
Answer: 10 kg of rice is required.

8. What happens when the same number is added to both terms of a ratio?
Answer: The ratio generally changes and proportionality is not necessarily preserved.

9. How many millilitres are there in one litre?
Answer: There are **1,000 mL** in one litre.

10. Write the formula for converting Celsius to Fahrenheit.
Answer: $F = \frac{9}{5}C + 32$.

C. Two-Mark Descriptive Questions:

1. How can a ratio be reduced to its simplest form?
Answer:

1. Find the **HCF** of the two terms.
2. Divide both terms by the HCF to obtain the simplest ratio.

2. Show that 18 : 24 and 27 : 36 are proportional.
Answer:

1. $18:24 = 3:4$and $27:36 = 3:4$.
2. Since their simplest forms are equal, the ratios are **proportional**.

3. Find $x$if $4:7::12:x$.
Answer:

1. Since 4 is multiplied by 3 to obtain 12, 7 must also be multiplied by 3.
2. Therefore, $x = 7 \times 3 = 21$.

4. Divide 42 in the ratio 4 : 3.
Answer:

1. Each ratio-part is $42 \div (4 + 3) = 6$.
2. Therefore, the two shares are **24 and 18**.

5. Why should units be made the same before forming a proportion?
Answer:

1. Corresponding quantities must be expressed in comparable units.
2. For example, 4 hours must be converted into **240 minutes** before comparing it with 150 minutes.

6. Find the quantities of sand and cement in a 40 kg mixture in the ratio 3 : 1.
Answer:

1. Sand $= \frac{3}{4} \times 40 = 30$kg.
2. Cement $= \frac{1}{4} \times 40 = 10$kg.

D. Three-Mark Descriptive Questions:

1. Explain proportional change using similar rectangles.
Answer:

1. Similar rectangles have the same ratio between corresponding dimensions.
2. If both dimensions are multiplied by the same factor, the ratio remains unchanged.
3. Therefore, the rectangle changes in size without becoming distorted.

2. Explain the lemonade example using proportional reasoning.
Answer:

1. Six glasses of lemonade require 10 spoons of sugar.
2. Eighteen glasses are three times 6 glasses, so the sugar must also be multiplied by 3.
3. Therefore, **30 spoons of sugar** are required to maintain the same sweetness.

3. Explain the cross-multiplication method.
Answer:

1. If $a: b: : c: d$, the two ratios are proportional.
2. Cross multiplication gives $ad = bc$.
3. Therefore, an unknown fourth term can be calculated using $d = \frac{bc}{a}$.

4. A car travels 90 km in 150 minutes. Find the distance travelled in 4 hours.
Answer:

1. Convert 4 hours into **240 minutes**.
2. Form $150: 90: : 240: x$, giving $x = \frac{90\times240}{150}$.
3. Therefore, the car travels **144 km**.

5. Explain how ₹4,000 profit is shared between partners whose investments are in the ratio 3 : 1.
Answer:

1. The total number of ratio-parts is $3 + 1 = 4$.
2. Each part of ₹4,000 equals **₹1,000**.
3. Therefore, the partners receive **₹3,000 and ₹1,000** respectively.

6. Why is speed and travel time for a fixed distance not a direct proportion?
Answer:

1. For a fixed distance, increasing speed reduces the time required.
2. Therefore, speed and time do not increase or decrease together by the same factor.
3. Hence, this situation cannot be represented using the chapter's **direct proportional model**.

E. Five-Mark Descriptive Questions:

1. Explain ratio and proportion with suitable examples.
Answer:

1. A ratio compares two quantities and is expressed as **a : b**.
2. Ratios can be simplified by dividing both terms by their HCF.
3. For example, $60:40$simplifies to **3 : 2**.
4. Two ratios are proportional if they have the same simplest form.
5. Thus, $60:40::30:20$, since both simplify to **3 : 2**.

2. Explain the Rule of Three using the school rice example.
Answer:

1. The Rule of Three finds a fourth quantity when three proportional quantities are known.
2. The chapter states that 120 students require **15 kg of rice**.
3. For 80 students, write the proportional relationship $120:15::80:x$.
4. Thus, $x = \frac{15\times80}{120} = 10\text{kg}$.
5. Therefore, **10 kg of rice** is required for 80 students.

3. Explain how a quantity is divided in a given ratio.
Answer:

1. Suppose $x$is divided in the ratio $m:n$.
2. The total number of ratio-parts is $m+n$.
3. The first share is $\frac{mx}{m+n}$.
4. The second share is $\frac{nx}{m+n}$.
5. For example, dividing 42 in the ratio 4 : 3 gives **24 and 18**.

4. A 40 kg sand-cement mixture is in the ratio 3 : 1. Find how much cement must be added to make the ratio 5 : 2.
Answer:

1. Initially, sand $= \frac{3}{4} \times 40 = 30\text{kg}$.
2. Cement $= \frac{1}{4} \times 40 = 10\text{kg}$.
3. For the required ratio 5 : 2, 30 kg of sand corresponds to **12 kg of cement**.
4. Additional cement required is $12 - 10 = 2\text{kg}$.

5. Therefore, **2 kg of cement must be added**.

5. A farmer's rectangular field measures 200 ft × 500 ft and requires manure at 10 tonnes per acre. Find approximately how much manure is required.
Answer:

1. The area of the field is $200 \times 500 = 100{,}000$ft².
2. Since 1 acre = 43,560 ft², the field is approximately $100{,}000/43{,}560 = 2.30$acres.
3. The manure requirement is **10 tonnes per acre**.
4. Therefore, manure required is approximately $2.30 \times 10 = 23$tonnes.
5. Hence, approximately **23 tonnes of manure** is required.

6. Compare two teas costing ₹200 for 200 g and ₹800 for 1 kg and determine which is cheaper.
Answer:

1. The two prices should first be compared for the **same quantity**.
2. Five packets of 200 g make 1 kg.
3. Therefore, the first tea costs $5 \times ₹200 = ₹1{,}000$per kg.
4. The second tea costs **₹800 per kg**.
5. Hence, the second tea is **cheaper by ₹200 per kg**.

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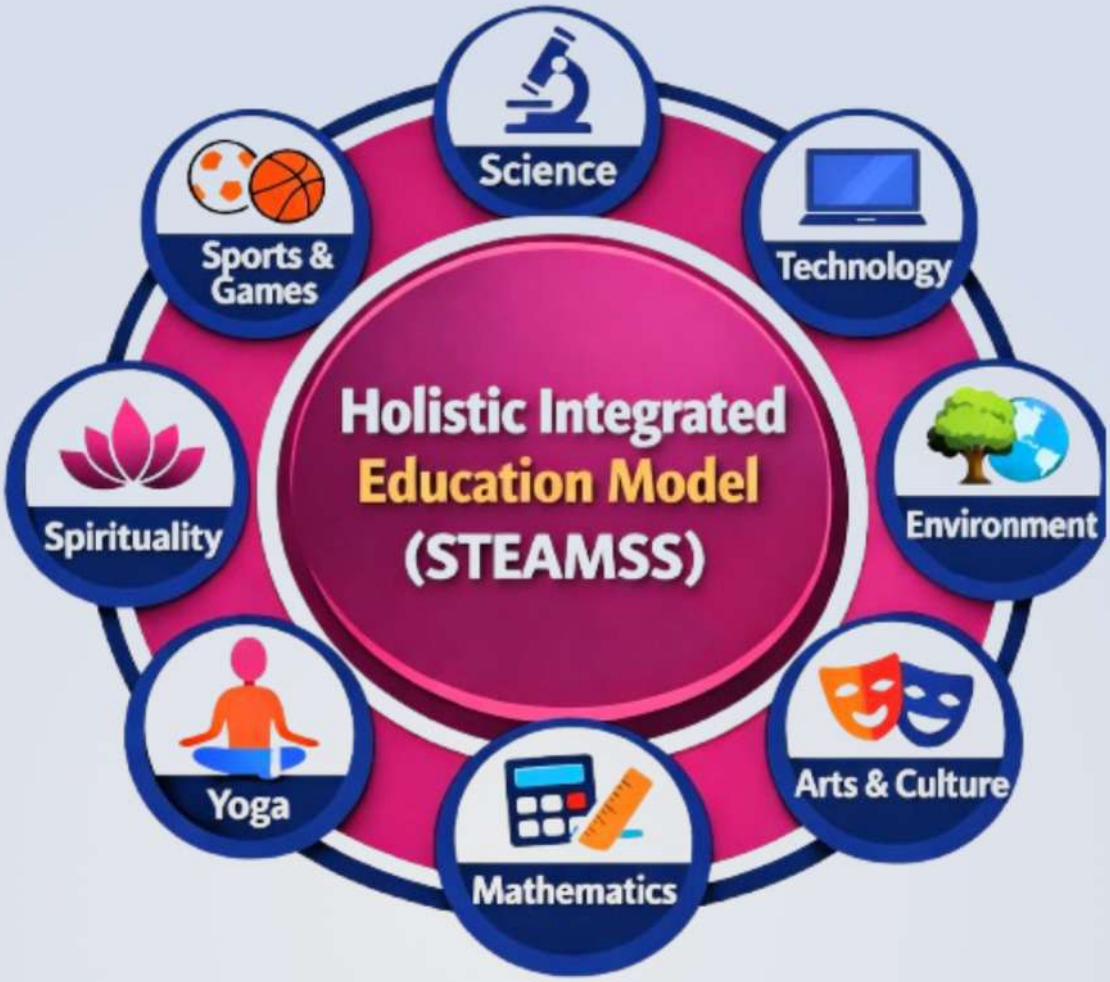

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